# Kinetic theory of situated agents applied to pedestrian flow in a corridor 

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#### Abstract

A situated agent-based model for simulation of pedestrian flow in a corridor is presented. In this model, pedestrians choose their paths freely and make decisions based on local criteria for solving collision conflicts. The crowd consists of multiple walking agents equipped with a function of perception as well as a competitive rule-based strategy that enables pedestrians to reach free access areas. Pedestrians in our model are autonomous entities capable of perceiving and making decisions. They apply socially accepted conventions, such as avoidance rules, as well as individual preferences such as the use of specific exit points, or the execution of eventual comfort turns resulting in spontaneous changes of walking speed. Periodic boundary conditions were considered in order to determine the density-average walking speed, and the density-average activity with respect to specific parameters: comfort angle turn and frequency of angle turn of walking agents. The main contribution of this work is an agent-based model where each pedestrian is represented as an autonomous agent. At the same time the pedestrian crowd dynamics is framed by the kinetic theory of biological systems.


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## 1. Introduction

In this work, a situated agent-based kinetic model for simulation of pedestrians is described. Agents are software programs developed in the eighties by the research community of Distributed Artificial Intelligence and Multi-agent Systems [1]. Agents act autonomously and are usually equipped with specific devices for data acquisition, communication and action execution, as well as a processor and memory data register for decision making.

Various approaches for agent definition have been proposed and investigated. In general, these can be divided into two categories: knowledge-based or deliberative agents, and behavior-based or situated agents. The former rely on plentiful resources for decision-making, such as preprocessed data, organized for instance in elaborate structures BDI (an acronym for Beliefs, Desires, Intentions) [2], as well as complex decision-making processes defined often under the influence of ideas from the fields of sociology and psychology. The latter do not process large volumes of data to make decisions, instead they acquire information from the problem domain directly, thanks to their inherent capability of being situated or embedded within an environment. In addition, situated agents act in response to preprocessed stimuli as defined in processes influenced mainly by ideas from the field of ethology [3].

In particular, situated agents are characterized by their simplicity. A situated agent needs a device enabling the perception of environmental conditions, generally within a spatially circumscribed area. In addition, a set of rules associating perceived stimuli and proven suitable responses is required. If we want an agent to exhibit an adaptive rather than a deterministic behavior, a memory as well as a form to evaluate and refine its rules from its own observation is also required.

[^0]We argue that modeling and simulation of pedestrian flows require tools that enable researchers to get closer to their natural dynamic counterpart: autonomous entities capable of making decisions. We also argue that deliberative agentbased models are not strictly required to achieve the capabilities involved in the decision making process for pedestrian transportation. For us, pedestrian walking behavior results from a set of simple well-defined rules. The previous rules are not fixed rules, but rules accomplished by pedestrians themselves according to the local circumstances.

Various simulation models for describing pedestrian flow in corridors and open spaces have been proposed. Each model deals differently with the complex behavior that is exhibited by pedestrians. Certain experimental observations have guided the development of some models. For instance, pedestrian flow has been compared in certain aspects with gas flow, but as long as pedestrians are self-propelled entities, specific maneuvers for avoidance and deceleration are considered in such models [4]. In this regard the social forces model should be mentioned [5] where pedestrians behave as particles subject to environmental influences in a stimulus-reaction process. It is worth mentioning that in these models pedestrians are unable to perceive their environment, and neither can they apply flexible behavioral strategies. Instead of that pedestrians have equal capabilities, and they are constrained to decelerate as a result of interactions with other pedestrians and with the borders of the free access area. The unique strategy that is applied by pedestrians is a tendency to reach certain positions of their environment and exit points. The functions that describe the temporal change of pedestrians are modeled as mechanical Langevin forces called behavioral forces [6].

The characterization of pedestrian flow in corridors depends on understanding the emergence of spatially distributed patterns. Such an understanding can be useful to predict the behavior of pedestrian flow under various conditions of density and size of free space areas. Moving crowds are considered to be a class of many body systems consisting of strongly interacting elements. Certain self-organizing spatio-temporal patterns resulting from non-linear interactions among pedestrians have been determined from the simulation of pedestrian counterflow. These patterns are related to the phenomenon of spontaneous symmetry-breaking, and with ordered phase transitions, particularly exhibited in paths formed by pedestrians walking in the same direction. These patterns change their appearance namely at a certain critical density value [7].

Other social force-based models have been presented. These include models that consider that interactions among pedestrians do not rely exclusively on their relative speed, but also on the spatial separation among them. The latter parameter produces a kind of centrifugal social force [8] that results in a simplified description of spatial patterns in crowds, in which, under certain critical densities, semi-circular jam formations are observed. The social force-based model has also been successfully applied to model intersecting pedestrian streams [9] in order to improve access facilities of real pedestrians. In more recent variations, a surrounding area that protects pedestrians from contact with other pedestrians, a "respect" mechanism, has been incorporated into the social force model [10].

A multiagent cellular automata model has also been proposed. This model is oriented to visualize pedestrian activity within urban spaces [11]. In this work, pedestrians are simulated as deliberative agents. They have cognitive skills to make decisions, and are also capable of pursuing particular goals of their own "agenda", such as stop and browse store windows, achieve particular subgoals, and reach specific exits. The environment is represented as a 3D cellular automata (CA), in which pedestrian movements as well as interesting points are registered. This representation is globally updated and it is accessible to pedestrians at specific iterations of decisions. Finally, a state combining pedestrian state and the state of CA is shown in a virtual reality interface. In this model, agents are capable of perceiving their environment locally and of updating their own state, whereas the state of the crowd is updated globally. As a remark, the latter is opposed to the sequential update of state, that is apparently more convenient to model observed interactions among real pedestrians [12].

Another interesting CA-based model combines lattice gas dynamics with a situated agent scheme. In this model, collective flow patterns emerge from basic pedestrian behaviors through a self-organizing process [13]. The agents perform three basic behaviors: walk, avoid and surround. The walking direction of pedestrians is influenced by weight functions associated with each behavior. In this work, the global state of pedestrians is updated sequentially, and different walking speeds and periodic boundary conditions are considered. The results reported in this work include the emergence of different collective patterns such as paths, and static jammed traffic distributions. The authors found that mean flow increases with an increase of walking velocity, and that phase transition happens almost at the same critical density.

More recently, a CA-based simulation model of pedestrian counterflow in a corridor has been presented. In this model, a speed for self-propulsion and basic behaviors are considered, as well as innovative pedestrian skills such as local perception and decision making skills [12]. Each pedestrian is a cell in the CA that has a flexible radius of interaction according to the corridor size, that enables an improvement of the pedestrians walking performance as the density is increased. In effect, it seems that increasing density has an effect in general pedestrian behavior. Thus, a gradual change from a situated towards a deliberative dynamics is necessary in order to deal with an increased frequency of multiple interactions, and with the eventual traffic jams. In this work, a competitive approach is applied. As long as pedestrians are able to widen their perceptual field, and they are able to move at different speeds, they are capable of reaching free space areas. As a result, pedestrian flow is efficient, and the number of traffic conflicts decreases. However, traffic jam probability is always present without regard to the corridor dimensions, namely whenever wide perceptual fields are used by pedestrians.

Experimental studies of the pedestrian dynamics in controlled conditions have also been performed [14-17]. In these works, the dynamics of real pedestrians is characterized by analyzing individual walking paths in different scenarios.

The literature concerning pedestrian models is quite extensive. Providing an exhaustive review of these models is beyond the reach of this article. For detailed surveys of both, the theoretical literature and the empirical aspects of pedestrian dynamics, see Refs. [9,18].

In contrast to pedestrian models previously cited, in this work a situated agent-based kinetic model is proposed in order to describe the pedestrian flow in a corridor. Our approach imports ideas from multi-agent systems to model self-propelled entities capable of making decisions about their own movement parameters, such as walking orientation and walking speed, taking into account certain local conditions in a corridor.

The subjacent idea of this work is to model pedestrians as autonomous agents that exhibit a more realistic behavior, in comparison with works where pedestrians are modeled as particles animated by generic motor principles. In the latter, pedestrians are deprived of autonomy, the competence of deciding how to move. Different approaches have been proposed to model autonomous agents. In this work, the situated agent approach is applied because it makes use of less expensive resources than deliberative agent approach.

Autonomy provides a platform to model specialized subgroups of pedestrian with particular features such as, for instance, pedestrian with multiple walking strategies, and fixed and mobile attractive/repulsive elements within the environment. The tendency to differentiate both individual pedestrians and specific elements of the environment has also been mentioned in previous works Refs. $[9,10]$.

In our model, local rules intended to solve collision conflicts are defined as a process of searching and competing for free space areas, that is combined with geometric criteria of walking comfort and a strategy to reach a goal position. As each pedestrian has a perception function that enables him to sense its surrounding environment, a combined model is required. On one hand, our pedestrians are self-propelled particles in a kinetic scheme, and on the other hand pedestrians are rulebased situated agents. Even though pedestrians have entry points and exit points in the corridor, their paths are generated according to the local circumstances of interaction. In this simulation model, pedestrians wander along the corridor and slightly modify their paths in the free space; pedestrians solve their collision conflicts by applying geometric criteria for avoidance.

## 2. Pedestrian model

This section describes our situated agent-based pedestrian model. Pedestrians are characterized by the following kinematic variables: location, walking speed, and orientation. A local perceptual field enables pedestrians to evaluate surrounding conditions. A specific reference direction is used to reach an exit point for each pedestrian. All these variables are iteratively updated during the simulation, according to the local conditions of pedestrians. Two remarkable aspects are considered in this model:

1. A kinetic model of acceleration defined through a transition function of kinematic states. The acceleration has two main contributions, on one hand a contribution generated by the decision function depending on the local density of pedestrians, and on the other hand a contribution of the acceleration towards the exit point, characterized by the unitary vector $v_{T}(t)$.
2. A situated agent based model, including a localized strategy, to solve collision conflicts by applying a well-defined set of walking rules. These rules update the values of the microscopic state variables.
The microscopic pedestrian state is defined as a tuple:

$$
\begin{equation*}
P_{\alpha}(t)=(\mathbf{x}, \mathbf{v}, u ; t)_{\alpha} \quad \alpha=1,2,3, \ldots, N . \tag{1}
\end{equation*}
$$

Let $N$ be the total pedestrian number within the corridor. Each pedestrian has information about its position $\mathbf{x}_{\alpha}=(r, \theta)_{\alpha}$ representing its location in the corridor. The walking speed is denoted by $\mathbf{v}_{\alpha}=(v, \omega)_{\alpha}$, that defines pedestrian orientation with respect to the borders. The unitary vector called heading is denoted by $\mathbf{h}_{\alpha}=\mathbf{v}_{\alpha}\left\|\mathbf{v}_{\alpha}\right\|^{-1}$, and is used to calculate the pedestrian orientation.

The activity function $u_{\alpha}$ represents the global average of pedestrians perceived by one pedestrian, that is,

$$
\begin{equation*}
u_{\alpha}=\frac{1}{\mathbb{N}} \sum_{\beta=1}^{\mathbb{N}} Z_{p}^{*}(\mathbf{x}, \mathbf{h})_{\alpha} \delta(t-\beta \tau) \tag{2}
\end{equation*}
$$

where $\mathbb{N}$ is the partial number of simulation iterations with time interval $\tau . Z_{p}^{*}(\mathbf{x}, \mathbf{h})_{\alpha}$ is the perception function $Z_{p}(\mathbf{x}, \mathbf{h})_{\alpha}$ constrained to pedestrians encounters. The latter is defined as,

$$
\begin{equation*}
Z_{p}(\mathbf{x}, \mathbf{h})_{\alpha}=\left[\sum_{\mu_{\alpha^{\prime}} \neq 0} \mu(\mathbf{x}, \mathbf{h})_{\alpha^{\prime}}+\sum_{\mu_{w} \neq 0} \mu(\mathbf{x}, \mathbf{b})_{w}\right]_{p} \tag{3}
\end{equation*}
$$

where the index $p$ is the frontal perceptual field of pedestrians (see Fig. 1) and $\mathbf{b}$ is an unitary vector parallel with the borders. The occupation function $\mu$ is 0 whether $p$ is empty, and 1 whether $p$ is occupied by another pedestrian or by the borders. In the latter case, $P_{\alpha}(t)$ perceives the location and orientation parameters from $P_{\alpha^{\prime}}(t)$, that are necessary for making walking decisions.

The unitary vector of orientation $\mathbf{h}_{\alpha}$ defines the walking direction and the relative orientation among pedestrians. The parameters walking speed and walking turn take values iteratively according to the state of the perceptual field. The sum in Eq. (3) has normally a unique value that enables pedestrians to update its kinetic state $(\mathbf{x}, \mathbf{v})_{\alpha}$ at time $t+\tau$.


Fig. 1. Perceptual field of pedestrians.


Fig. 2. Composition of a pedestrian (left) and of a crowd (right).
The crowd is represented as a "supertuple" $C(t)$. It contains the total of pedestrians, the decision and execution functions, $\varphi(t)$ and $A_{i}^{j}(t)$ respectively, that is,

$$
\begin{equation*}
C(t)=\left(P_{\alpha}, \varphi, A_{i}^{j} ; t\right) . \tag{4}
\end{equation*}
$$

The decision function $\varphi(t)$ represents socially accepted walking rules. These are applied for all agents when walking along the corridor. Even though such rules are individually applied by agents, according to local circumstances, they are equally interpreted by agents because they summarize appropriate actions to generic situations.

The execution function $A_{i}^{j}(t)$ represents the state transition $\left(\mathbf{x}_{i}, \mathbf{v}_{i} ; t\right) \rightarrow\left(\mathbf{x}_{j}, \mathbf{v}_{j} ; t+\tau\right)$, that depends on the previous decision function. The execution function implements pedestrian decisions generated by $\varphi(t)$, by updating the location and walking speed of pedestrians.

It is worth mentioning that pedestrian perception is constrained to a local frontal area, and lateral collisions eventually happen. The so-called pseudo-collisions describe the encounters between two pedestrians where at least one of them is able to perceive the other, and changes its heading as a result. In this process there is no direct exchange of information.

## 3. Agent model

An agent is a software module for solving a simple particular problem, e.g. to cross a corridor. A multi-agent system consists of a number of these modules jointly involved in the process of solving a complex problem.

Our agents are individually modeled as a hierarchy of subsystems. The first level concerns the agent perception. In this level basic agent skills concerning acquisition and processing of data about corridor conditions, including empty or occupied corridors, were defined. An interface for decision-making, a process that is in charge of the subsequent level, was also defined. The second level refers to an agent's actuation. A set of common conventions or so-called walking rules was programmed. These rules are implemented by updating specific state variables in the third level. The agent's composition is illustrated in Fig. 2 and a detailed description is given below.

### 3.1. Level I. Perception

In contrast to other pedestrian models in which a pedestrian takes into account data of the whole corridor to calculate his movements, our pedestrian does not have access to global data, he bases his decisions on neighborhood data only, that is acquired by himself. For that, a perceptual field $Z_{p}$ was defined. The perceptual field consists of three cells in the direction of the pedestrian heading. Pedestrians are capable of perceiving three categories of objects within their perceptual field: free access areas and borders of the corridor, and other pedestrians (see Fig. 1).

The corridor was divided into three strips: two lateral strips and one central strip, as illustrated in Fig. 3. This division into strips is consistent with observations reported in previous work [4], according to which human pedestrians prefer the center of a corridor when they move along it.


Fig. 3. Virtual division of the corridor into two lateral strips and one central strip.

a

b


C

d

Fig. 4. Free walking in free access areas.
The expected result of the division of the corridor into strips was to enable pedestrians to differentiate areas of the corridor in order to take into account in their decisions, preferences and strategies of walking (see Section 3.2 for a detailed description). It is worth mentioning that the division of the corridor is virtual, and that any explicit landmark is used for this purpose in the environment.

The state of his perceptual field is continuously evaluated by a pedestrian. The content of the perceptual field is progressively examined, starting with the nearest cell until the furthest cell. If a cell of his perceptual field is occupied, by a border of the corridor or by another pedestrian, a pedestrian no longer evaluates the rest of the cells of his perceptual field.

In addition to the content of his perceptual field, a pedestrian perceives the strip of the corridor where he is located, and whether or not he is within the frontiers of the corridor. This information is perceived by pedestrians from their coordinates. Thus, the location of a pedestrian within the corridor $\mathbf{x}_{\alpha}$ and his walking speed $\mathbf{v}_{\alpha}$ are determined by himself locally.

The actions performed by pedestrians are selected by considering local information of the corridor through a decision process that is continually repeated. This process can be summarized in this sequence: perceive - decide - turn at time $t$, then perceive - decide - move forward at time $t+\tau$. For that, a sequential self-assignment of new kinematic states of pedestrians is required.

The decision process, that is repeated by each pedestrian every step $\tau$, is described below.

1. Perceive: Evaluate $Z_{p}(\mathbf{x}, \mathbf{h} ; t)_{\alpha}$ to get $\mu_{\alpha^{\prime}} \neq 0$ or $\mu_{w} \neq 0$.
2. Decide turn size: Determine $\varphi(t)$ to generate the value of the turn $\omega_{j}(t)$ by applying the walking rules.
3. Turn: Apply $\omega_{i}(t) \rightarrow \omega_{j}(t)$ by using $\left(A_{i}^{j}, \boldsymbol{\varphi} ; t\right)$.
4. Perceive: Evaluate $Z_{p}(\mathbf{x}, \mathbf{h} ; t+\tau)_{\alpha}$ to get $\mu_{\alpha^{\prime}} \neq 0$ or $\mu_{w} \neq 0$.
5. Decide step value: Determine $\varphi(t+\tau)$ and generate the transition value $v_{j}(t+\tau)$ by applying the walking rules.
6. Move forward: Apply $v_{i}(t+\tau) \rightarrow v_{j}(t+\tau)$ by using $\left(A_{i}^{j}, \boldsymbol{\varphi} ; t+\tau\right)$.

Note that the process of updating the pedestrian position is generated by the transition $\mathbf{x}_{i} \rightarrow \mathbf{x}_{j}=\mathbf{x}_{i}+\mathbf{v}_{i} \tau$.

### 3.2. Level II. Walking rules

The walking rules are socially accepted conventions that are applied by pedestrians in two general cases. On one hand, they apply these rules to move in both, free access areas and areas with obstacles. And on the other hand, they apply these rules to solve pseudo-collisions encountered while walking. A pseudo-collision refers, in the context of this work, an encounter between two pedestrians in which at least one of them perceives the other, and he acts in consequence by applying an avoidance strategy.

### 3.2.1. Free walking

Free walking rules are applied in cases where pedestrians do not perceive other pedestrians. Six cases are identified, four of them comprise situations where pedestrians do not perceive obstacles, and two of them comprise situations where they do perceive obstacles. These cases are illustrated in Figs. 4 and 5.

Pedestrians that do not perceive obstacles and do not perceive other pedestrians either stay aligned with the borders of the corridor, as illustrated in Fig. 4(a). Occasionally, pedestrians turn towards a specific exit point. In effect, each pedestrian has a "favorite" but not mandatory exit point that he tries to reach while walking along the corridor, see Fig. 4(b). Eventually, pedestrians are willing to break the border-alignment, and they turn randomly by doing a so-called comfort turn, as illustrated in Fig. 4(c). When pedestrians perceive they are walking in the lateral strips of the corridor, they turn towards the central strip, see Fig. 4(d).


Fig. 5. Free walking in areas with obstacles.


Fig. 6. Solving pseudo-collisions in the central strip of the corridor.
When walking in lateral strips, pedestrians can perceive the borders of the corridor as obstacles. In these cases, illustrated in Fig. 5(a) and (b), they turn to be aligned with respect to the borders of the corridor.

### 3.2.2. Pseudo-collisions

Even though sporadic collisions are allowed and solved in our model as hard-sphere collisions, the general case is that pedestrians look for free-access areas from the conditions perceived within their perceptual field. To solve pseudo-collisions, the encounter between two pedestrians where at least one of them perceives the other, an avoidance strategy is required. This strategy is based on a change of the degree of a turn, and the size of the walking step.

The geometric model of activity is very important in solving traffic conflicts or pseudo-collisions among pedestrians. From there, criteria to solve different situations produced during pedestrian encounters can be defined. The general rule is the alignment and the alignment breaking of walking orientations: if a pedestrian perceives another pedestrian who is non-aligned with respect to him, he aligns himself with the other, whereas if they are aligned both of them decide to turn to break their alignment.

Pedestrians are able to identify pedestrians in counter-flow, that satisfy the condition $\mathbf{h}_{\alpha} \cdot \mathbf{h}_{\alpha^{\prime}}<0$, from pedestrians in flow, that satisfy the condition $\mathbf{h}_{\alpha} \cdot \mathbf{h}_{\alpha^{\prime}}>0$. There are also encounters between two pedestrians aligned with each other, in flow or in counter-flow, characterized because the conditions $\mathbf{h}_{\alpha} \cdot \mathbf{h}_{\alpha^{\prime}}= \pm 1$ and ( $\left.\left(\mathbf{x}_{\alpha^{\prime}}-\mathbf{x}_{\alpha}\right) /\left\|\mathbf{x}_{\alpha^{\prime}}-\mathbf{x}_{\alpha}\right\|\right) \cdot \mathbf{h}_{\alpha^{\prime}}=1$, are verified. In the rest of pedestrian encounters the condition $\mathbf{h}_{\alpha} \cdot \mathbf{h}_{\alpha^{\prime}} \neq 1$ is verified. As explained below, the traffic conflict resolution depends on the specific patterns produced during pedestrian encounters.

A classification of the most common pedestrian encounters was done. This classification comprises encounters of pedestrians walking in flow and in counter-flow, and pedestrian encounters within lateral and central strips.

While in the central strip, three pseudo-collisions can be produced among pedestrians walking in counter-flow. The first case, illustrated in Fig. 6(a), is seen at any density. This case is solved by a mutual turn of both pedestrians, who turn in opposite directions. In the second case, illustrated in Fig. 6(b), pedestrian $P_{\alpha}(t)$ reacts to the situation by turning to align itself with $-\mathbf{h}_{\alpha^{\prime}}$. This turn is defined by $\theta=\cos ^{-1}\left(\mathbf{h}_{\alpha} \cdot \mathbf{h}_{\alpha^{\prime}}\right)$. The third case, illustrated in Fig. 6(c), is solved by a turn of both pedestrians in order to align themselves with the borders of the corridor.

There are three pseudo-collisions among pedestrians walking in flow in the central strip. These cases are solved by the pedestrian who perceives another one in front of him. The first case, illustrated in Fig. 6(d), is solved by a random turn in order to break the alignment with the "leader". In contrast, in the second case illustrated in Fig. 6(e), the "follower" tries to align itself with respect to $\mathbf{h}_{\alpha^{\prime}}$. In the latter case, the "follower" aligns itself with the borders of the corridor, see Fig. 6(f).

While in the lateral strips, pedestrian encounters are produced in the proximity of the borders. The first three cases concern pedestrian encounters in counter-flow. The first and second cases, illustrated in Fig. 7(a) and (b), are solved by applying the special rule called to give way according to which the first pedestrian who perceives the situation stops. The third case is solved by a turn to break the alignment, see Fig. 7(c). These rules are intended to promote a wall-following tendency of pedestrians.


Fig. 7. Solving pseudo-collisions in the lateral strips of the corridor.
The last three cases happen while pedestrians walk in flow in the lateral strips. Thus, these cases are solved by the pedestrian who perceives the situation. In the first case, illustrated in Fig. 7(d), the pseudo-collision is solved by a turn towards the center of the corridor. In the second case, illustrated in Fig. 7(e), the "follower" stops and gives the other the right of way. In the latter case, illustrated in Fig. 7(f), the "follower" aligns itself with respect to the borders of the corridor.

It is worth mentioning that previous conventions capture the essence of general known cases, identified in the observed social behavior of pedestrians [7,9]. Different conventions such as, for instance, a wall-following preference instead of the actual tendency of our pedestrians for walking along the central strip of the corridor can be implemented to solve collision conflicts under specific high-density conditions.

### 3.3. Level III. Implementation

### 3.3.1. State-variable domains

The domains of variation of the state variables are such that the location $\mathbf{x} \in D_{\mathbf{x}}=D_{r} \times D_{\theta}=\cup_{S} D_{S}$ there can be on the strips $S=$ top, center, bottom of the corridor. The speed $\mathbf{v} \in D_{\mathbf{v}}=D_{v} \times D_{\omega}$, being $\omega=\frac{\theta}{\tau}$ the changes of orientation of the walking speed are defined in terms of the simulation time $\tau$. The domain of the step speed is separated in low and high in such a way that $D_{v}=D_{v(\text { low })} \cup D_{v(\text { high })}$, where $D_{v(\text { low })}=\{0.0,0.5\}$ and $D_{v(h i g h)}=\{1.0,2.0\}$. These values are normalized with respect to $v_{0}=1.34 \mathrm{~m} / \mathrm{s}$, the average walking speed, a known value of the free speed of pedestrians with different purposes of walking $[14,15]$.

The orientation $\omega$ takes values in a low and high domain, so that $D_{\omega}=D_{\omega(l o w)} \cup D_{\omega(h i g h)}$ as indicated in expressions 5 and 6. For convenience we define the domain $D_{\omega(\text { medium })}$ as only the positive part of the interval of variation of the low domain $D_{\omega(l o w)}$, in order to consider the deviations that happen to pedestrians nearing the edges of the corridor. A threshold of $\frac{\pi}{4}$ was considered for all domains of $\omega$ for simulation purposes.

$$
\begin{align*}
& D_{\omega(l o w)}=\left\{\omega=\frac{\theta(u)}{\tau} \left\lvert\, \theta(u)=u \frac{\pi}{4}-(1-u) \frac{\pi}{4}\right., u \in \operatorname{Rand}[0,1)\right\}  \tag{5}\\
& D_{\omega(h i g h)}=\left\{\left.\omega=\frac{\theta}{\tau} \right\rvert\, \theta=\text { Rand }\left\{-\frac{\pi}{4}, \frac{\pi}{4}\right\}\right\} \tag{6}
\end{align*}
$$

### 3.3.2. Assignment of values of turning and advance

The specific rules of assignment by the functions of decision and execution ( $\varphi, A_{i}^{j} ; t$ ), that correspond to the process generated by the set of rules of the agents, detailed in the previous section are:

1. If $\mathbf{x}_{i} \in D_{S(\text { top })} \cup D_{S(\text { bottom })}$ and $\mu_{\alpha^{\prime}}=\mu_{w}=0$ for all $\alpha^{\prime}$ at time $t$, then $\varphi(t): \omega_{j} \in D_{\omega(\text { medium })}$ towards the central strip, and the transition $\left(\varphi, A_{i}^{j} ; t\right): \omega_{i}(t) \rightarrow \omega_{j}(t)$ is performed. If now $\mu_{\alpha^{\prime}}=\mu_{w}=0$ for all $\alpha^{\prime}$ at time $t+\tau$, then $\varphi(t+\tau): v_{j} \in D_{v(h i g h)}$, whereas if $\mu_{\alpha^{\prime}}=1, \mu_{w}=0$ or $\mu_{\alpha^{\prime}}=0, \mu_{w}=1$ then $\varphi(t+\tau): v_{j} \in D_{v(l o w)}$, and the transition $\left(\varphi, A_{i}^{j} ; t+\tau\right): v_{i} \rightarrow v_{j}, \mathbf{x}_{i} \rightarrow \mathbf{x}_{j}$ is performed.
2. If $\mathbf{x}_{i} \in D_{S(\text { top })} \cup D_{S(\text { bottom })}$ and $\mu_{\alpha^{\prime}}=1, \mu_{w}=0$ or $\mu_{\alpha^{\prime}}=0, \mu_{w}=1$ for some $\alpha^{\prime}$ at time $t$, then $\boldsymbol{\varphi}(t): \omega_{j} \in D_{\omega(\text { medium })}$ towards the central strip, and the transition $\left(\varphi, A_{i}^{j} ; t\right): \omega_{i}(t) \rightarrow \omega_{j}(t)$ is performed. If now $\mu_{\alpha^{\prime}}=\mu_{w}=0$ for all $\alpha^{\prime}$ at time $t+\tau$, then $\varphi(t+\tau): v_{j} \in D_{v(h i g h)}$, whereas if $\mu_{\alpha^{\prime}}=1, \mu_{w}=0$ or $\mu_{\alpha^{\prime}}=0, \mu_{w}=1$ then $\varphi(t+\tau): v_{j} \in D_{v(l o w)}$, and the transition $\left(\varphi, A_{i}^{j} ; t+\tau\right): v_{i} \rightarrow v_{j}, \mathbf{x}_{i} \rightarrow \mathbf{x}_{j}$ is performed.
3. If $\mathbf{x}_{i} \in D_{S(c e n t e r)}$ and $\mu_{\alpha^{\prime}}=\mu_{w}=0$ for all $\alpha^{\prime}$ at time $t$, then $\varphi(t): \omega_{j} \in D_{\omega(l o w)}$ towards the goal position or towards a random orientation determined by the comfort turn, the transition $\left(\varphi, A_{i}^{j} ; t\right): \omega_{i}(t) \rightarrow \omega_{j}(t)$ is performed. If now $\mu_{\alpha^{\prime}}=\mu_{w}=0$ for all $\alpha^{\prime}$ at time $t+\tau$, then $\varphi(t+\tau): v_{j} \in D_{v(h i g h)}$, whereas if $\mu_{\alpha^{\prime}}=1, \mu_{w}=0$ or $\mu_{\alpha^{\prime}}=0, \mu_{w}=1$ then $\varphi(t+\tau): v_{j} \in D_{v(l o w)}$, and the transition $\left(\varphi, A_{i}^{j} ; t+\tau\right): v_{i} \rightarrow v_{j}, \mathbf{x}_{i} \rightarrow \mathbf{x}_{j}$ is performed.
4. If $\mathbf{x}_{i} \in D_{S(\text { center })}$ and $\mu_{\alpha^{\prime}}=1, \mu_{w}=0$ for some $\alpha^{\prime}$ at time $t$, then $\varphi(t): \omega_{j} \in D_{\omega(h i g h)}$ towards a random orientation, or if $\mu_{\alpha^{\prime}}=0, \mu_{w}=1$ then $\varphi(t): \omega_{j} \in D_{\omega(l o w)}$ to be aligned respect to the border, and the transition $\left(\varphi, A_{i}^{j} ; t\right): \omega_{i}(t) \rightarrow \omega_{j}(t)$ is performed. If now $\mu_{\alpha^{\prime}}=\mu_{w}=0$ for all $\alpha^{\prime}$ at time $t+\tau$, then $\varphi(t+\tau): v_{j} \in D_{v(h i g h)}$, whereas if $\mu_{\alpha^{\prime}}=1, \mu_{w}=0$ or $\mu_{\alpha^{\prime}}=0, \mu_{w}=1$ then $\varphi(t+\tau): v_{j} \in D_{v(l o w)}$, the transition $\left(\varphi, A_{i}^{j} ; t+\tau\right): v_{i} \rightarrow v_{j}, \mathbf{x}_{i} \rightarrow \mathbf{x}_{j}$ is performed.

## 4. Balance equations of pedestrians

The kinetic description of pedestrian flow in a corridor is represented as a function of probability of a particle, such that $\eta(\mathbf{v}, t)=\eta(\mathbf{x}, \mathbf{v}, u ; t)=\eta(\mathbf{x}, \mathbf{v} ; t) \mathbb{P}(u)$ denotes the average number of pedestrians occupation around ( $\mathbf{x}, \mathbf{v}$ ), with activity function $u$ at time $t$ [19-21].

The changes of the density of probability are given as a balance between pedestrians entering and leaving the volume of the cell of observation in the corridor, this is,

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{~d} t} \eta(\mathbf{v}, t)=\left(\frac{\mathrm{d}}{\mathrm{~d} t} \eta(\mathbf{v}, t)\right)_{+}-\left(\frac{\mathrm{d}}{\mathrm{~d} t} \eta(\mathbf{v}, t)\right)_{-} . \tag{7}
\end{equation*}
$$

Each of the terms on the rhs of Eq. (7) can be specified by considering that the agent state variables transition of the reference pedestrian, $P_{\alpha}(t)=\left(\mathbf{x}_{\mathbf{i}}, \mathbf{v}_{\mathbf{i}}, u_{i} ; t\right)_{\alpha} \rightarrow P_{\alpha}(t+\tau)=\left(\mathbf{x}_{j}, \mathbf{v} \mathbf{x}_{j}, u_{j} ; t+\tau\right)_{\alpha}$ is a consequence of colliding with $P_{\alpha^{\prime}}(t)=\left(\mathbf{x}_{\mathbf{k}}^{\prime}, \mathbf{v}_{\mathbf{k}}^{\prime}, u_{k}^{\prime} ; t\right)_{\alpha^{\prime}}$. In addition, the following notation can be used for the distribution function $\eta\left(\mathbf{v}_{i}, t\right)=\left(\mathbf{x}_{\mathbf{i}}, \mathbf{v}_{\mathbf{i}}, u_{i} ; t\right)$ and $\eta\left(\mathbf{v}_{k}^{\prime}, t\right)=\left(\mathbf{x}_{\mathbf{k}}^{\prime}, \mathbf{v}_{\mathbf{k}}^{\prime}, u_{k} ; t\right)$ of the pedestrian $P_{\alpha}(t)$ and $P_{\alpha^{\prime}}(t)$ respectively.

In order to evaluate the number of pedestrians entering $(+)$ to the cell, we consider the product of the frequency of collisions $\gamma\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime}\right)$ multiplied by the probability of occupation $\eta\left(\mathbf{v}_{k}^{\prime}, t\right)$, multiplied by the probability of transition of kinetic states $B\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime} ; \mathbf{v}_{j}\right)$ and by the probability that the arrival position is empty $\left(1-\eta\left(\mathbf{v}_{j}, t\right)\right.$ ), that is:

$$
\begin{equation*}
\left(\frac{\mathrm{d}}{\mathrm{~d} t} \eta(\mathbf{v}, t)\right)_{+}=\sum_{i j k}\left(1-\eta\left(\mathbf{v}_{j}, t\right)\right) \eta\left(\mathbf{v}_{k}^{\prime}, t\right) \gamma\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime}\right) B\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime} ; \mathbf{v}_{j}\right) \delta\left(\mathbf{v}-\mathbf{v}_{j}\right) \tag{8}
\end{equation*}
$$

The values of the walking speed depend on the decision function $\varphi(t)$ applied by pedestrians themselves, through a kinetic interaction of self-propelled particles. In a similar way, it is possible to evaluate the number of pedestrians leaving $(-)$ the volume of observation, that is,

$$
\begin{equation*}
\left(\frac{\mathrm{d}}{\mathrm{~d} t} \eta(\mathbf{v}, t)\right)_{-}=\sum_{i j k}\left(1-\eta\left(\mathbf{v}_{k}^{\prime}, t\right)\right) \eta\left(\mathbf{v}_{j}, t\right) \gamma\left(\mathbf{v}_{j}, \mathbf{v}_{k}^{\prime}\right) B\left(\mathbf{v}_{j}, \mathbf{v}_{k}^{\prime} ; \mathbf{v}_{i}\right) \delta\left(\mathbf{v}-\mathbf{v}_{j}\right) \tag{9}
\end{equation*}
$$

Note that the frequency of pseudo-collisions among pedestrians is calculated by recording the changes of the perceptual field of encountered pedestrians, that is,

$$
\begin{equation*}
\gamma\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime}\right)=\frac{1}{2} \frac{\mathrm{~d}}{\mathrm{~d} t}\left(Z p^{*}\left(\mathbf{x}_{i}, \mathbf{h}_{i} ; t\right)+Z p^{*}\left(\mathbf{x}_{k}^{\prime}, \mathbf{h}_{k}^{\prime} ; t\right)\right) \delta\left(\mathbf{x}_{i}-\mathbf{x}_{k}^{\prime}\right) \tag{10}
\end{equation*}
$$

that measures changes of the perceptual field of pedestrians $P_{\alpha}(t)$ with walking speed $\mathbf{v}_{i}$ because of the presence of $P_{\alpha^{\prime}}(t)$ with walking speed $\mathbf{v}_{k}^{\prime}$.

By substitution of Eqs. (8) and (9) in Eq. (7), the kinetic equation of the crowd is obtained as,

$$
\begin{align*}
\frac{\mathrm{d}}{\mathrm{~d} t} \eta(\mathbf{v}, t)= & \sum_{i j k}\left(1-\eta\left(\mathbf{v}_{j}, t\right)\right) \eta\left(\mathbf{v}_{k}^{\prime}, t\right) \gamma\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime}\right) B\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime} ; \mathbf{v}_{j}\right) \delta\left(\mathbf{v}-\mathbf{v}_{j}\right) \\
& -\sum_{j k} \eta\left(\mathbf{v}_{j}, t\right)\left(1-\eta\left(\mathbf{v}_{k}^{\prime}, t\right)\right) \gamma\left(\mathbf{v}_{j}, \mathbf{v}_{k}^{\prime}\right) \delta\left(\mathbf{v}-\mathbf{v}_{j}\right) \tag{11}
\end{align*}
$$

The state transition is a function of marginal probability of the activity, that is,

$$
\begin{equation*}
B\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime} ; \mathbf{v}_{j}\right)=\int B\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime} ; \mathbf{v}_{j} \mid u_{i}, u_{k}^{\prime}\right) \mathbb{P}\left(u_{i}\right) \mathbb{P}\left(u_{k}^{\prime}\right) \delta\left(u-u_{i}\right) \delta\left(u^{\prime}-u_{k}^{\prime}\right) \mathrm{d} u \mathrm{~d} u^{\prime} \tag{12}
\end{equation*}
$$

Due to the fact that the walking rules are associated with the crowd, they are known by all the pedestrians. The function $B\left(\mathbf{v}_{i}, \mathbf{v}_{k}^{\prime} ; \mathbf{v}_{j}\right)$ summarizes the rules of pedestrians and the criteria for solving pseudo-collisions in the corridor through the application of the function of the crowd $C(t)$.


Fig. 8. Average speed (a) and average activity (b) vs density by varying the range of comfort turn, 4-350 pedestrians, 350 iterations, frequency of comfort turn $1 / 5$.

As it was previously mentioned, the function of transition $A_{i}^{j}$ depends on the activation of the perceptual field $Z_{p}$ of the pedestrian $P_{\alpha}(t)$ and on its orientation to the goal position $\boldsymbol{v}_{T}$. As the direct encounters or pseudo-collisions among pedestrians are not an one-to-one function with respect to the inverse pseudo-collisions, and that encounters among pedestrians concern only the one perceiving the encounter, the transition function does not satisfy any rule of symmetry. Namely, the condition $A_{i}^{j} \neq A_{j}^{i}$ is verified.

Once Eq. (11) is solved for the density of probabilities $\eta(\mathbf{v}, t)$, we can evaluate the quantities of interest that describe the macroscopic conditions of the collective behavior such as, the density,

$$
\begin{equation*}
\langle n(t)\rangle=\int \eta(\mathbf{v}, t) \mathrm{d} \mathbf{x} \mathrm{~d} \mathbf{v} \mathrm{~d} u \tag{13}
\end{equation*}
$$

the rate of flow,

$$
\begin{equation*}
\langle\mathbf{v}(t)\rangle=\int \mathbf{v} \eta(\mathbf{v}, t) \mathrm{d} \mathbf{x} \mathrm{~d} \mathbf{v} \mathrm{~d} u \tag{14}
\end{equation*}
$$

and the mean activity function or activation,

$$
\begin{equation*}
\langle u(t)\rangle=\int u \eta(\mathbf{v}, t) \mathrm{d} \mathbf{x} \mathrm{~d} \mathbf{v} \mathrm{~d} u \tag{15}
\end{equation*}
$$



Fig. 9. Average speed (a) and average activity (b) vs density by varying the frequency of the comfort turn, 4-350 pedestrians, 350 iterations, range of comfort turn $\pm 12.5^{\circ}$.

The results are shown as curves of the average flow of the values in a stationary state. In the phase transition analysis of the crowd, the density is considered as the control variable of the flow speed and average activity of pedestrians.

## 5. Results and discussion

The results of the simulation are presented as the rate of flow and average activity depending on the number density in stationary conditions, and are illustrated with snapshots of the pedestrian distribution captured during the simulation.

The activity represents, as it was previously mentioned, the global rate of pedestrian collisions measured from the variability of the perceptual field of pedestrians. This measure reflects the activity in terms of the application of the walking rules of pedestrians: the more active the perceptual field is, the more the rules are used.

This work is, as far as we know, the first one considering the dynamics of pedestrians in terms of both, the individual activity and the crowd activation. In recent works [19-21], the former has been defined as the microscopic social condition of agents, a measure of the individual dynamics. And the latter has been considered as the macroscopic biological condition of the crowd, a measure of the collective dynamics.

In Fig. 8, the variation of the average rate of flow in terms of the number density of pedestrians is presented. Different values of the interval in the comfort turn are considered in these experiments. The variation of the average activity function, which exhibits an inverse tendency to that of the speed curve, is also presented. In Fig. 9, these variations have been calculated for different values of comfort turn frequency.

(a) Begin of phase I, 12p.

(b) Phase I, 80p.

(c) Transitory state, 150p.

(d) Phase II, 250p.

(e) Enf of phase II, 320p.

Fig. 10. Flow patterns observed in the corridor at different densities, range of comfort turn $\pm 20^{\circ}$.
In Figs. 8 and 9, three regions of flow behavior are presented: the first one for scarce flow, the second one for a transition zone, and the third one for heavy flow. At low densities, as illustrated in Fig. 10(a) and (b), the crowd move uniformly and the avoidance rules enable to solve traffic conflicts, in such a way that, eventually, a pattern consisting of two opposite waving lines emerge. However, some flow patterns where the walking speed decreases may appear in the low density region. These patterns can be explained as an early manifestation of the heavy flow, as illustrated in Fig. 10(c). Accordingly, in the graphs shown in Figs. 8(b) and 9(b), the average activity function increases linearly as the density changes.

Starting from a specific density value, isolated traffic jams may appear as shown in Fig. 10(d) and (e). Note that the free access areas decrease drastically in this region, without regard to the walking rules. This section of the graph is dominated by the heavy flow, but sometimes the flow increases in some points. These patterns can be explained as a late persistence of the scarce flow. Accordingly, the activity function takes high and uniform values. In the images of the corridor frantic activity is observed while pedestrians solve collision conflicts, resulting in the emergence of pedestrians piles.

In the transition zone, illustrated in Fig. 10(c), known in the literature as the zone of phase transition, both phases of the flow cohabit, as illustrated in Fig. 11(a). As the number density increases, the scarce flow alternates with the heavy flow resulting in a decrease in the average pedestrian flow. This pattern can also be explained as a unique state of variation of flow whose amplitude increases considerably in the zone of phase transition, see Fig. 11(b).

The phase transition in Figs. 8 and 9, in analogy to some continuous equilibrium systems, exhibits critical values of the pedestrian density for both, the average velocity and the average activity. These values are necessary for obtaining a power law and the associated critical exponents as shown in Figs. 12 and 13. In these Figures the order parameters obtained by applying logarithm scale transformation near of the critical values were plotted. The critical exponents were obtained from the fit of average speed and average activity data. The former yields a slope of $0.45 \pm 0.2$ for a comfort turn of $12.5^{\circ}$, and a slope of $0.43 \pm 0.15$ for a frequency of comfort turn of $1 / 7$. The second yields a slope of $0.41 \pm 0.12$ for a comfort turn of $12.5^{\circ}$, and a slope of $0.44 \pm 0.10$ for a frequency of comfort turn of $1 / 7$.

The main contribution of this work concerns an analysis of an agent-based model of pedestrians in terms of the kinetic theory of biological systems. In that sense, self-propelled particles capable of perception and decision-making are simulated.


Fig. 11. Two phases of flow in the average speed (a) and flow behavior under fluctuation effects (b), 4-350 pedestrians, 350 iterations, range of comfort turn $\pm 12.5^{\circ}$, frequency of comfort turn $1 / 5$.


Fig. 12. Critical exponents of the average speed (a) and the average activity (b) for comfort turn of $\pm 12.5^{\circ}$, as function of density ( $\rho$ ) close to the critical density $\left(\rho_{c}\right)$.

In contrast to related works we combine a fine-grain model, the situated agent one, where pedestrians are simulated as autonomous entities; and at the same time a high-level descriptive model, the kinetic biological one, to describe pedestrian dynamics.


Fig. 13. Critical exponents of the average speed (a) and the average activity (b) for frequency of comfort turn of $1 / 7$, as function of density ( $\rho$ ) close to the critical density $\left(\rho_{c}\right)$.

Pedestrians in our model are autonomous entities capable of perceiving and making decision instead of being inanimate objects whose motion is entirely governed by external forces. For that, pedestrians of our model apply socially accepted conventions, such as the avoidance rules, as well as individual preferences such as the use of specific exit points, or the execution of eventual comfort turns resulting in spontaneous changes of walking speed.

In that way, a very flexible simulation model is achieved. On one hand pedestrians exhibit goal-oriented behavior, and on the other hand they are susceptible of being customized. Thus, pedestrians can be extended, in the near future, for having specific profiles or walking features, such as, for instance, pedestrians moving in groups or pedestrians willing to break the walking rules.

A global observer is in effect present in our simulation, but its role is to be a data recording agent instead of a controller agent. This agent is in charge of collecting data concerning walking parameters, generated by the decisions of agents.

A second contribution of this work is the consideration of an activity function to measure the collective dynamics. The activity function can also be used to evaluate, for instance, the impact in pedestrian dynamics of different walking strategies, such as the frequency of comfort turns and even, the "loyalty" of pedestrians to socially accepted walking rules.

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