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Self-imaging and caustics in two-dimensional surface plasmon optics

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Abstract

We study theoretically the surface plasmon electromagnetic field in the plane of the interface along which it propagates. Arbitrary surface plasmon fields are expressed by a linear superposition of elementary surface plasmon modes, thus obtaining an expression for the in-plane components similar to the angular spectrum model, which establishes the formal foundations of a two-dimensional surface plasmon optics. From this representation, we obtain the general description for surface plasmon modes be having as in-plane diffraction free beams. These new modes with their corresponding phase parameters are used to study surface plasmon self-imaging phenomenon and the synthesis of surface plasmon singularity regions (caustics) of surface plasmon fields, proposing experimental means to observe both.

1. Introduction

Surface-plasmon-polaritons (SPs) are electromagnetic waves due to oscillations of the electron plasma in the metal that propagate along a metal-dielectric interface to which they are bounded [1]. The intrinsic properties of surface-plasmon-polaritons (SPs) on nanostructured metal surfaces make them especially suitable for applications in bi-dimensional optics [2,3]. A few years ago, the concept of surface plasmon optics was suggested on the basis of manipulating SPs in the plane of the interface as classical light in free space [4,5]. The rich phenomenology exhibited by SPs has been explored in a variety of configurations since [3–6]. However, most of the effort has focused on the guiding and control of SPs in a quasi one-dimensional manner, eluding the 2D character of the in-plane SPs propagation; except for a very recent work [7], revealing the ability of oil drops to act as lens/mirrors for SPs. Recall that due to the mixed character of SPs, 2D manipulation of the optical properties conveys in turn a 2D control of the surface charge oscillations. In general, the spatial fluctuations of the electric field associated with charges confined within small regions, can have very high values locally distributed. In principle, the control of the electromagnetic field on surfaces allows us to generate and to control some important physical behavior, for example, induced transparency, control of fluorescence lifetime, generation of quantum dots, the possibility of generating surface tunable photonic crystals, synthesis of meta-materials, etc. [8–15]. In this letter, we will investigate formally the in-plane, 2D propagation of SPs, which paves the way to a full 2D surface plasmon optics, in fact showing the conditions to obtain SP diffraction free beams, self-imaging and the synthesis of surface singularities. This point of view implies the description of elementary SP solutions whose phase function is determinate by the dispersion relation; these solutions are analogous to plane waves for free space. Therefore, arbitrary SP fields can be described by means of the coherent superposition of elementary SP modes and the mathematical representation thus obtained is the angular spectrum model for SP fields. This representation allows us to obtain a more general one for surface plasmon modes which are analogous to diffraction free beams [16,17], having the property that the phase parameter along the coordinate of propagations is smaller than the one determinate by the dispersion relation. With the help of the general surface mode solutions, we describe surface

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self-imaging fields: this analysis is performed in the frequency space and the condition obtained is matched with the Montgomery's condition for homogeneous media [18,19]. Finally, the surface mode solution allows the research of new features such as the description of surface singularities, which occurs in caustic regions [20–22]. The phase function on these regions presents an adiabatic behavior, and it is deeply related with the charge re-distribution. The construction of caustic region and some of its behavior is associating a catastrophe function to the phase function.

2. Description of general surface plasmon modes

We start the study by describing the mode solutions for homogeneous media, considering the z coordinate as the propagation coordinate, the scalar amplitude representation is given by $\phi(x,y,z)=f(x,y)\exp i\beta z$, where $f(x,y)$ describes the profile of the amplitude and it satisfies the eigenvalue equation $\nabla_\perp^2 f(x,y)+K^2 f(x,y)=\beta^2 f(x,y)$. Prototype mode solutions are the plane waves and Bessel waves [12]. In present paper, the mode solutions for homogeneous media are used as a fundamental structure for the research of general vector mode solutions propagating in the neighborhood of a smooth surface. By analogy with homogeneous mode, the simplest mode has a vector structure of the form

$$\mathbf{E}_1(x,z)=\left(\vec{i}a+\vec{k}b\right)\exp(-\alpha_1 x)e^{i\beta z},$$
$$\mathbf{E}_2(x,z)=\left(\vec{i}\frac{\varepsilon_1}{\varepsilon_2}a+\vec{k}b\right)\exp(-\alpha_2 x)e^{i\beta z}; \quad (1)$$

where $\mathbf{E}$ represents the electric field and the indices are associated with the electric field in each media. We are considering surface free of charge and currents, which implies that $\nabla\cdot E_{1,2}=0$, thus the attenuation rates for electric fields in each media are related by $\varepsilon_1\alpha_2=\varepsilon_2\alpha_1$. The possible values for the phase parameter β can be obtained by direct substitution of Eq. (1) into the Helmholtz equations, $\nabla^2 E_{1,2}+K_{1,2}^2 E_{1,2}=0$, obtaining $K_i^2=\beta^2-\alpha_1^2$, $i=1,\ 2$. Solving explicitly for the phase parameter β, we have: $\beta=\frac{w}{c}\left(\frac{\varepsilon_{1r}\varepsilon_{2r}}{\varepsilon_{1r}+\varepsilon_{2r}}\right)^{1/2}$, where $\varepsilon_{ir}=\varepsilon_i/\varepsilon_0$ refers to the relative permittivity. It should be noted that the expression for β represents the possible values for the phase function associated with surface waves. This function is known as the dispersion relation [1]. Surface mode solutions given by Eq. (1) are analogous to plane waves for homogeneous media. For this reason they can be implemented to describe arbitrary SP fields of the form

$$\mathbf{E}_1(x,y,z)=\int_{-\infty}^{\infty}\left(\vec{i}A(u)+\vec{j}B(u)+\vec{k}C(u)\right)\exp(-\alpha_1 x)\times\exp\left\{i2\pi\left(\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)^{1/2}(yu+zp)\right\}du$$
$$\mathbf{E}_2(x,y,z)=\int_{-\infty}^{\infty}\left(\vec{i}\frac{\varepsilon_1}{\varepsilon_2}A(u)+\vec{j}B(u)+\vec{k}C(u)\right)\exp(-\alpha_2 x)\times\exp\left\{i2\pi\left(\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)^{1/2}(yu+zp)\right\}du, \quad (2)$$

where $u=\frac{\cos\theta}{\lambda_0}$ corresponds to spatial frequencies, λ_0 represents the wavelength in vacuum and $A(u),B(u),C(u)$ are the amplitude functions which are related to the transmittance function by means of a Fourier transform. Essentially, Eq. (2) have the same mathematical structure as that of the angular spectrum model [23]. The electric field given by Eq. (2) generates surface charge redistribution; however, the spatial average of the charge must be zero to satisfy the condition of media free of charge. This is the manifestation of the interference fringes between the elementary SP modes emerging from the transmittance function. Next point of the analysis consists of finding a general expression for SP modes, from which Eq. (1) is a particular case. By analogy with diffraction free beams, the general structure of the mode must be of the form $\mathbf{E}_1(x,y,z)=\left(\vec{i}a(y)+\vec{j}b(y)+\vec{k}c(y)\right)\exp(-\alpha x)e^{i\Omega z}$, where the problem to be solved consists in finding the expression for the $[a(y),b(y),c(y)]$ functions and phase parameter Ω. By substitution in the Helmholtz equation these functions satisfy $\frac{\partial^2(a_n(y),b_n(y),c_n(y))}{\partial y^2}+\left(K^2+\alpha^2-\Omega^2\right)(a_n(y),b_n(y),c_n(y))=0$, and the general expression for mode solution is

$$\mathbf{E}_1(x,y,z)=\left(\vec{i}\xi_1+\vec{j}\xi_2+\vec{k}\xi_3\right)\cos(py+\eta)\exp(-\alpha x)e^{i\Omega z}, \quad (3)$$

where $p=\sqrt{\left(K^2+\alpha^2-\Omega^2\right)}$ and $(\xi_{1,2,3})$, η are arbitrary constants. Eq. (3) can be obtained by means of the interference between two elementary plasmon modes of the form (1). The parameter Ω is related to the dispersion relation by means of $\Omega=\beta\sin\theta$. Until this point, we have shown that it is possible to generate new modes by interfering elementary SP modes. This construction from the angular spectrum model is analogous to diffraction free beams for free space, a prototype of that being the Bessel beams; for such modes, the spatial frequency representation should be on a circle. For general SP modes the corresponding representation consists of two points as can be deduced from Eq. (3). These two points are generated by the intersection of the circle for homogeneous media with the (u,p) plane. More details about the frequency representation for diffracting free beams can be found in [12,13,15].

3. Surface plasmon self-imaging

A natural extension of the previous analysis consists in generating a coherent interaction between two or more modes. This can be obtained by a superposition of modes propagating in different directions with different phase parameters. An interesting case occurs in some regions where a consonance between the phase parameters occurs. This consonance generates SP fields which exhibit the self-imaging phenomenon as follows. The self-imaging condition means that the amplitude function for the optical field must be periodic along the propagation coordinate and it can be represented as a Fourier series. Considering again the z axis as coordinate of propagation, the vector amplitude distribution (on media 1) has the representation

$$\mathbf{E}_1(x,y,z) = \mathrm{e}^{-\alpha x}\sum\left(\vec{i}a_n+\vec{j}b_n+\vec{k}c_n\right)\mathrm{e}^{\mathrm{i}2\pi zn/d} = \int_{-\infty}^{\infty}\left(\vec{i}A(u)+\vec{j}B(u)+\vec{k}C(u)\right)\mathrm{e}^{-\alpha x} \times \exp\left\{\mathrm{i}2\pi\left(\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)^{1/2}(uy+pz)\right\}\mathrm{d}u \quad (4)$$

where "d" is the period of self-imaging along z-coordinate, and the coefficients (a_n, b_n, c_n) may be functions of y coordinate. To find the general structure of the amplitude frequency functions $(A(u), B(u), C(u))$, we substitute expression (4) in SP Helmholtz equation. Solving for y-variable, we obtain that solutions are given by $(a_n(y), b_n(y), c_n(y)) = (a\mathrm{e}^{\mathrm{i}\delta_n y}, b\mathrm{e}^{\mathrm{i}\delta_n y}, c\mathrm{e}^{\mathrm{i}\delta_n y})$ being (a,b,c) arbitrary constants, and the phase term satisfies $\delta_n = \sqrt{\left(B^2+\alpha^2-\frac{4\pi^2n^2}{d^2}\right)}$. Using this representation in the series of Eq. (4), and considering $z = 0$, we obtain that the amplitude frequency representation may be obtained by means of a Fourier transform having the structure of a Dirac-δ function of the form $(A(u), B(u), C(u)) = (A_n(u), B_n(u), C_n(u))\delta(u-\beta-\delta_n)$, where $(A_n(u), B_n(u), C_n(u))$ are arbitrary functions. This consists of a set of points in frequency space; this result is equivalent to the Montgomery condition for free space [14]. The spatial analysis of the self-imaging fields can be obtained by using the paraxial approximation for δ_n. This corresponds with the weak self-imaging condition, and following the Montgomery's treatment it can be written as

$$\delta_n \approx \frac{\lambda}{\left(\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)^{1/2} 2d^2}\sqrt{n}, \quad (5)$$

A particular case of Eq. (5) is when $n = 0, 1, 4, 9, \ldots$; this is known as Talbot's effect, and the transmittance function corresponds to a periodical object of period "a" which is related to the self-imaging period by

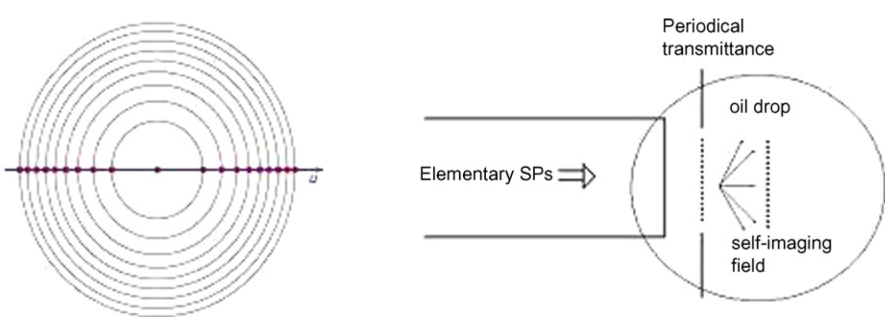

Fig. 1. (a) Montgomery rings and its intersection with u axis. The highlighted points are the frequency condition for the plasmon fields presents the "weak self-imaging". (b) Schematic set up for the generation of self-imaging by diffraction of SPs.

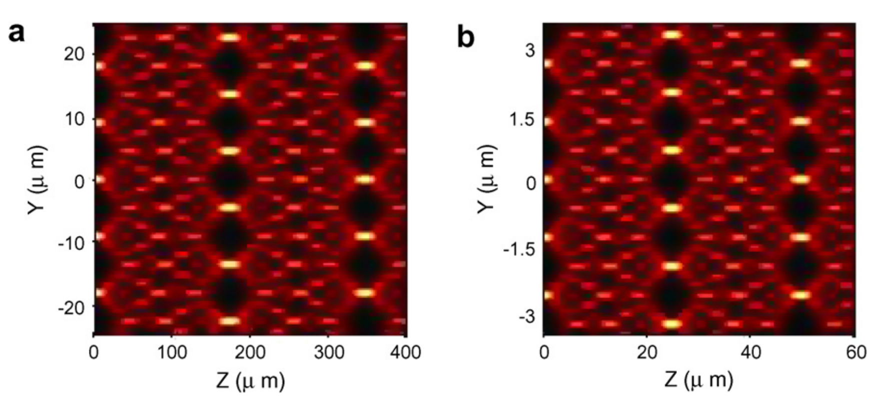

Fig. 2. Irradiance distribution for a weak SP self-imaging field. The field was generated by interfering elementary 7 SPs, propagating along directions making the following with angle respect to the positive z-axis $\theta_{SP} = (\pm 9°, \pm 6°, \pm 3°, 0)$. The wavelength is vacuum is $\lambda = 2\pi c/w = 502$ nm. (a) On a gold surface for which the SP wavelength is $\lambda = 476$ nm; (b) on the surface of a Au thin film with thickness $d = 40$ nm, covered by oil drop of glycerin ($n = 2.1$), for which $\lambda = 68$ nm.

$$d \approx \frac{2a^2}{\lambda\left(\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)^{1/2}}.$$

Eq. (5) represents a set of points whose structure is a one-dimensional zone plate as it is sketched in Fig. 1a. They can be obtained intersecting the traditional Montgomery's rings with u-axe.

The self-imaging field is obtained by means of the diffraction field generated by illuminating a transmittance whose Fourier transform corresponds with a set of "points" which must satisfy Eq. (5). For a periodical transmittance with period of 10 μm, and for a wavelength of 476 nm, the period of self-imaging is about 300 μm. For this propagation length, the absorption plays an important role and the self-imaging phenomenon is not possible. However, by considering a reduction of the period about 1.5 μm deposited on a gold film of thickness $d = 40$ nm and covered with oil of refractive index $n = 2.1$, the effective refractive index is of $n_{\mathrm{eff}} = \lambda_0/\lambda_{\mathrm{SP}} = 7$. For these configurations, the self-imaging field has a period of 50 μm, which is feasible to implement experimentally avoiding absorption analysis. These comments are sketched in Fig. 2a and b just to compare the scale reduction in the self-imaging period. Similar configurations have been proposed recently to implement an oil drop as a mirror/lens [4].

4. Surface plasmon singularities

In order to have a complete description of the surface optical field it is necessary to describe the singular regions. On these regions, the amplitude distribution has adiabatic features; it means that the spatial/temporal amplitude function changes very slowly. These features are deeply connected with charge re-distribution. The simplest case to generate a singularity is by interfering two elementary contra-propagating plasmon waves as is sketched in Fig. 3a, the nodes corresponds to singular points. Performing the calculus only for the electric field amplitude on media 1, we find that the electric field satisfies $\mathbf{E}_1(x,z) = 2\exp(-\alpha_1 x)(\vec{i}a\cos\beta z + \vec{k}ib\sin\beta z)$. A similar expression is obtained for media 2. To associate a physical meaning to the electric field, the parameters (a, b) must be imaginary. The expression corresponds to a standing wave. Taking the divergence of the electric field we obtain $\nabla\cdot\mathbf{E}_1 = 2\mathrm{e}^{-\alpha x}(a\alpha_1 - \mathrm{i}b\beta)\cos\beta z$, which is in general non-zero. This means that the superposition of two contra-propagating SP waves generates a periodic stationary charge redistribution density function, where the charge density function of media 1 is given by $\rho_{d1} = \varepsilon_1 2\mathrm{e}^{-\alpha x}(a\alpha_1 - \mathrm{i}b\beta)\cos\beta z$. Again a similar expression can be obtained for media 2. It must be noted that the period "a" of the charge array is proportional to the inverse value of the dispersion relation given by $a = \frac{2\pi}{\beta} = \lambda\left(\frac{\varepsilon_{1r}+\varepsilon_{2r}}{\varepsilon_{1r}\varepsilon_{2r}}\right)^{1/2}$. For the SP fields, the charge period satisfies $a > \lambda$. Analysis previous has associated a discrete set of singular points, the continuous case is analogous to the envelope region for homogeneous media [18].

The singular regions associated to the diffraction field emerging from a transmittance function of the form $t(y,z) = \delta(y - g(z))$ means that parameterized amplitude function can be represented as

$$E(x,y,u) = \left(\vec{i}A(u) + \vec{j}B(u) + \vec{k}C(u)\right)\exp(-\alpha_1 x)\mathrm{e}^{\mathrm{i}\beta(yu+g(y)p)}, \tag{6}$$

where u is considered as a parameter. From this represen-

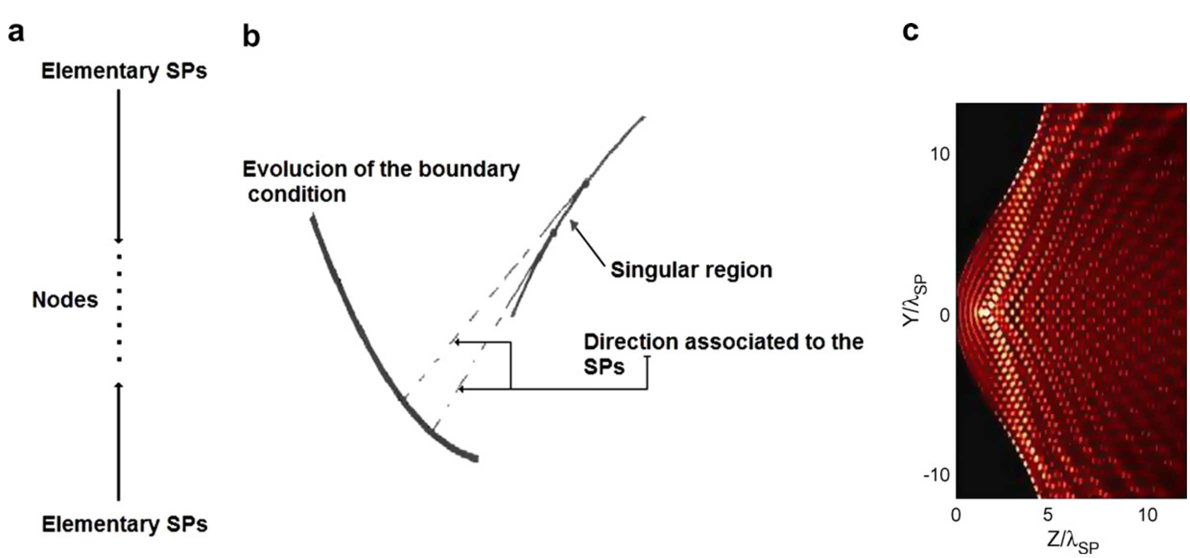

Fig. 3. Geometrical description of singular regions. (a) Generation of discrete singular points by means of the interference between two contra-propagating elementary SPs. (b) Singular region by means of the envelope of elementary SPs. The modes emerge in a perpendicular direction tot the curve. (c) Focusing region obtained by illuminating with a elementary surface plasmom wave a reflecting surface with Gaussian shape with $\sigma = 10\lambda_{\mathrm{SP}}$ and depth $5\lambda_{\mathrm{SP}}$.

tation, the extremal features of the mode trajectories whose phase function is $L(x,y,u)=\beta(yu+g(y)p)$ are given by $\frac{\partial L}{\partial y}=0$ and the calculus of the envelope implies that the phase function satisfies $\frac{\partial g}{\partial y}=\frac{p}{u}=\tan\theta$. This is the tangential condition for envelope regions of mode trajectories. Since the geometrical point of view means that trajectories are tangent to the singular regions as is sketched in Fig. 3b. In Fig. 3c, we show the computational simulations for the focusing region for the case when the geometry of the boundary condition corresponds to a Gaussian profile.

Due to the tangent property, generic features for the phase function can be implemented. This means that a catastrophe function for the phase function can be used [20,22]. By considering a curve where the trajectories emerge in a perpendicular way, it is easy to show that singularities correspond with the envelope of curvature centers; this curve is known as evolute curve. On this region adiabatic features are presented, which means that the phase function changes very slowly and charge re-distribution is generated. More details concerning this representation can be found in [22]. Associating to the set of surface modes a parametric representation as a catastrophe function, the surface plasmon field has an structure of the form

$$\mathbf{E}_1(x,z)=\left(\vec{i}a+\vec{j}b+\vec{k}c\right)\exp(-\alpha_1 x)\times\exp\mathrm{i}(\text{catastrophe function}(y,z,u)) \quad (7)$$

Using as prototype a function of catastrophe kind cusped given by $L(y,z)=\frac{y^4}{4}-\frac{\xi^2}{2}y+\eta y$, where $\xi=\xi\ (z),\eta=\eta(z)$, the envelope of critical points generates a charge distribution of the form $\rho(\xi,\eta)=\varepsilon\delta\left(\xi-3\sqrt{3}\eta^{3/2}\right)$, where δ is the Dirac-δ function.

5. Conclusions

In conclusion, using the representation for optical modes in homogeneous media, we can incorporate the treatment for the description of elementary surface wave mode vectors. These kinds of solutions are analogous to plane waves and they can be used to describe arbitrary surface plasmon fields. The representation obtained corresponds with the angular spectrum model for surface plasmon fields. Using this representation, we obtain a more general mode solution for surface plasmon fields having the property that the phase function along coordinate of propagation is smaller than the dispersion relation function $\beta>\Omega$. This behavior of the phase function allows one to describe the self-imaging phenomenon for surface plasmon fields. We find that the sufficient condition for self-imaging
is
a-
n-
a-
l-
o-
g-
ous to the Montgomery's condition for homogenous media. Finally, we show that singular points can be generated experimentally by mean of the interference between two plane plasmon modes propagating in opposite directions, The nodes corresponds with stationary charge distribution, having a periodically representation. General singular regions were described using a parameterization for the phase function by means of a catastrophe function. The mode analysis presented allows incorporation of other interesting features related to self-imaging, such as the Lau effect also as the generation of partially coherent effects. Recall that self-imaging reproduce periodical patterns of relatively intense surface electromagnetic fields, which could be exploited to generate surface optical lattices and surface optical tweezers. The associated arrays of surface charge distributions can be also of fundamental interest in a number of fields [1–4]. Finally, the description of singularities on the plane offers interesting applications to design and control of nano-optics.

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