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Spatial filtering dark hollow beams

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Abstract

We describe the synthesis of diffraction free beams and dark hollow beams with features easily tunable using a holographic transmittance. The hologram is generated by interfering two zero order Bessel beams with non-common axis. Spatial filtering techniques are implemented by controlling the illumination during the reconstruction process. The experimental results are for illumination similar to the one used in the recording process, obtaining a set of diffracting free beams. One of these beams propagates quasi-parallel to the surface hologram. For illumination with a plane wave we obtain a dark hollow beam propagating in the same direction as the reconstructions beam. Experimental results are shown in both cases.

1. Introduction

A contemporary topic research consists in the trapping and manipulation of particle conductors and dielectrics by means of optical fields [1,2]. This problem implies the generation of diffraction free beams [3,4] and dark hollow beams with geometries and phase easily tunable [5]; some applications concerning these beams can be founded in [6]. In the present paper, we describe a holographic technique to generate these kind of optical fields by incorporating spatial filtering techniques. The resulting optical fields can be diffracting free beams and/or dark hollow beams. The spatial filtering is implemented by controlling the kind of illumination which allows us to add or avoid some spatial frequencies. The transmittance function for the optical field is obtained by interfering two zero order Bessel beams with non-common axis and recording the irradiance distribution on a holographic plate. In this configuration it is indistinct which the object beam is and which is the reference beam. This configuration is analogous to one point hologram as was described by Hariharan [7]. By illuminating the hologram with the reference beam, the resulting optical field consists in a set of diffraction free beams propagating in different directions. We show that, for certain values of the recording angle a beam emerges that propagates in the neighborhood of the hologram surface. The holographic reconstruction with other kinds of illumination generates a variety of optical fields. The simplest case is for illumination with a plane wave illuminating in the same direction as the recording Bessel beams. In this configuration, we are adding the zero order spatial frequency. In consequence contrast reverse in the optical field respect to a zero order Bessel beam can be expected. This simple technique allows us to generate dark hollow beams. Changing the kind of illumination we can generate arrays of diffraction free fields and/or dark hollow beams whose profile and interference features are easily tunable. Also, the kind of illumination allows us to incorporate partially coherent features.

The experimental results are for a holographic transmittance generated with the interference between two zero order Bessel beams and reconstructing with the same reference beam. The experimental results consist in a set of beams where one of them emerges quasi-parallel to the

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surface. The other result is by illuminating with a plane wave, where the resulting optical field consists in a dark hollow beam.

2. Holographic recording with two zero order bessel beams

It is a fact known that the spatial frequencies representation for diffraction free fields must be on some regions of a circle of radius "d" as was appointed by Durnin [3]. With this condition, the synthesis of diffraction free beams using a Fourier transform system is possible, two of the simplest diffraction free beams are a plane wave and a zero order Bessel beam. The mathematical representation for the zero order Bessel beam propagating along the z-coordinate is given by $\phi(x,y,z) = AJ_0(2\pi d\sqrt{x^2+y^2})\exp(i2\pi\gamma z)$, where $\gamma = \sqrt{\frac{1}{\lambda^2} - d^2}$, $u^2 + v^2 = d^2$, being (u,v,γ) the spatial frequencies.

The amplitude function for the superposition of two zero order Bessel beams with non common axis propagating on x–z plane is given by

$$\phi(x,y,z) = AJ_0\left(2\pi d\sqrt{(ax+bz)^2+y^2}\right)\exp i\xi(-ax+bz) + AJ_0\left(2\pi d\sqrt{(ax-bz)^2+y^2}\right)\exp i(\alpha x+\beta z), \quad (1)$$

where $a = \cos\theta$ and $b = \sin\theta$, $\alpha = \gamma\cos\theta$; $\beta = \gamma\sin\theta$. The irradiance associated takes the form

$$\begin{aligned} I(x,y,z) &= J_0^2\left(\mu\sqrt{(ax+bz)^2+y^2}\right) + J_0^2\left(\mu\sqrt{(ax-bz)^2+y^2}\right) \\ &+ J_0\left(\mu\sqrt{(ax+bz)^2+y^2}\right)J_0\left(\mu\sqrt{(ax-bz)^2+y^2}\right)e^{-i2\xi ax} \\ &+ J_0\left(\mu\sqrt{(ax+bz)^2+y^2}\right)J_0\left(\mu\sqrt{(ax-bz)^2+y^2}\right)e^{i2\xi ax} \end{aligned} \quad (2)$$

where $\mu = 2\pi d$. The irradiance distribution was recorded on a photorefractive crystal of $Fe{:}LiNbO_3$ using a He–Ne laser of 25 mw with a wavelength of 543 nm. We assume that the transmittance function is proportional to the irradiance distribution $t(x,y) = hI(x,y)$, where h is a constant. For further analysis this constant is not relevant and it will be omitted. The holographic plate is localized on $z = 0$ plane, consequently the transmittance function can be approximated by the following representation

$$\begin{aligned} t(x,y) &= J_0^2\left(\mu\sqrt{(ax)^2+y^2}\right) + J_0^2\left(\mu\sqrt{(ax)^2+y^2}\right) \\ &+ J_0\left(\mu\sqrt{(ax)^2+y^2}\right)J_0\left(\mu\sqrt{(ax)^2+y^2}\right)\exp -i2\xi ax \\ &+ J_0\left(\mu\sqrt{(ax)^2+y^2}\right)J_0\left(\mu\sqrt{(ax)^2+y^2}\right)\exp i2\xi ax. \end{aligned} \quad (3)$$

The experimental set up for the generation of the hologram and the parameters involved are shown in (Fig. 1a).

3. Holographic reconstruction

During the reconstruction process, we illuminate with a zero order Bessel beam, the resulting optical field can be obtained from the diffraction integral representation, where a set of optical beams may be identified. The amplitude optical field representation is

$$\begin{aligned} \phi = &\int\int 2J_0^2\left(\mu\sqrt{(ax)^2+y^2}\right)J_0\left(\mu\sqrt{(ax)^2+y^2}\right)e^{-i\xi ax} \times \frac{e^{ikr}}{r}dxdy \\ &+ \int\int J_0^2\left(\mu\sqrt{(ax)^2+y^2}\right)J_0\left(\mu\sqrt{(ax)^2+y^2}\right)e^{i\xi ax} \times \frac{e^{ikr}}{r}dxdy \\ &+ \int\int J_0^2\left(\mu\sqrt{(ax)^2+y^2}\right)J_0\left(\mu\sqrt{(ax)^2+y^2}\right)e^{-i3\xi ax} \times \frac{e^{ikr}}{r}dxdy \end{aligned} \quad (4)$$

The first integral corresponds to the reference beam, whose profile is an optical field similar to the zero order Bessel beam. The second integral represents the "object beam" and the third term corresponds to a beam forming an angle $\omega = \arcsin(3\sin\theta)$. These beams are sketched in (Fig. 1b). One interesting feature is that we can generate a beam, quasi-parallel to the surface transmittance. This is obtained if the recording angle is in the range [15°–20°] and $\omega \approx 90°$. In (Fig. 2a) we show the experimental result for the quasi-parallel beam, it has the structure of the speckle pattern and it was obtained for a reconstruction angle ≈20°. The origin of a speckle pattern is because we have a volume hologram, in this way from each small region of the hologram emerges a set of beams with different values in the phase function. These beams are sketched in (Fig. 2b). The non-diffracting features associated to such speckle-beams may be founded in [8]. A very interesting fact consists in the speckle pattern can be controlled by a tilt in the angle of the reconstruction beam. In (Fig. 2c) we show the experimental result for an angle ≈ 15°. In this case, the beam propagates no more parallel to the surface and its profile resembles a zero order Bessel beam.

4. Diffraction free features of dark hollow beams

An interesting feature is that spatial filtering techniques can be implemented if we change the kind of illumination. From Eq. (4) it is clear that the optical field does not have the zero order diffraction. If we illuminate the holographic transmittance with a plane wave, we are adding the zero order diffraction to the optical field, consequently the new

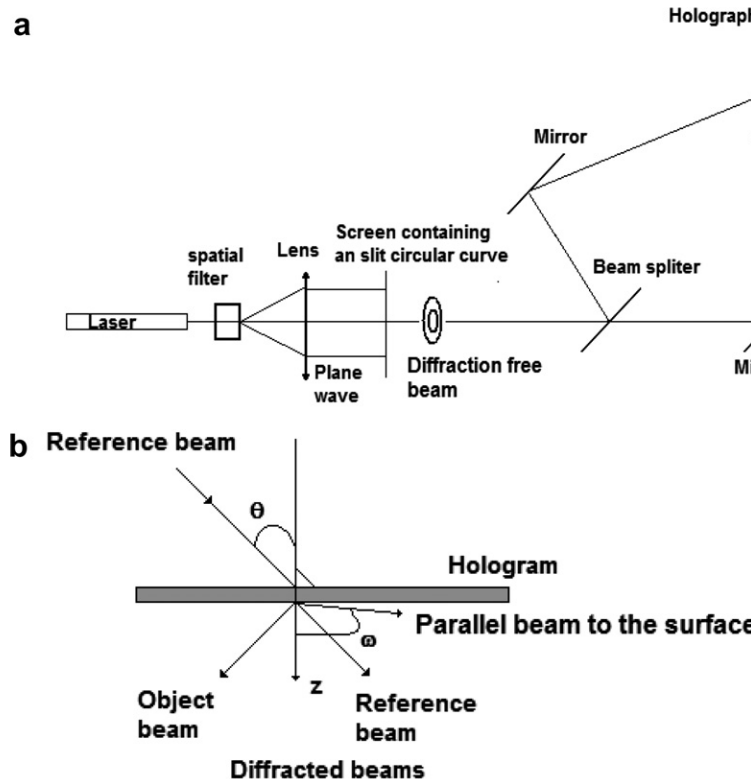

Fig. 1. In (a) Experimental set up, the radius of the circular slit is 2 mm. approximately and the recording angle θ is in the range (15°–20°), the holographic plate consists in a photorefractive crystal of Fe:$LiNbO_3$. In (b), beams expected during the reconstruction process.

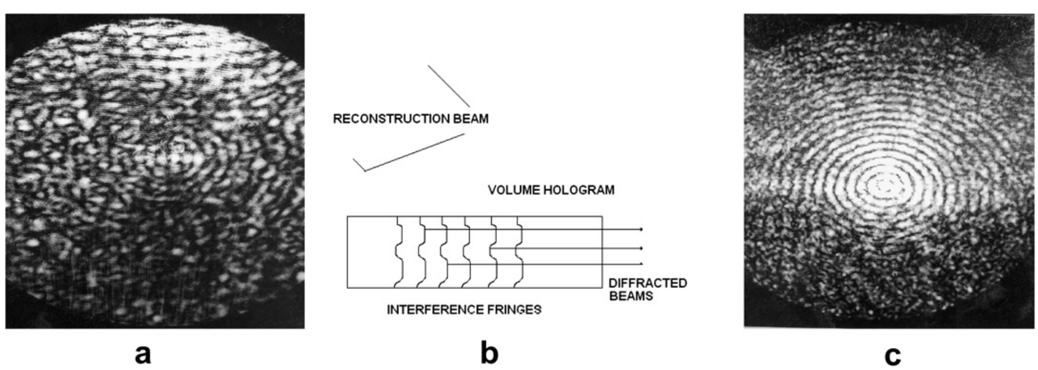

Fig. 2. Experimental results for the beams propagating quasi-parallel to the hologram surface. In (a) the angle of the illumination beam is ≈20° and a diffraction free speckle pattern is obtained, in c) the illumination beam is ≈15° and a zero order Bessel beam is obtained.

optical field must vary in its geometry, having in particular reverse contrast. In Fig. 3 we show the experimental results obtained when a plane wave illuminates the hologram, in the same direction as the reference beam. For this case a dark hollow beam is obtained, but it is no more a diffraction free beam, however it keeps some features. The shape of the beam does not change under propagation; however the size of the beam increases. When the distance is approximately 5 cm. from the hologram the diameter of the dark region is 1.2 mm. When the distance of propagation is 2 m, the diameter of the dark region is approximately 1 cm. The spatial evolution was measured with a commercial CCD camera and transforming the number of pixels.

The amplitude representation for a dark hollow beam in a point $X(x,y,z)$ can be obtained approximately from the angular spectrum model, given by

$$\begin{aligned}\phi(X) &= \int\int_{-\infty}^{\infty}(\delta(u,v) - \delta(u^2+v^2-d^2))\exp i2\pi(xu \\ &\quad + yv + zp)\mathrm{d}u\mathrm{d}v \\ &= \mathrm{e}^{ikz} - \left(J_0(2\pi d\sqrt{x^2+y^2})\right)\mathrm{e}^{i\gamma z}. \qquad (5)\end{aligned}$$

A caution note is necessary because we are dealing with an volume hologram. This means that the frequency representation given by $\delta(u,v) - \delta(u^2 + v^2 - d^2)$is an approximation to the Fourier transform of the transmittance function.

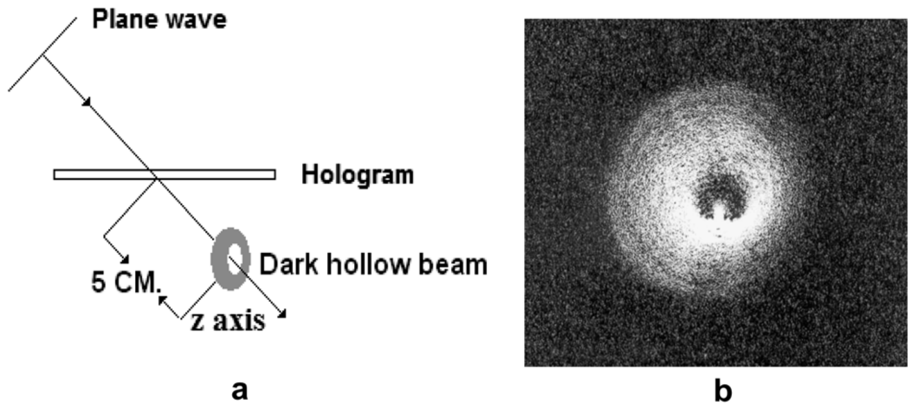

Fig. 3. In (a), Schematic set up to implement spatial filtering by illuminating the hologram with a plane wave. In (b), the experimental result corresponds with a dark hollow beam. The radius of the dark region is about 1.2 mm. It was measured from 5 cm. approximately from the hologram.

The exact representation must contain a term depending of z coordinate. This justifies the increasing size of the beam during its propagation. However, the approximation is good enough to explain the generation of the dark region. The amplitude term in the negative sign between the δ-functions corresponds with a phase change between diffraction beams emerging from the transmittance function and the illumination beam. This is a consequence of the Fresnel equations. The irradiance associated is given by

$$I(X) = 1 + J_0^2(2\pi d\sqrt{x^2+y^2}) - 2J_0(2\pi d\sqrt{x^2+y^2}) \times \cos(k - 2\pi\gamma)z. \quad (6)$$

The period of the cosine term is very large and it is given $P = \frac{2\pi}{k-2\pi\gamma} \approx \frac{2}{\lambda d^2}$. In $z \geqslant 0$ region, the irradiance can be approximated as $I(X) = 1 - J_0^2(2\pi d\sqrt{x^2+y^2})$, that explains the contrast reverse. An alternative treatment of dark hollow beams can be found in [9].

5. Conclusions

We have described a simple method using holographic techniques in order to generate diffraction free beams and dark hollow beams. A fundamental part of the study consists in the generation of the boundary condition for the optical field characterized by a transmittance function obtained with holographic techniques. The transmittance function was illuminated with different kinds of optical fields that allowed us to incorporate spatial filtering techniques. The analysis shown allowed us to generate a set of diffraction free beams/dark hollow beams with profiles/phases easily tunable. An important fact is that we can generate diffracting free beams quasi-parallel to the hologram surface. This beam offers the possibility to be implemented as surface optical twisters and to transfer angular momentum perpendicular to the surface for the development of micro-motors. The experimental results shown in Fig. 2 are very interesting from the theoretical point of view because we can investigate the features of the electromagnetic field implicit in the self re-generation phenomenon. The control of the illumination implies that partially coherent features may be implemented easily. The experimental results are in very good agreement with the theory. This work was supported by Consejo Nacional de Ciencia y Tecnologia (CONACyT) trough research Project 47325.

References

[1] J. Arlt, V. Garces-Chavez, W. Sibbett, K. Dholakia, Optical micromanipulation using a Bessel light beam, Opt. Comm 197 (2001) 239.

[2] D.P. Rhodes, G.P.T. Lancaster, J. Livesey, D. Mcgloin, J. Arlt, K. Dholakia, Guiding a cold atomic beam along a co-propagating and oblique hollow light guide, Opt. Commun. 214 (2002) 247.

[3] J. Durnin, Exact solutions for diffraction free beams I: the scalar theory, J. Opt. Soc. Am. A 4 (1987) 651.

[4] Andrey S. Ostrovsky, Gabriel Martinez Niconoff, Julio C. Ramirez San-Juan, Generation of light string and light capillary beams, Opt. Commun. 207 (2002) 131.

[5] Gabriel Martinez Niconoff, Julio C. Ramirez San-Juan, Patricia Martinez Vara, Adrian Carbajal Dominguez, Andrey S. Ostrovsky, Generation of partially coherent diffraction free fields with tunable geometry, JOSA A vol. 21 (4) (2004).

[6] Jianping Yin, Weijian Gao, YifunZhu, Generation of dark hollow beams and their applications. Progress in Optics, vol. 45, Elsevier Science, North Holland, 2003 (chapter 3), p. 119.

[7] P. Hariharan, Optical Holography: Principles, Techniques, and Applications, Cambridge University Press, New York, USA, 1984, p. 23.

[8] Gabriel Martinez-Niconoff, J.C. Ramirez San Juan, J. Muñoz-Lopez, Patricia Martinez Vara, Incoherent convergence of diffraction free fields, Opt. Commun. (2007).

[9] Andrey S. Ostrovsky, Gabriel Martinez Niconoff, Julio C. Ramirez San Juan, Coherent mode representation of propagation-invariant fields, Opt. Comm 195 (2001) 27.