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Automatic Simulation of 1D and 2D Chaotic Oscillators

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Abstract. A new method is introduced for automatic simulation of three kinds of chaotic oscillators: Chua's circuit, generalized Chua's circuit and chaotic oscillator implemented with saturated functions. The former generates the double-scroll, and the others 1D n-scroll attractors. The third chaotic oscillator is modified to generate 2D n-scrolls attractors. The oscillators are modelled by applying state variables and piecewise-linear approximation. Basically, the method computes the eigenvalues of the oscillators to begin time simulation and to make control of step-size automatically.

1. Introduction

This work introduces guidelines for the development of a synthesis approach to design chaotic oscillators, by beginning with high-level simulations [1-4], and ending with the synthesis of individual blocks using practical electronic devices [5-6]. The proposed automatic simulation approach is performed by applying multistep algorithms [2], from which values of circuit elements can be determined, and the simulation is executed by automatic determination and control of step-size using [3]. Further, this process can be improved by exploring on the measurement of parameters between external signals and chaotic orbits [4]. Additionally, chaotic dynamical behaviors and basic dynamical properties can also be explored by performing high-level numerical simulations [4-5].

Basically, the methods introduced in [2-3] are used herein to compute the eigenvalues and to control step-size automatically of three chaotic oscillators: Chua's circuit, generalized Chua's circuit, and chaotic oscillator implemented using saturated functions. The smaller eigenvalue is computed to determine the step-size (h), and to automatically begin numerical simulation. For instance, to compute the eigenvalues the chaotic oscillators are described by state variables and piecewise-linear (PWL) approximation [7]. It is worthy to mention that the three chaotic oscillators can be implemented using commercially available electronic devices [8-10], and they are quite useful for signals transmission applications [8]. Besides, in electronics the majority of the research efforts have been focused to design Chua's circuit [6], and some variants of it to generate one dimensional (1D) n-scrolls attractors [9-10]. Furthermore, by applying state variables and PWL approximations one can also simulate 2D (as shown in this work), 3D [11], and 4D-scrolls attractors [12].

2. High-level modeling of Chua's circuit, generalized Chua and saturated functions

This section describes the modeling of chaotic oscillators by state-variables and PWL approach.

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2.1. Chua's circuit chaotic oscillator

Chua's circuit is shown in Fig. 1(a). It consists of five elements: a resistor R , an inductor L , two capacitors C , and Chua's diode (N_R). N_R is modeled by PWL approximation, as shown in Fig. 1(b), where i_{NR} is described by (1). Also, (1) can be represented by (2) to derive (3), where m is updated to (g_1, g_2, g_3) and I_x is updated according to (1). Eq. (3) has the general form $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$, which has been solved in [2], where time-simulation is minimized by automatic control of h [3].

$$i_{NR} = \begin{cases} -g_2 V_{C1} + (g_1 - g_2)BP1 & V_{C1} < -BP1 \\ -g_1 V_{C1} & -BP1 \leq V_{C1} \leq BP1 \\ -g_2 V_{C1} + (g_2 - g_1)BP1 & V_{C1} > BP1 \end{cases} \quad (1)$$

$$i_{NR} = -mV_{C1} + I_x \quad (2)$$

$$\begin{bmatrix} \dot{V}_{C1} \\ \dot{V}_{C2} \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} + \frac{m}{C_1} & \frac{1}{RC_1} & 0 \\ \frac{1}{RC_2} & -\frac{1}{RC_2} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ I_L \end{bmatrix} + \begin{bmatrix} -\frac{I_x}{C_1} \\ 0 \\ 0 \end{bmatrix} \quad (3)$$

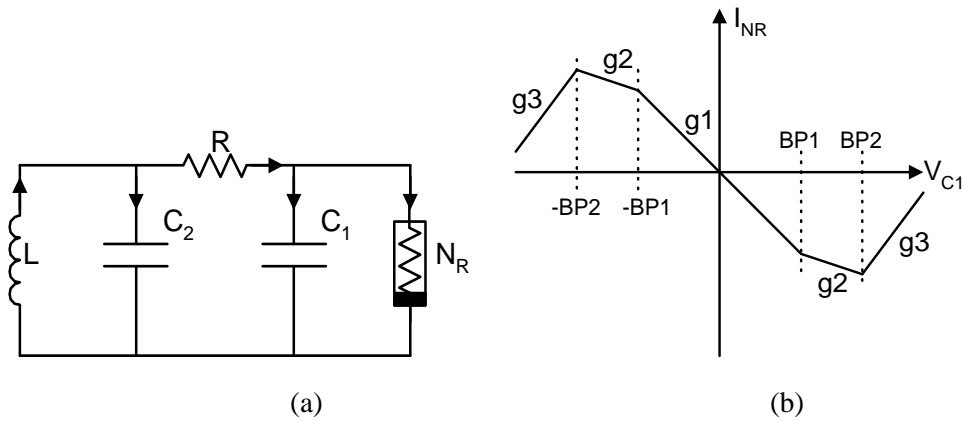


Figure 1. (a) Chua's circuit, and (b) I-V PWL characteristic of Chua's diode (N_R).

2.2. Generalized Chua's circuit using VCVSs

In [10] is shown the generalization of Chua's circuit whose behavioral equations are given by (4), where $h(x)$ describes a PWL characteristic given by (5). q is a natural number adjusted to generate even- or odd-scrolls attractors, m are the slopes of the PWL characteristic, and c are the breaking points. The values of m and c for $q=1,2,3$, have been determined in [10].

$$\begin{aligned} \dot{x} &= \alpha[y - h(x)] \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \quad (4)$$

$$h(x) = (m_{2q-1})x + \frac{1}{2} \sum_{i=1}^{2q-1} (m_{i-1} - m_i) \times (|x + c_i| - |x - c_i|) \quad (5)$$

In Fig. 2 is shown Chua's circuit implemented with a voltage-controlled voltage source (VCVS) [10]. The state-variables equations are derived in (6). By setting $C_1=C_2=R=R_G=L=1$, and by choosing $V_{C1} = x$, $V_{C2} = y$, $I_L = z$, (6) can be described by (7), where $f(x) = -h(x) + (1 + \delta)x$ and it depends on the number of slopes and breaking points of the nonlinear function $h(x)$ in (5).

By selecting $q = 2$, $m = [0.9/7, -3/7, 3.5/7, -2.4/7]$, and $c = [1; 2.15; 4]$, with $\alpha = 9$, $\beta = 14.28$ and $\delta = 1$ ($C1=1/9F$, $C2=1F$, $L=70mH$, $R=1\Omega$ and $R_G=1\Omega$), the PWL graph of (5) is shown in Fig. 3(a). If $q = 3$, $m = [0.9/7, -3/7, 3.5/7, -2.7/7, 4/7, -2.4/7]$, and $c = [1; 2.15; 3.6; 6.2; 9]$, with $\alpha = 9$, $\beta = 14.28$ and $\delta = 1$, the PWL graph is shown in Fig. 3(b).

$$\begin{aligned} \frac{dV_{C1}}{dt} &= -\frac{V_{C1}}{RR_GC_1}(R+R_G) + \frac{V_{C2}}{RC_1} + \frac{f(V_{C1})}{C_1} \\ \frac{dV_{C2}}{dt} &= \frac{V_{C1}}{RC_2} - \frac{V_{C2}}{RC_2} + \frac{I_L}{C_2} \\ \frac{dI_L}{dt} &= -\frac{V_{C2}}{L} \end{aligned} \quad (6)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -\alpha(1+\delta) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} \alpha f(x) \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

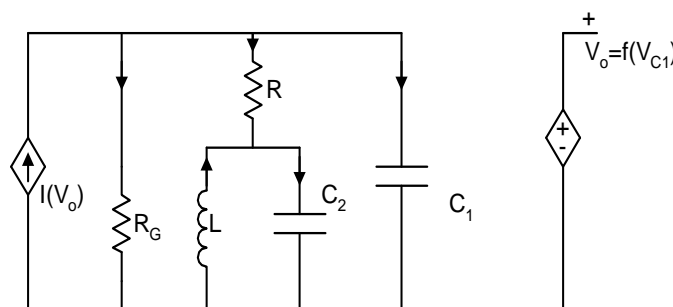


Figure 2. Generalized Chua's circuit using a VCVS.

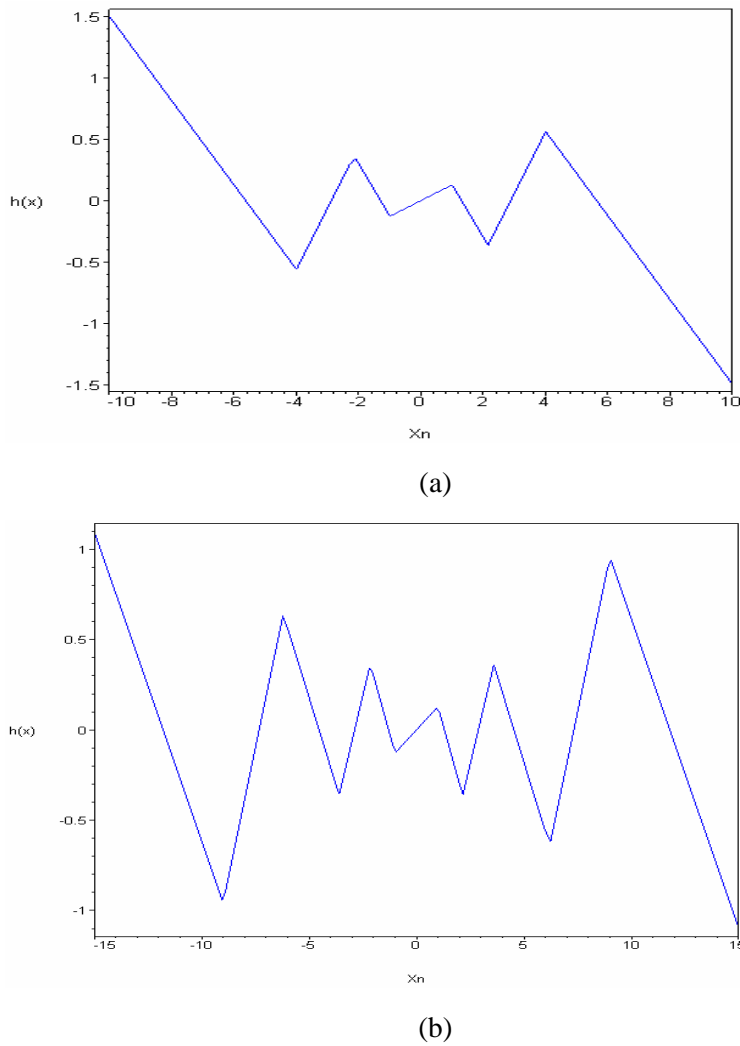


Figure 3. PWL approximation of $h(x)$: (a) for $q=2$, and (b) for $q=3$.

2.3. Saturated Functions

A chaotic oscillator implemented with saturated circuits is well suited to implement PWL approximations. For instance, the behavior of the opamp and OTA can be modeled by using saturated functions [7]. In Fig. 4(a) is shown a saturated function with three-segments, where the segments b are called saturated plateaus and m saturated slope. In (8) is described a PWL approximation called series of a saturated function, where $k > 0$ is the slope of the saturated function, $h > 2$ is the saturated delay, p and q are positive integers. To generate n-scrolls attractors, a controller is added to a system of state-variables equations, as shown in (9), where $f(x; k, h, p, q)$ is defined by (10), and a, b, c, d are positive constants.

$$f(x; k, h, p, q) = \sum_{i=-p}^q f_i(x; k, h) \quad (8)$$

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -ax - by - cz + df(x; k, h, p, q)\end{aligned}\quad (9)$$

$$f(x; k, h, p, q) = \begin{cases} (2q+1)k & x > qh+1 \\ k(x-ih)+2ik & |x-ih| \leq 1, -p \leq i \leq q \\ (2i+1)k & ih+1 < x < (i+1)h-1, -p \leq i \leq q-1 \\ -(2p+1)k & x < -ph-1 \end{cases} \quad (10)$$

In (9) there are $2(p+q)+3$ equilibrium points on the x axis, called *saddle points* of index 1 and index 2. Since the scrolls are generated only around the *saddle points of index 2* [9-10], (9) has the potential to generate $(p+q+2)$ -scrolls. However, the $(p+q+1)$ *saddle points of index 1* are responsible to connect the $(p+q+2)$ -scrolls to generate the attractor. Additionally, the *saddle points of index 2* correspond to a unique *saturated plateau* and to a unique attractor, while the *saddle points of index 1* correspond to a unique *saturated slope* and connection among the neighbors-scrolls.

3. Numerical simulation of chaotic oscillators

This section shows simulation results to generate 1D and 2D n-scrolls attractors.

3.1. Chua's circuit

By choosing $C1=450\text{pf}$, $C2=1.5\text{nf}$, $L=1\text{mH}$, $R=1655$, $g1=1/1358$, $g2=1/2464$, $g3=1/1600$, $BP1=0.114\text{V}$, $BP2=0.4\text{V}$, in Fig. 1, with $V_{C1}(0)=0.01\text{V}$, $V_{C2}(0)=0\text{V}$, $I_L(0)=0\text{A}$, then the simulation of Chua's circuit is shown in Fig. 4(b). A sequence of chaotic behaviors by varying R is shown in [6].

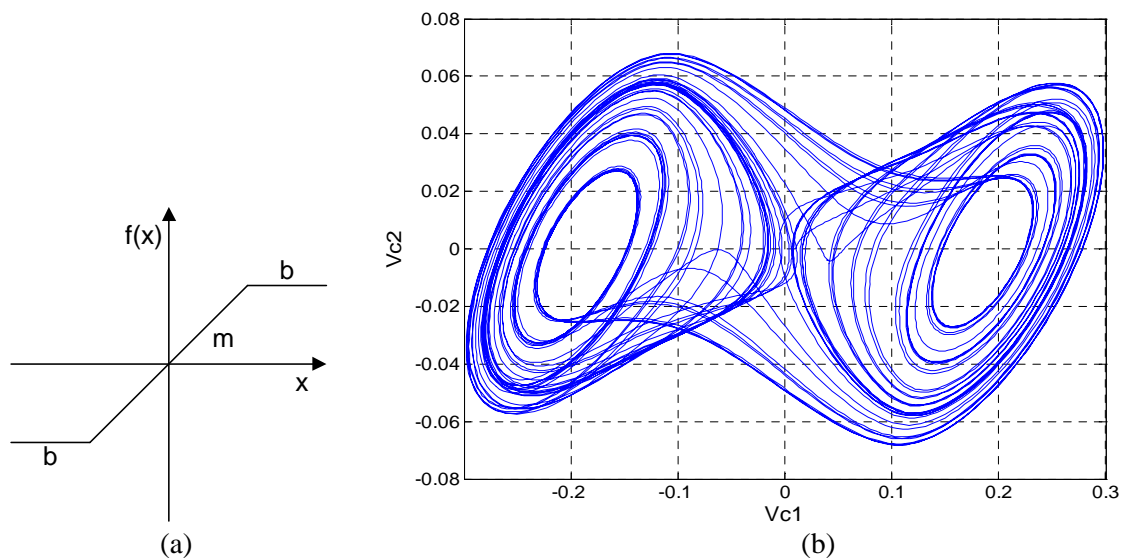


Figure 4. (a) Description of a saturated function, and (b) double scroll-attractor generated by Chua's circuit.

3.2. Generalized Chua's circuit

To generate 3- and 5-scrolls attractors from Fig. 2, the simulation of (7) is performed by using the PWL approximations shown in Fig. 3(a) and 3(b), respectively. The results are shown in Fig. 5.

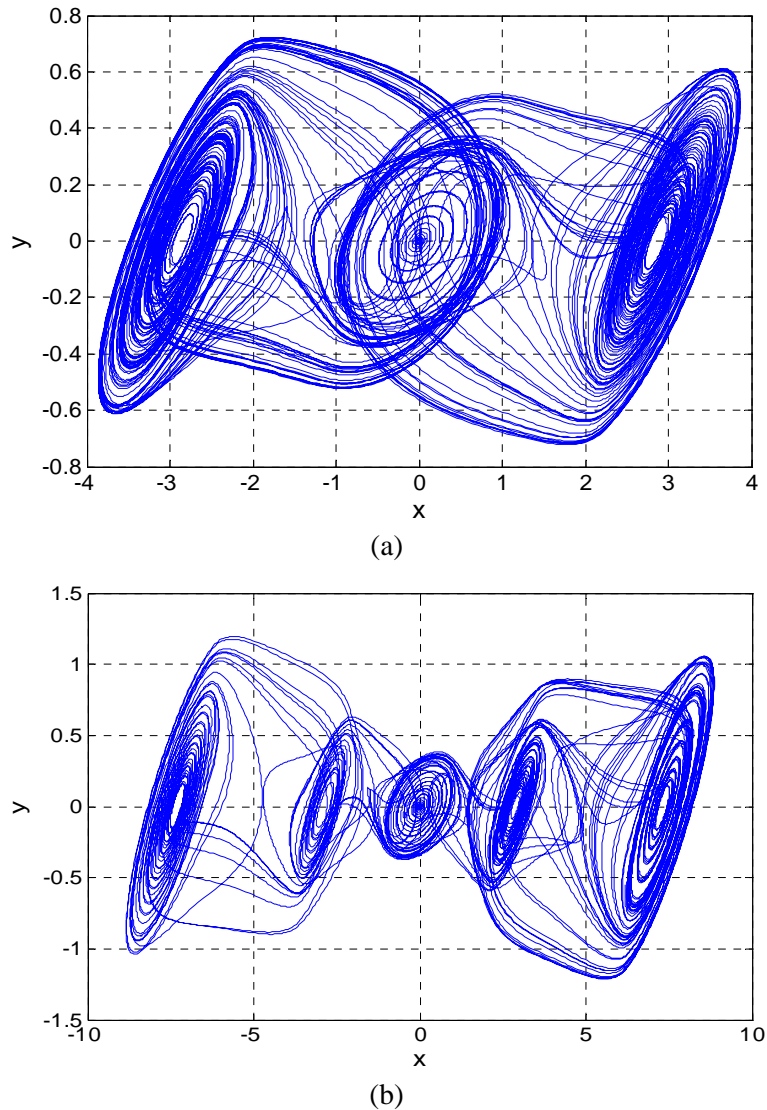


Figure 5. Generation of (a) 3-scrolls, and (b) 5-scrolls attractors with the generalized Chua's circuit.

3.3. Saturated Functions

By setting $a = b = c = d = 0.7$, $k = 9$, $h = 18$ to evaluate (9), 4-scroll and 6-scrolls attractors are generated, as shown in Fig. 6(a) and Fig. 6(b), with $p = q = 1$ and $p = q = 2$, respectively.

3.4. Simulation of 2D-scroll attractors

To generate 2D-scroll attractors, it is necessary to add two controllers to (9), which results in (11), where $f(x; k_1, h_1, p_1, q_1)$ and $f(y; k_2, h_2, p_2, q_2)$ are defined by (8), and a, b, c, d_1, d_2 are positive constants. Eq. (11) has $(2p_1 + 2q_1 + 3) \times (2p_2 + 2q_2 + 3)$ equilibrium points located in the $x - y$ plane, to generate a 2D-mesh of $(p_1 + q_1 + 2) \times (p_2 + q_2 + 2)$ -scrolls attractors. Therefore, if $k_1 = k_2 = 50$, $a = b = c = d_1 = d_2 = 0.7$, $h_1 = h_2 = 100$ and $p_1 = q_1 = p_2 = q_2 = 1$, the simulation of (11) generates the 2D-mesh of 4-scroll attractors, as shown in Fig. 7.

$$\begin{aligned}\dot{x} &= y - \frac{d_2}{b} f(y; k_2, h_2, p_2, q_2) \\ \dot{y} &= z \\ \dot{z} &= -ax - by - cz + d_1 f(x; k_1, h_1, p_1, q_1) + d_2 f(y; k_2, h_2, p_2, q_2)\end{aligned}\tag{11}$$

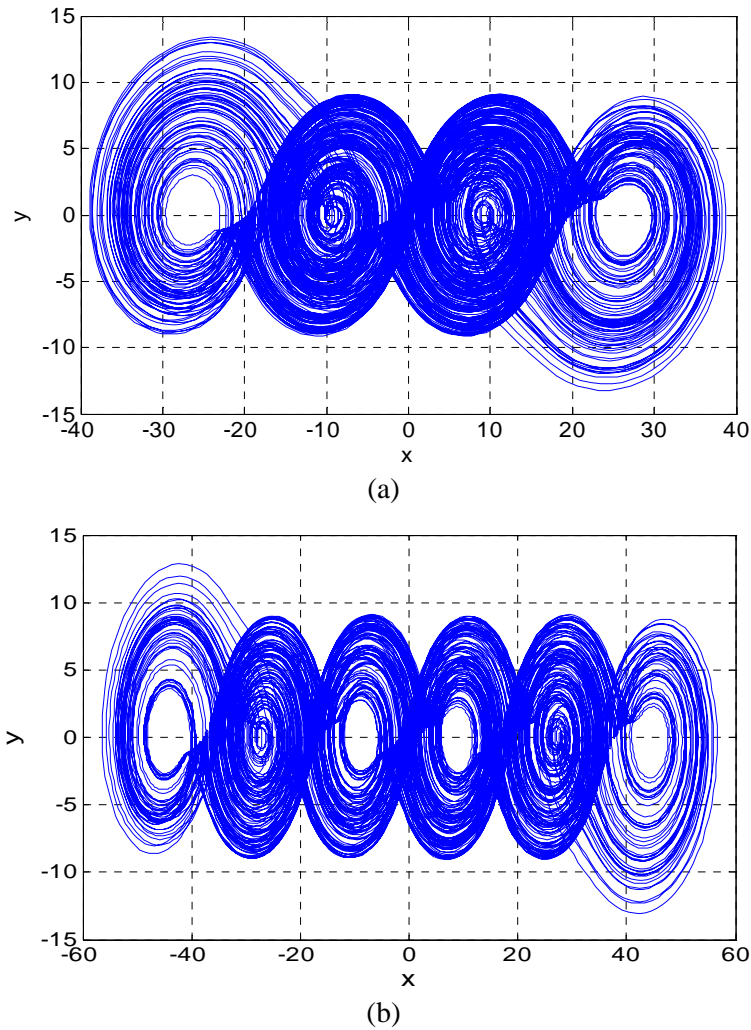


Figure 6. Generation of (a) 4-scrolls, and (b) 6-scrolls attractors.

4. Conclusion

It has been shown the simulation of three kinds of chaotic oscillators by state variables and PWL approximations. It was shown that the generalized Chua's circuit and the oscillator implemented with saturated functions generate 1D and n-scrolls attractors. Furthermore, the addition of two controllers to the oscillator based on saturated functions generates 2D and n-scrolls attractors. Elsewhere, the simulation was performed by automatic control and determination of h using multistep algorithms.

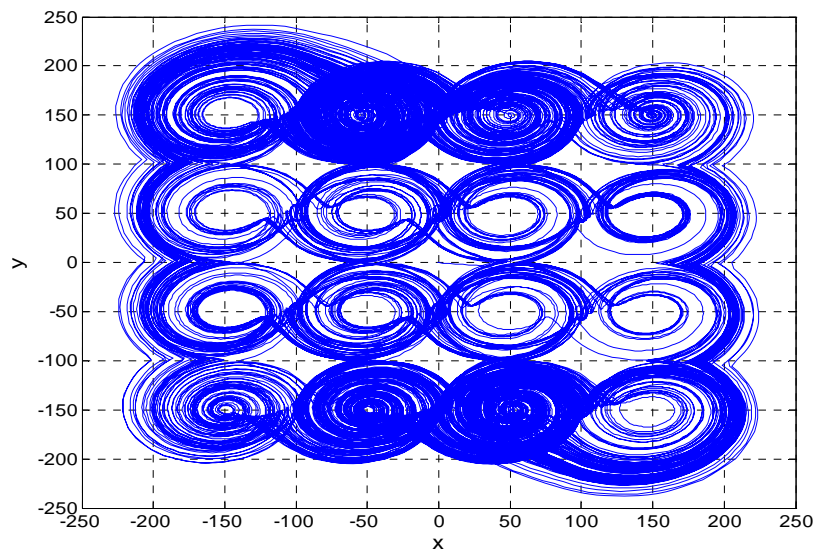


Figure 7. Generation of 2D-scroll attractors with a mesh of 4-scrolls.

Acknowledgment

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