Conversion from *n* Bands Color Space to HSI*n* Color Space

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This paper presents the method to get the equations that transform a color space of *n* independent primary colors to the HSI*n* color system (H: hue, S: saturation, I: intensity); *n* indicates the number of bands and the shape of the HSI*n* space. For $n = 3$ the structure is a double triangular pyramid, for $n = 4$ it is the structure of the double pyramid tetrangular, and so on. © 2009 The Optical Society of Japan

1. Introduction

The *n*-bands color space and HSI*n* space are mathematical entities that we have extended from RGB and HSI color systems. The *n*-bands color space is a generalization of the RGB additive system, which indicates that a color point belonging to the *n*-bands color space is the result of the mixture of *n* additive primary colors. This affirmation is supported by the following definition of additive mixture:

Definition 1.1. *Additive mixture means a color stimulus for which the radiant power in any wavelength interval, small or large, in any part of the spectrum is equal to the sum of the powers in the same interval of the constituents of the mixture.*[1]

This definition allows us to see that a color can be matched by different number of primary colors, not necessarily three, which is the case found in most books or papers related to the study of color.

On the other hand, the HSI*n* space has the same characteristics of the HSI: hue, saturation and intensity. These features that are in the most natural terms to an artist. Also, the HSI system is based on the RGB and the HSI*n* color model is in terms of the *n*-bands space. The *n* is related to the shape of the space; HSI*n* takes the form of an *n*-angular double pyramid.

The HSI color space is widely used to generate high quality computer graphics in digital image processing. With the development of technology in the capture and display of multispectral images (images based on more than three primary colors), we require a space that generalizes to the HSI space and use it, for example in segmentation or restoration of such images.[2] At the Tokyo Institute of Technology along with government organizations of Japan, devices have been developed for the capture and visualization of multispectral images in the visible with 8, 16, and 32 bands. These have been designed, built and patented by the Natural Vision Research Laboratory, with the main objective of reproducing the real color of the objects.[3]

2. *n*-Bands Color Space

The *n*-bands color model is a generalization of the RGB

additive model, so we can say that an *n*-bands color space is determined by a basis vector, whose linear combinations generate all elements of the space. As the number of vectors increases, the space will include a larger amount of real colors. Examples: Space of one dimension, the grayscale; two-dimensional spaces, the RG, GB, and BR planes that are respectively variations of the yellow, cyan and magenta; three-dimensional spaces, RGB, HSV (H: hue, S: saturation, V: value), and HSI.

The *n*-bands color space is an *n*-dimensional subspace contained in Euclidean space of dimension *n*. The subspace is generated by a basis of *n* unit vectors. The colors appear with their primary components, namely with the *n* independent primary colors, which we will denote as C_1, C_2, ..., C_n. The values given in real numbers of C_1, C_2, ..., C_n range from zero to one, and they are respectively over the *n* axes. In other words, in the C_i-axis ($i = 1, ..., n$) is the scale of color C_i; black is located at the origin $(0, ..., 0)$, and white is in $(1, ..., 1)$; the levels of gray extend from black to white, over a hyper-line (line defined in the Euclidean *n*-dimensional space). Any gray intensity is obtained by adding equal amounts of each primary color, so the colors are points of the space. For $n = 3$, the color space is a unit color cube as shown in Fig. 1, this space is well-known as RGB space.[4]

3. HSI*n* Color Space

The HSI*n* color model is the generalization of the HSI space, in which the colors can be distinguished from each other by their hue (H), intensity (I), and saturation (S).[5]

The *hue* is associated with the dominant wavelength in a mixture of light waves. Thus, the hue represents the dominant color as we perceive it, when we say that an object is red, green or brown, we are indicating its hue [Fig. 2(a)].

The *intensity* represents the perceived illumination. The intensity conveys the extent to which light is reflected on an object. We can see this attribute clearly in a white and black television [Fig. 2(b)].

The *saturation* refers to the amount of white light mixed with the dominant color. The saturation is an attribute that makes us distinguish between a deep and a pale color. Each of the primary colors has its greatest value saturation before being mixed with others. This way, the sky blue is clear (less saturated), while the navy blue is more opaque (more saturated). Another example is the pink (combination of red
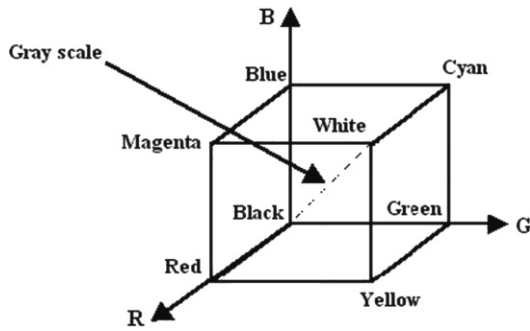
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Fig. 1. The RGB unit color cube. The horizontal G-axis as green values increasing to the right, R-axis as red increasing to the lower left, and the vertical B-axis as blue increasing towards the top. The color black is in the origin, at the vertex $(0, 0, 0)$, the color white at the vertex $(1, 1, 1)$. The grayscale is in the line that joins the point black with the point white.
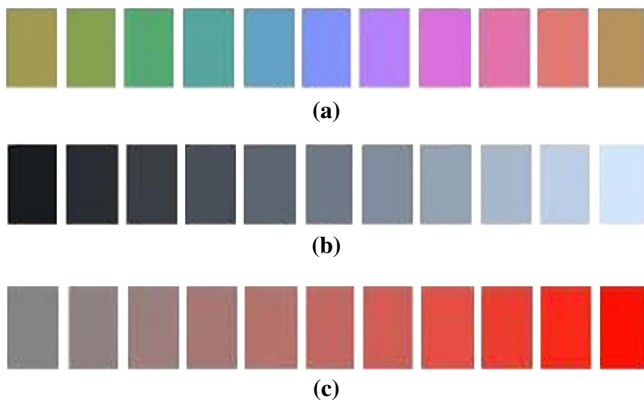

Fig. 2. (Color online) Hue, saturation and intensity. (a) Different hues (dominant color); (b) Different intensities represented by this grayscale; (c) A variety of saturation of red color.

and white), it is less saturated, while the color red is totally saturated [see Fig. 2(c)].

Hue and saturation are defined by the regular polygon of n sides (Fig. 3). The hue H of color point P is the angle of the vector regarding the axis C_1. The saturation S of the point P is proportional to the distance measured from P up to the center of the polygon W. As the distance is greater, the saturation will be greater.

The value of the intensity of point P is proportional to the distance (on the perpendicular straight line to the plane of the polygon, which passes through its center) measured from the black point to starting point of the vector. Thus for any color point in the polygonal pyramid, if its intensity tends to the white point, then the color will be clearer. But if it tends to black point, the color will be darker. Joining hue, saturation and intensity we have the n-sided double structure of polygonal pyramid (Fig. 4).

4. Conversion from the n-Bands Color Space to HSI$_n$ Color Space

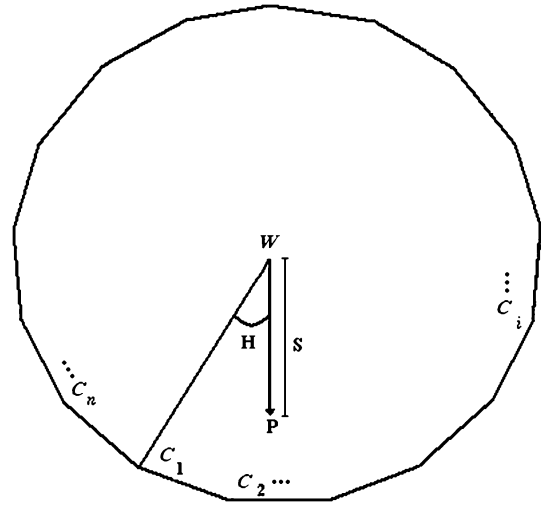The H and the S are given in terms of equations called chromatic coordinates, which are defined as

Fig. 3. Hue and saturation are defined by the color regular polygon of n sides. The Hue is a angular measure and the saturation is a longitudinal measure.
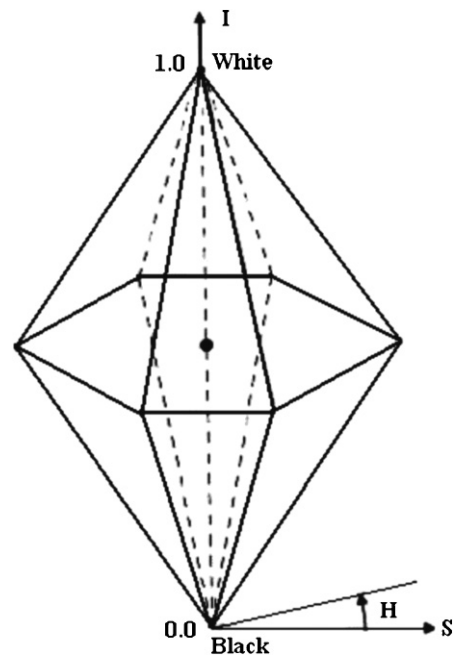

Fig. 4. Color space HSI$_n$. Hue (H), saturation (S), and intensity (I) form the n-sided double structure of polygonal pyramid.

$$c_i = \frac{C_i}{\sum_{j=1}^{n} C_j} ; \quad i = 1, \ldots, n. \tag{1}$$

These are the normalized values of each C_i. Adding the c_i's, we find the equation of a plane in the Euclidean n-dimensional space,[6] also called hyperplane for $n > 3$, with coordinate axes $c_1, c_2, \ldots, c_n$ and passing through the points $\mathbf{e_1} = (1, 0, \ldots, 0)$, $\mathbf{e_2} = (0, 1, \ldots, 0), \ldots, \mathbf{e_n} = (0, 0, \ldots, 1)$:
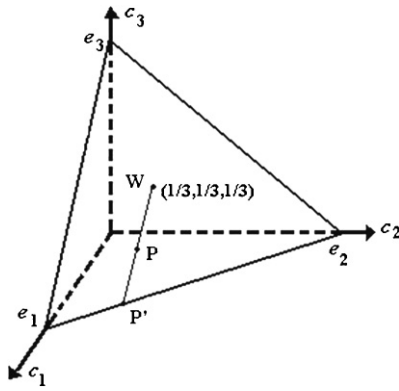
$$\sum_{i=1}^{n} c_i = 1. \tag{2}$$

Fig. 5. Color triangle. The equation $\sum_{i=1}^{3} c_i = 1$, c_i within the interval $[0, 1]$ represents a plane bounded by $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ giving us the form of an equilateral triangle centered on $(1/3, 1/3, 1/3)$.

In the case of two-dimensional, eq. (2) is the representation of a line through $\mathbf{e_1}$ and $\mathbf{e_2}$, with midpoint at coordinates $(1/2, 1/2)$. In the case of three dimensions, it represents a plane bounded by $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ giving the form of an equilateral triangle centered on $(1/3, 1/3, 1/3)$, Fig. 5. Thus we assume that to n dimensions, the hyper-plane defined by $\mathbf{e_1}, \mathbf{e_2} \ldots, \mathbf{e_n}$ is a regular polygon of n sides with center in $(1/n, \ldots, 1/n)$. It can be verified that this hyper-plane delimited fulfills with the definition of a regular polygon, taking as distance to the Euclidean norm for n-dimensional spaces; the coordinates $(1/n, \ldots, 1/n)$ as the center of this polygon; and to measure the angles, use the equation of the dot product of spatial vectors,[6] $\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta$, for $0 \leq \theta \leq 180$.

Definition 4.1. *A polygon is considered regular when all sides and angles are equal.*[7]

Definition 4.2. *The center of the regular polygon is the equidistant point from the vertices and sides at the center of a regular polygon.*[7]

4.1 Intensity and Hue

The intensity I is defined as the average number of C_i's

$$I = \frac{1}{n}\left(\sum_{i=1}^{n} C_i\right). \tag{3}$$

In Fig. 3, we see that the H is the angle between the line defined by the center of the polygon W and C_1 and the line that is defined by W and the color point P. These points on the plane given in terms of chromatic coordinates have coordinates $\mathbf{w} = (1/n, \ldots, 1/n)$, $\mathbf{p} = (c_{01}, c_{02}, \ldots, c_{0n})$ and C_1 becomes $\mathbf{e_1} = (1, 0, \ldots, 0)$. As an example, see Fig. 6 for $n = 3$. Therefore, using the equation of the dot product the H is the equation:

$$H = \cos^{-1}\left[\frac{(\mathbf{p} - \mathbf{w}) \cdot (\mathbf{e_1} - \mathbf{w})}{\|\mathbf{p} - \mathbf{w}\|\|\mathbf{e_1} - \mathbf{w}\|}\right], \text{ for } 0 \leq H \leq 180. \tag{4}$$

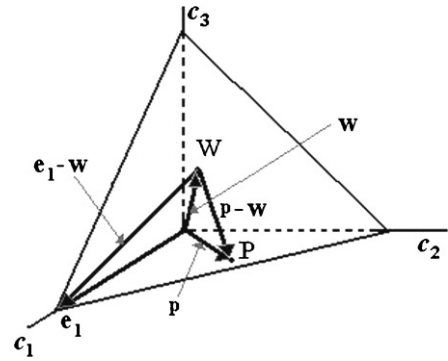$\|\mathbf{p} - \mathbf{w}\|$ can be expressed as,

Fig. 6. Vectors involved in the calculus of hue. The subtraction vector gives us vectors in which the hue is determined.

$$\|\mathbf{p} - \mathbf{w}\| = \sqrt{\sum_{i=1}^{n} c_{0i}^2 - \frac{1}{n}}. \tag{5}$$

Now, Substituting the values c_{0i} into eq. (5):

$$\|\mathbf{p} - \mathbf{w}\| = \sqrt{\frac{n\left(\sum_{i=1}^{n} C_{0i}^2\right) - \left(\sum_{i=1}^{n} C_{0i}\right)^2}{n\left(\sum_{i=1}^{n} C_{0i}\right)^2}}. \tag{6}$$

On the other hand,

$$\|\mathbf{e_1} - \mathbf{w}\| = \sqrt{1 - \frac{1}{n}}, \tag{7}$$

and

$$(\mathbf{p} - \mathbf{w}) \cdot (\mathbf{e_1} - \mathbf{w}) = \frac{(n-1)C_{01} - \sum_{i=2}^{n} C_{0i}}{n\left(\sum_{i=1}^{n} C_{0i}\right)}. \tag{8}$$

Finally

$$H = \cos^{-1}\left[\frac{(n-1)C_{01} - \sum_{i=2}^{n} C_{0i}}{\sqrt{n-1}\sqrt{n\left(\sum_{i=1}^{n} C_{0i}^2\right) - \left(\sum_{i=1}^{n} C_{0i}\right)^2}}\right]. \tag{9}$$

In Fig. 3 we notice that the angle formed by each C_i W C_{i+1} ought to be $360°/n$, since adding this quantity n times, the result is $360°$. Using eq. (9), that angle has a value of $\cos^{-1}[-1/(n-1)]$. For $n = 3$ this value is matched with $360°/n$. Nevertheless, for $n > 3$, the value is not equal to $360°/n$, and adding n times the result is greater than $360°$. To fix this problem we made a scaling with the equation:

$$H = \left[\frac{360}{n\cos^{-1}\left(\frac{-1}{n-1}\right)}\right]H. \tag{10}$$

So, the addition will be $360°$. Applying this new formulation on numerical experiments for $n > 3$, we realized
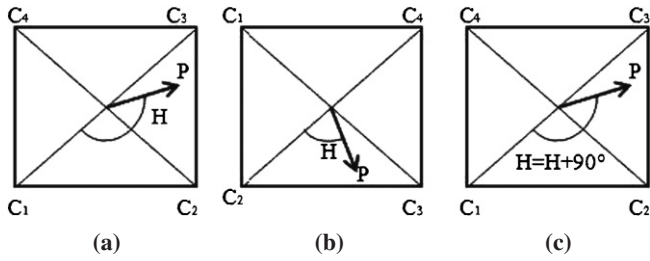
Fig. 7.   Squares color. The figure shows the method to find the hue in any color regular polygon. (a) Original position of color point; (b) Translation of the color point towards area 1, and on this area is calculated the hue; (c) Color point has been returned to the original area, and its hue is calculated by adding to the previous hue the ninety degrees that were needed for the translation.
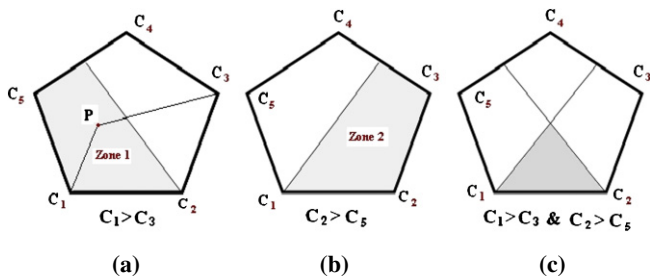

Fig. 8.   (Color online) Description of the area of the color point. Any color area i-th, zone bounded by C_i W C_{i+1} (c), can be described by the intersection of two regions that contains the zone and divide the polygon in two equal parts (a, b). Mathematically, each area i-th can be expressed in terms of C_{i-1}, C_i, C_{i+1}, C_{i+2} (c).

that for points belonging to the area bounded by C_1 W C_2 (we call it area 1), the resulting hue was correct. However, for color points outside the area 1 and that had an angle greater than $100°$, the H calculated was incorrect. Hence, the solution to find the true angle of H in any other area i-th (area bounded by C_i W C_{i+1}, $i = 2, \ldots, n$) for any plane polygonal of n sides, is moving this up to the area 1; find the hue, and return to the original area, adding to the H the degrees that are needed for the translation (Fig. 7). Therefore, for any regular polygon we must know how to characterize the area where is the color point.

For this purpose, let us analyze the color pentagon of Fig. 8. The findings can be extended to any regular polygon. Suppose that the color point is in area 1 of the pentagon; from Fig. 8, we see that this area can be obtained as the intersection of the shaded zone 1 of Fig. 8(a) and the shaded zone 2 of Fig. 8(b). So respectively, each shaded zone is achieved if $C_1 > C_3$, and $C_2 > C_5$. The explanation is this: If we take a color point P in the shaded zone 1, the distance measured from P to C_1 is less than the distance measured from P to C_3 [Fig. 8(a)], which means the color point P has a greater amount of color C_1 than color C_3. For the remaining areas, a similar analysis provides explanation.

Generalizing for any regular polygon, let us call area i-th to the area bounded by C_i W C_{i+1}.
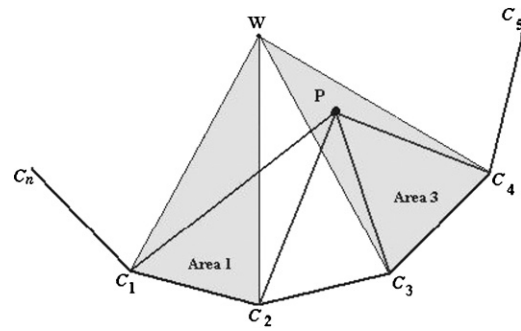

Fig. 9.   Virtual areas. The method for distinguishing the color area of the color point P among others, which comply with the restriction $C_i > C_{i+2}$ and $C_{i+1} > C_{i-1}$ is to add $C_i + C_{i+2}$; the largest amount corresponds to the color area of P. This is equivalent to the distance from P to the vertices of the color area of P are less than or equal to the distance from P to the vertices of any other.

Area 1 is obtained when $C_1 > C_3$ and $C_2 > C_n$.
Area i-th is obtained when $C_i > C_{i+2}$ and $C_{i+1} > C_{i-1}$, $i = 2, \ldots, n - 2$.
Area $n - 1$ is obtained when $C_{n-1} > C_1$ and $C_n > C_{n-2}$.
Area n-th is found when we have $C_n > C_2$ and $C_1 > C_{n-1}$.

However, as we shall see in §5, it may happen that occur more than one of these conditions. In other words, that some color point P takes more areas in a virtual way. For example it can happen, that $C_1 > C_3$ and $C_2 > C_n$; $C_3 > C_5$ and $C_4 > C_2$, where each C_i is a component of P. So, P would seem to be in two areas, area 1 and area 3. Moreover, suppose that the real area of the color point P is the third. From Fig. 9, which is portion of the polygon of Fig. 3, note that if P is in area 3, then P is closer to the vertexes C_3, C_4 than to the vertexes C_1, C_2. Therefore, we can conclude that if the sum of the values C_3 and C_4 is greater than the sum of C_1 and C_2, then P is in the area 3. For the foregoing, we deduce that on a color polygon of n-sides, the area of a color point P can be located as follows.

Algorithm 4.1.1.

- *Area 1 is obtained if:* $C_1 > C_3$ *and* $C_2 > C_n$; *and* $C_1 + C_2 > C_j + C_{j+1}$ *where* $C_j > C_{j+2}$ *and* $C_{j+1} > C_{j-1}$, *for* $j \neq 1$.
- *Area i-th is obtained when* $C_i > C_{i+2}$ *and* $C_{i+1} > C_{i-1}$, $i = 2, \ldots, n - 2$; *and* $C_i + C_{i+1} > C_j + C_{j+1}$ *where* $C_j > C_{j+2}$ *and* $C_{j+1} > C_{j-1}$, *for* $j \neq i$.
- *Area $n - 1$ is obtained when* $C_{n-1} > C_1$ *and* $C_n > C_{n-2}$; *and* $C_{n-1} + C_n > C_j + C_{j+1}$ *where* $C_j > C_{j+2}$ *and* $C_{j+1} > C_{j-1}$, *for* $j \neq n - 1$.
- *Area n-th is found when* $C_n > C_2$ *and* $C_1 > C_{n-1}$; *and* $C_n + C_1 > C_j + C_{j+1}$ *where* $C_j > C_{j+2}$ *and* $C_{j+1} > C_{j-1}$, *for* $j \neq n$.

Finally, to get the hue of the color point P, the procedure to follow is to transfer the area where P is, toward the area 1; calculate the hue in eq. (9); make an escalation in the interval from $0°$ to $360°/n$ [eq. (10)], and to finish, calculate the hue on the original area adding to H the grades needed for translation.

Algorithm 4.1.2.

1. *Finding the i-th area where* P *belongs through the algorithm 4.1.1.*
2. *Move to Area 1:* $C_1 = C_i$, $C_2 = C_{i+1}, \ldots, C_{n-1} = C_{i-2}$, $C_n = C_{i-1}$.
3. *Finding* H:

$$H = \cos^{-1}\left[\frac{(n-1)C_{01} - \sum_{i=2}^{n} C_{0i}}{\sqrt{n-1}\sqrt{n\left(\sum_{i=1}^{n} C_{0i}^2\right) - \left(\sum_{i=1}^{n} C_{0i}\right)^2}} \right].$$

4. *Carry out an escalation in the interval from* $0°$ *to* $360°/n$:

$$H = \left[\frac{360}{n\cos^{-1}\left(\frac{-1}{n-1}\right)} \right] H.$$

5. *Return to the area i-th:* $H = H + (i-1)\,(360/n)$.

For those points of color that are in the diagonals, H is easily found with the following:

If $C_i \neq 0$ *and* $C_j =$ constant, *for any* $j \neq i$, $H = (i-1)(360/n)$.

4.2 Saturation

To calculate the S, we use Fig. 3. Extend the line defined by the points W and P until it intersects a side of the polygon, the point where it is cut is denoted by P′. Since the saturation is in the closed interval $[0,1]$ and is proportional to the distance from W up to P ($|WP|$), this is defined as:

$$S = \frac{|WP|}{|WP'|}. \tag{11}$$

This idea applied to our polygon contained in n-dimensional space,

$$S = \frac{\|\mathbf{w} - \mathbf{p}\|}{\|\mathbf{w} - \mathbf{p}'\|}. \tag{12}$$

We know $\mathbf{w}$ and $\mathbf{p}$, but not $\mathbf{p}'$. Let us suppose that we are working in three-dimensional Euclidean space. Without loss of generality let us suppose that P′ is in the plane defined by the vectors $\mathbf{e_1}$ and $\mathbf{e_2}$ (Fig. 5). In this figure we realize that P′ is the intersection of the line defined by W and P with the plane. Generalizing for $n > 3$, P′ is the intersection of the hyper-line defined by W and P with the hyper-plane defined by the vectors $\mathbf{e_1}, \mathbf{e_2}, \ldots, \mathbf{e_{n-1}}$.

Solving the system of equations, which is the result of the intersection of the hyper-line through W and P, that it is expressed mathematically

$$c_i = k\left(c_{0i} - \frac{1}{n}\right) + \frac{1}{n}; \quad i = 1, \ldots, n; \tag{13}$$

k within the interval $[0,1]$,

and the hyper-plane passing through $\mathbf{e_1}, \mathbf{e_2}, \ldots, \mathbf{e_{n-1}}$, whose parametric equation is

$$c_1 = 1 - t_2 - t_3 - \cdots t_{n-1},$$
$$c_2 = t_2,$$
$$\vdots \tag{14}$$
$$c_{n-1} = t_{n-1},$$
$$c_n = 0;$$

each t_i lies into $[0,1]$, it is found that:

$$\mathbf{p}' = (1 - nc_{0n})^{-1}$$
$$\times \left(1 - \sum_{i=2}^{n-1} c_{0i} - 2c_{0n}, c_{02} - c_{0n}, \ldots, c_{0n-1} - c_{0n}, 0\right). \tag{15}$$

By replacing eqs. (5) and (15) in eq. (12),

$$S = \frac{\sqrt{\left(\sum_{i=1}^{n} c_{0i}^2\right) - \frac{1}{n}}}{(1 - nc_{0n})^{-1}\sqrt{\left(\sum_{i=1}^{n} c_{0i}^2\right) - \frac{1}{n}}}, \tag{16}$$

that becomes:

$$S = 1 - nc_{0n}, \tag{17}$$

where c_{0n} is the minimum of all the c_{0i}'s, because c_{0n} is in $[0, 1/n]$ and the c_{0i}'s belong to $[1/n, 1]$; hence

$$S = 1 - \frac{n\min\{C_{01}, C_{02}, \ldots, C_{0n}\}}{\sum_{i=1}^{n} C_{0i}}. \tag{18}$$

Performing the same process for P′, when this is in all other hyper-planes passing through $(n-1)$ $\mathbf{e_i}$'s, we get to eq. (18).

5. Numerical Experiments

In this section we show some results after applying the formulas to find the hue, saturation and intensity on one digital image based on 16-color bands, Fig. 10. This figure is the result of a computational program that allows us to visualize that multispectral image on a monitor based in the RGB color space. The images are pictures of some codices obtained from National Library of Anthropology and History (BNAH, acronym from spanish: Biblioteca Nacional de Antropología e Historia).[8] Natural Vision Research Laboratory (NVRL), National Institute of Astrophysics, Optics and Electronics (INAOE, acronym from spanish: Instituto Nacional de Astrofísica Óptica y Electrónica), and BNAH signed a collaboration agreement for the digitalization of the codices, and analysis of data obtained through devices which capture multispectral images.

The images are composed of 16 matrices of size 768×768. Each matrix has the scale of a color band. Just as in the RGB images its maximum value for each matrix is 255, the maximum value of the scale of the 16 matrices is 38160.

From Fig. 10, we have selected shades of yellow, green and red that are enclosed inside a blue box (Fig. 11). The

Fig. 10. (Color online) Test image. Image based on 16-color bands. We can look at it because of a computer program that converts a multispectral image to a RGB image.

Fig. 11. (Color online) Test areas. We have been taken pixels inside blue boxes to calculate its hue, saturation and intensity.

results are shown in tables. Since each pixel in the image has 16 values, we occupy only one table to display all the values concerning a pixel; just in case the reader is interested in verifying the results. The others tables contain the coordinates of the pixels, the values of area, H, S, and I.

Band	Signal level	Wave length	Shutter speed
1:	3632(14/256)	418-35	1/60
2:	6816(27/256)	446-21	1/60
3:	8992(35/256)	465-17	1/60
→ 4:	8912(35/256)	481-16	1/60
5:	10880(43/256)	497-16	1/60
6:	11040(43/256)	513-16	1/60
7:	12800(50/256)	530-17	1/60
8:	14048(55/256)	548-18	1/60
9:	13120(51/256)	566-17	1/60
10:	14928(58/256)	584-18	1/60
11:	13808(54/256)	602-18	1/60
12:	14624(57/256)	621-19	1/60
13:	13920(54/256)	640-20	1/60
14:	11488(45/256)	662-22	1/60
15:	12144(47/256)	686-26	1/60
16:	9392(37/256)	717-36	1/60

Fig. 12. (Color online) Sixteen primary colors. These colors represent the 16 vertices of color polygon of space HSI_{16}.

Table 1. C_i's data corresponding to a pixel; values of the area of color point, and of HSI.

Color point green	
i	641
j	536
C_1	8832
C_2	14784
C_3	19872
C_4	19008
C_5	21456
C_6	20464
C_7	24688
C_8	25648
C_9	24368
C_{10}	25456
C_{11}	24192
C_{12}	25488
C_{13}	24256
C_{14}	20736
C_{15}	20752
C_{16}	15552
Color areas found	7, 12
Area of color point	7
H	153.656431
S	0.578867
I	0.549581

The areas color were found by means of Algorithm 4.1.1; the value of H was found by Algorithm 4.1.2; the value of S with eq. (18) and I with eq. (3). In Fig. 12 we see the 16 primary colors ranging from blue to red, passing through the green hues. So, the color polygon of space HSI_{16}, has 16 sides. Therefore, the color labeled with the number one has $H = 0°$, the color labeled with the number two has $H = 22.5°$, the third color has $H = 45°$, and so on. Area 1 is determined by color 1 and color 2; area 2 by color 2 and color 3, etc.

Table 2. Values of color point's area and HSI, obtained from a green zone.

		Green zone			
i	j	H	S	I	Area of the color point
636	531	153.732218	0.544172	0.592374	7
636	532	153.893505	0.531093	0.572275	7
636	533	220.180124	0.514441	0.536242	10
636	534	219.898929	0.496002	0.485010	10
636	535	219.786693	0.522078	0.464099	10
637	531	153.718112	0.532521	0.582993	7
637	532	219.914997	0.521789	0.553249	10
637	533	219.953434	0.502308	0.499581	10
637	534	220.041620	0.504807	0.447065	10
637	535	220.088802	0.550059	0.440776	10

Table 4. Values of color point's area and HSI, obtained from a reddish zone.

		Reddish zone			
i	j	H	S	I	Area of the color point
144	550	265.110261	0.665532	0.215618	12
144	551	265.051124	0.645743	0.222510	12
144	552	264.890945	0.633560	0.231132	12
144	553	219.832862	0.651056	0.269156	10
145	550	264.635055	0.624161	0.226468	12
145	551	265.373731	0.624413	0.234434	12
145	552	265.164669	0.613050	0.229717	12
145	553	265.024537	0.634239	0.255634	12
146	550	265.290433	0.650961	0.233045	12
146	551	219.842611	0.644133	0.239177	10

Table 3. Values of color point's area and HSI, obtained from a yellow zone.

		Yellow zone			
i	j	H	S	I	Area of the color point
635	314	220.295169	0.610610	0.491012	10
635	315	219.885758	0.604556	0.487736	10
635	316	220.065209	0.624114	0.506420	10
635	317	219.809990	0.592663	0.497170	10
635	318	220.178402	0.523319	0.416929	10
636	314	220.119336	0.608831	0.486635	10
636	315	219.982936	0.611496	0.487814	10
636	316	220.144948	0.614538	0.502542	10
636	317	220.137425	0.577006	0.471829	10
636	318	219.853621	0.557763	0.401048	10

Table 5. Values of color point's area and HSI, obtained from a dark zone.

		Dark zone			
i	j	H	S	I	Area of the color point
31	694	264.119223	0.671795	0.005110	12
31	695	240.996097	0.551402	0.005608	11
31	696	151.707692	0.507692	0.005110	7
31	697	240.792967	0.454545	0.004612	11
31	698	354.973827	0.437186	0.005215	16
32	694	239.895048	0.384615	0.005451	11
32	695	128.518875	0.381215	0.004743	6
32	696	286.080858	0.558011	0.004743	13
32	697	84.017547	0.448276	0.004560	4
32	698	217.429438	0.322751	0.004953	10

Table 1 shows the data of the 16 color bands of a pixel, which has a greenish hue. This pixel is taken from the rectangular region with hues of green, and it is shown below Fig. 11. In this table, we found two possible areas to the color point, namely, areas 7 and 12; but by means of Algorithm 4.1.1 we find that area 7 is the correct.

The pixels of Table 2 are taken from the rectangular region that lies below the image and is a green zone (Fig. 11). From the results, we note that only three pixels are in area 7, and the most pixels are in area 10, which means that it is an area of shades of yellow. So, why do we see green hues? There are two answers: First, we must remember that the image of Fig. 10 is based on three basic colors, while the multispectral image is based on sixteen. The second answer is the colors of area 10 are less saturated, and are less intense than those in area 7.

For Table 3, the pixels were selected from the rectangular region displayed below the image on the bird (Fig. 11). In this region, hues of yellow shades can be observed, and Table 3 confirms this. Much of the pixels have a greater degree of saturation than those from the previous table and have an intensity of less than 50%.

The Table 4 shows the coordinates of pixels that were taken at the top of Fig. 11, inside the shield of the image.

The results show that indeed we are in a reddish zone, in which case we are in area 12. In contrast to the previous table, here the colors are more saturated because exceed more than 50%, but they are darker because the values are closer to zero than one.

From the Table 5, the pixels have been obtained from top, right of Fig. 11. We observed a dark area with a little color. We have eight different hues; the data from saturation are contrasting; and as expected, the intensity is nearly zero.

6. Conclusions

Through this manuscript we have explained the method to obtain the equations that lead us from n bands color space to HSI_n color space. The n bands color space is the generalization of the RGB system additive. In this way, the n bands color space is a n-dimensional vector subspace, and each element of color is defined by a vector. HSI_n color space is just the n bands color space given in a different coordinate system. The intensity I within the n bands color, is defined along the line defined in Euclidian n-dimensional space, from the black $(0, \ldots, 0)$ to the white $(1, \ldots, 1)$. The intensity range is between $[0, 1]$ and 0 means black, 1 means white. HS are the polar coordinates of a plane in n dimensions, and it is normal to I. The saturation component

shows how much the color is diluted with white color. The range of the S component is [0, 1], and it is measured by the Euclidean distance. H is the angle starting from the first color band vector, given in degrees. The hue component describes the color itself in the form of an angle between [0, 360] deg.

The conversion from n bands color space to HSI_n is given in terms of the values of the n primaries bands of a color, we can find its HSI representation using the hyper-plane defined by the chromatic coordinates and using n dimensional geometry. To show the functionality of the formulas found, we apply these to a multispectral image of 16 color bands. The image, among others, was obtained through an agreement between Natural Vision Research Laboratory (NVRL), National Library of Anthropology and History (BNAH, acronym from spanish: Biblioteca Nacional de Antropología e Historia), and National Institute of Astrophysics, Optics and Electronics (INAOE, acronym from spanish: Instituto Nacional de Astrofísica Óptica y Electrónica). The results of hue, saturation and intensity are displayed in five tables.

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