Modulation of coherence and polarization using liquid crystal spatial light modulators

Andrey S. Ostrovsky[1]*, Gabriel Martínez-Niconoff [2], Victor Arrizón [2], Patricia Martínez-Vara[1], Miguel A. Olvera-Santamaría[1], and Carolina Rickenstorff-Parrao[1]

[1]Universidad Autónoma de Puebla, Facultad de Ciencias Físico Matemáticas, Puebla 72000, Mexico
[2]Instituto Nacional de Astrofísica Óptica y Electrónica, Departamento de Óptica, Puebla 72000, Mexico
**Corresponding author: andreyo@fcfm.buap.mx*

Abstract: We propose a method for modulation of coherence and polarization of electromagnetic fields, employing two crossed zero-twisted nematic liquid crystal spatial light modulators. In contrast to a similar technique analyzed by Shirai and Wolf [J. Opt. Soc. Am A, **21**, 1907, (2004)] our method provides a wide range simultaneous modulation of coherence and polarization. The dependence of the obtained results on different definitions of electromagnetic coherence is considered.

OCIS codes: (030.1640) Coherence; (030.6600) Statistical optics; (260.5430) Polarization

References and links

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1. Introduction

When using laser illumination, it is frequently needed to destroy completely or partially the spatial coherence of an optical field. Such a situation arises, for example, in coherent imagery when it is necessary to eliminate undesirable speckle patterns [1] or in the problem of

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generating propagation-invariant fields when it is necessary to create a secondary source with a special structure of spatial coherence [2,3]. The most efficient method to produce the desired changes of coherence is based on the use of a computer controlled liquid-crystal (LC) spatial light modulator (SLM) [4,5]. In Ref. 5 it was shown that using a zero-twisted nematic LC SLM, under certain conditions one can obtain not only the changes of coherence but also the changes of polarization of the incident optical field. This technique can be efficiently employed to modulate coherence or polarization each taken separately. However it can not provide a wide range simultaneous modulation of both. To solve this problem, in the present paper we propose to use two crossed zero-twisted nematic LC SLMs with the different control signals. We also discuss the dependence of the obtained results on different definitions of the electromagnetic coherence. To ensure a thorough coverage of the total scope of the subject, we start the paper with a brief summary of the main results reported in Ref. 5.

2. Statistical properties of the electromagnetic field passed through a zero-twisted nematic LC SLM with random transmittance

We consider a stochastic stationary electromagnetic field propagating within a narrow solid angle around the z axis. According to the coherence theory in the space-frequency domain [6] such a field can be represented, at any typical point $\mathbf{x} = (x, y)$ in some plane $z = \text{const}$ and at any frequency ν, by the statistical ensemble

$$\left\{ \mathbf{E}(\mathbf{x}, \nu) \right\} = \begin{Bmatrix} E_x(\mathbf{x}, \nu) \\ E_y(\mathbf{x}, \nu) \end{Bmatrix}, \tag{1}$$

where E_x and E_y are the Cartesian components of the electric field vector $\mathbf{E}$. For the sake of simplicity from now on we will omit the explicit dependence of the considered quantities on frequency ν. The second order correlation properties of the electromagnetic field can be completely characterized by the so-called cross-spectral density matrix [7,8]

$$\mathbf{W}(\mathbf{x}_1, \mathbf{x}_2) = \begin{bmatrix} \left\langle E_x^*(\mathbf{x}_1) E_x(\mathbf{x}_2) \right\rangle & \left\langle E_x^*(\mathbf{x}_1) E_y(\mathbf{x}_2) \right\rangle \\ \left\langle E_y^*(\mathbf{x}_1) E_x(\mathbf{x}_2) \right\rangle & \left\langle E_y^*(\mathbf{x}_1) E_y(\mathbf{x}_2) \right\rangle \end{bmatrix}, \tag{2}$$

where the asterisk and the angle brackets denote the complex conjugation and the ensemble average, respectively. According to Wolf [7], the degree of coherence and the degree of polarization of the electromagnetic field are defined by the formulas

$$\eta(\mathbf{x}_1, \mathbf{x}_2) = \frac{\operatorname{Tr} \mathbf{W}(\mathbf{x}_1, \mathbf{x}_2)}{[\operatorname{Tr} \mathbf{W}(\mathbf{x}_1, \mathbf{x}_1) \operatorname{Tr} \mathbf{W}(\mathbf{x}_2, \mathbf{x}_2)]^{1/2}}, \tag{3}$$

$$P(\mathbf{x}) = \left(1 - \frac{4 \operatorname{Det} \mathbf{W}(\mathbf{x}, \mathbf{x})}{[\operatorname{Tr} \mathbf{W}(\mathbf{x}, \mathbf{x})]^2} \right)^{1/2}, \tag{4}$$

respectively, where Tr stands for the trace and Det denotes the determinant.

Now we consider the propagation of the electromagnetic field through a polarization-dependent device with a random transmittance given by the Jones matrix $\mathbf{T}(\mathbf{x})$. Let $\mathbf{W}_{\text{in}}(\mathbf{x}_1, \mathbf{x}_2)$ be the cross-spectral density matrix of the incident field. Then the cross-spectral density matrix of the transmitted field is given by the expression

$$\mathbf{W}_{\text{out}}(\mathbf{x}_1, \mathbf{x}_2) = \left\langle \mathbf{T}^{\dagger}(\mathbf{x}_1) \mathbf{W}_{\text{in}}(\mathbf{x}_1, \mathbf{x}_2) \mathbf{T}(\mathbf{x}_2) \right\rangle, \tag{5}$$

where the dagger denotes the Hermitian conjugation. Hence, taking into account the definitions given by Eqs. (3) and (4), one can conclude that, in the general case, the degree of coherence and the degree of polarization change on propagation through a polarization-

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dependent device with a random transmittance. To evaluate these changes, one has to specify the form of the incident electromagnetic field and the type of the polarization-dependent device. Following Ref. 5, as an incident field we will consider the linearly polarized spatially coherent Gaussian field given by the cross-spectral density

$$\mathbf{W}_{\text{in}}(\mathbf{x}_1,\mathbf{x}_2)=E_0^2\exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{4\varepsilon^2}\right)\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}, \tag{6}$$

where E_0 and ε are positive constants, and θ is the angle that the direction of polarization makes with the x axis. It can be readily verified that for such a field $\eta_{\text{in}}(\mathbf{x}_1,\mathbf{x}_2)=1$ and $P_{\text{in}}(\mathbf{x})=1$. As a polarization-dependent device in Ref. 5 it was employed a zero-twisted (or parallel-aligned) nematic LC SLM with the extraordinary axis aligned along the y direction. The transmittance of such a modulator is given by the matrix

$$\mathbf{T}_1(\mathbf{x})=\begin{bmatrix}1 & 0\\ 0 & \exp[\mathrm{i}\beta_1(\mathbf{x})]\end{bmatrix}, \tag{7}$$

where

$$\beta_1=\frac{\pi d}{\lambda}(n_{\text{e}}-n_{\text{o}}) \tag{8}$$

is the so-called birefringence, n_{o} and n_{e} are ordinary and extraordinary indices of refraction, respectively, λ is the wavelength, and d is the thickness of the crystal. It is assumed that the birefringence has the form

$$\beta_1(\mathbf{x})=\varphi_0+\varphi(\mathbf{x}), \tag{9}$$

where φ_0 is a constant which is much lager than $\pi/2$ and $\varphi(\mathbf{x})$ is a zero mean random variable which is characterized by the Gaussian probability distribution density

$$p[\varphi(\mathbf{x})]=\frac{1}{\sqrt{2\pi}\sigma_\varphi}\exp\left(-\frac{\varphi^2(\mathbf{x})}{2\sigma_\varphi^2}\right), \tag{10}$$

with variance $\left\langle\varphi^2(\mathbf{x})\right\rangle=\sigma_\varphi^2$ and cross-correlation defined for two different points as

$$\left\langle\varphi(\mathbf{x}_1)\,\varphi(\mathbf{x}_2)\right\rangle=\sigma_\varphi^2\exp\left(-\frac{\xi^2}{2\gamma^2}\right), \tag{11}$$

where $\xi=\left|\mathbf{x}_1-\mathbf{x}_2\right|$ and γ is a positive constant characterizing the correlation width of function $\varphi(\mathbf{x})$.

By substituting Eqs. (6) – (9) into Eq. (5) and making use of Eqs. (A3) and (A8) from Appendix, one obtains

#106643 - $15.00 USD Received 21 Jan 2009; revised 22 Feb 2009; accepted 12 Mar 2009; published 18 Mar 2009

$$\mathbf{W}_{\text{out}}(\mathbf{x}_1,\mathbf{x}_2) = E_0^2 \exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{4\varepsilon^2}\right) \times \begin{bmatrix} \cos^2\theta & \exp(i\varphi_0)\exp\left(-\frac{\sigma_\varphi^2}{2}\right)\sin\theta\cos\theta \\ \exp(-i\varphi_0)\exp\left(-\frac{\sigma_\varphi^2}{2}\right)\sin\theta\cos\theta & \exp\left\{-\sigma_\varphi^2\left[1-\exp\left(-\frac{\xi^2}{2\gamma^2}\right)\right]\right\}\sin^2\theta \end{bmatrix}. \quad (12)$$

Then, substituting Eq. (12) into Eqs. (3) and (4), one finds

$$\eta_{\text{out}}(\xi) = \cos^2\theta + \exp\left\{-\sigma_\varphi^2\left[1-\exp\left(-\frac{\xi^2}{2\gamma^2}\right)\right]\right\}\sin^2\theta, \quad (13)$$

$$P_{\text{out}}(\mathbf{x}) = \left\{1-[1-\exp(-\sigma_\varphi^2)]\sin^2 2\theta\right\}^{1/2}. \quad (14)$$

To provide a physical insight into these results, let us discuss two special cases considered in Ref. 5, namely $\theta = 0$ and $\theta = \pi/2$. In the first case $\eta_{\text{out}} = 1$ and $P_{\text{out}} = 1$, i.e. no modulation of coherence or polarization occur. In the second case

$$\eta_{\text{out}}(\xi) = \exp\left\{-\sigma_\varphi^2\left[1-\exp\left(-\frac{\xi^2}{2\gamma^2}\right)\right]\right\}, \quad (15)$$

and $P_{\text{out}} = 1$. This time, the LC SLM provides the modulation of coherence without changing the polarization of the incident field. In the intermediate case, i.e., when $0 < \theta < \pi/2$, the spectral degree of coherence and the spectral degree of polarization of the field transmitted through the zero-twisted LC SLM decrease simultaneously in accordance with Eqs. (13) and (14). In this case the particular choice $\theta = \pi/4$ provides the minimum possible value of the spectral degree of polarization, namely $\exp(-\sigma_\varphi^2/2)$, and the degree of coherence

$$\eta_{\text{out}}(\xi) = \frac{1}{2} + \frac{1}{2}\exp\left\{-\sigma_\varphi^2\left[1-\exp\left(-\frac{\xi^2}{2\gamma^2}\right)\right]\right\}. \quad (16)$$

The degree of coherence given by Eq. (16) is illustrated in Fig. 1 for several values of parameter γ. The width of function $\eta_{\text{out}}(\xi)$, which according to Ref. 1 is evaluated by

$$\Delta\eta_{\text{out}} = \int_0^\infty \eta_{\text{out}}^2(\xi)\,d\xi, \quad (17)$$

may be associated with the so-called transverse coherence length of the field [6]. As it is obvious from Fig. 1, in the considered case $\Delta\eta_{\text{out}} = \infty$ for any value of parameter γ, i.e. the transmitted field is practically coherent. Therefore, we can conclude that the technique analyzed in Ref. 5 does not provide the appropriate simultaneous modulation of coherence and polarization.

#106643 - $15.00 USD Received 21 Jan 2009; revised 22 Feb 2009; accepted 12 Mar 2009; published 18 Mar 2009

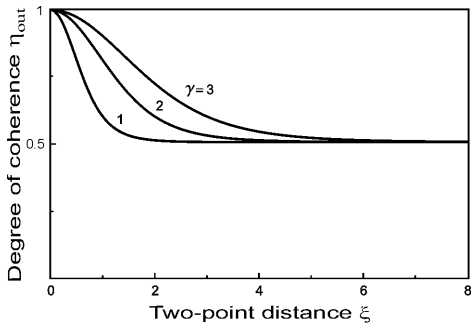

Fig. 1. Degree of coherence given by Eq. (16), $\sigma_\varphi = 2$ and $\gamma = 1,2,3$.

3. Statistical properties of the electromagnetic field passed through two crossed zero-twisted nematic LC SLMs with random transmittance

To attain a wide range simultaneous modulation of coherence and polarization, we will place at the output of the first LC SLM, considered in the previous section, a second SLM (identical to the first one) with the extraordinary axis aligned along the *x* direction. The transmission matrix of the second SLM is

$$\mathbf{T}_2(\mathbf{x}) = \begin{bmatrix} \exp[\,\mathrm{i}\beta_2(\mathbf{x})] & 0 \\ 0 & 1 \end{bmatrix}, \tag{18}$$

where the birefringence $\beta_2(\mathbf{x})$ has the form

$$\beta_2(\mathbf{x}) = \varphi_0 - \varphi(\mathbf{x}), \tag{19}$$

with φ_0 and $\varphi(\mathbf{x})$ of the same meaning as stated in the context of Eq. (9). The transmittance of the system formed by the crossed LC SLMs is given by

$$\mathbf{T}(\mathbf{x}) = \mathbf{T}_2(\mathbf{x})\mathbf{T}_1(\mathbf{x}) = \exp(\mathrm{i}\varphi_0)\begin{bmatrix} \exp[-\mathrm{i}\,\varphi(\mathbf{x})] & 0 \\ 0 & \exp[\mathrm{i}\,\varphi(\mathbf{x})] \end{bmatrix}. \tag{20}$$

At first sight it seems that the above system of two crossed SLMs can be replaced by a sole 90°-twisted LC SLM operating in the adiabatic limit [9]. But it is not the case since the Jones matrix for the twisted LC SLM differs from the one in Eq. (20) by the presence of a common exponential factor depending on birefringence too (see, e.g., Ref. 10).

By substituting Eqs. (6) and (20) into Eq. (5) and making use of formula (A8) from Appendix, we obtain

$$\mathbf{W}_{\mathrm{out}}(\mathbf{x}_1,\mathbf{x}_2) = E_0^2 \exp\left(-\frac{\mathbf{x}_1^2+\mathbf{x}_2^2}{4\varepsilon^2}\right)\exp(-\sigma_\varphi^2)$$
$$\times\begin{bmatrix} \exp\left[\sigma_\varphi^2\exp\left(-\frac{\xi^2}{2\gamma^2}\right)\right]\sin^2\theta & \exp\left[-\sigma_\varphi^2\exp\left(-\frac{\xi^2}{2\gamma^2}\right)\right]\cos\theta\sin\theta \\ \exp\left[-\sigma_\varphi^2\exp\left(-\frac{\xi^2}{2\gamma^2}\right)\right]\sin\theta\cos\theta & \exp\left[\sigma_\varphi^2\exp\left(-\frac{\xi^2}{2\gamma^2}\right)\right]\cos^2\theta \end{bmatrix}. \tag{21}$$

#106643 - $15.00 USD Received 21 Jan 2009; revised 22 Feb 2009; accepted 12 Mar 2009; published 18 Mar 2009

Then, substituting Eq. (21) into Eqs. (3) and (4), we find

$$\eta_{\text{out}}(\xi) = \exp\left\{-\sigma_\varphi^2\left[1-\exp\left(-\frac{\xi^2}{2\gamma^2}\right)\right]\right\}, \tag{22}$$

$$P_{\text{out}}(\mathbf{x}) = \left\{1-[1-\exp(-4\sigma_\varphi^2)]\sin^2 2\theta\right\}^{1/2}. \tag{23}$$

As can be seen from Eq. (22) and (23), the output degree of coherence in this case does not depend on the direction of polarization of the incident field and the output degree of polarization reaches it minimum value of $\exp(-2\sigma_\varphi^2)$ with the choice $\theta = \pi/4$. The degree of coherence given by Eq. (22), for $\sigma_\varphi = 2$ and different values of γ, is plotted in Fig. 2. As can be seen from this figure, the curve $\eta_{\text{out}}(\xi)$ is approximately asymptotic to the ξ axis, and hence the transverse coherence length given by Eq. (17) tends to zero with the decrease of parameter γ. This result confirms that our object is successfully achieved.

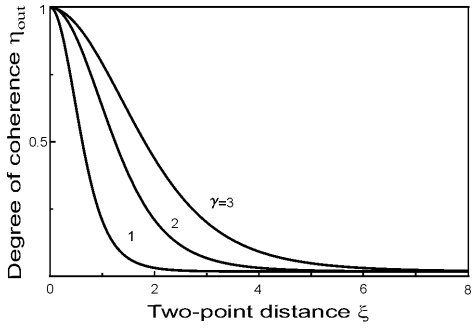

Fig. 2. Degree of coherence given by Eq. (22), $\sigma_\varphi = 2$ and $\gamma = 1,2,3$.

Concluding this section, we note that the random function $\varphi(\mathbf{x})$, which serves to change the birefringence of the LC SLM, can readily be generated with the help of the control computer. When doing this one has to calculate the needed value of the parameter σ_φ corresponding to the desired value of $P_{\text{out}}(\mathbf{x})$ and then to choose the needed value of the parameter γ from the desired plot of function $\eta_{\text{out}}(\xi)$.

4. Dependence of the results on alternative definition of the degree of coherence

The above analysis of coherence modulation was done using Wolf's definition of electromagnetic coherence given by Eq. (3). However, there are other possible definitions of the electromagnetic coherence [11-14]. It is obvious that applying these alternative definitions, one can develop the analysis done in previous sections, obtaining different results. Here, we do not set ourselves the task of recalculating the results for all known definitions of electromagnetic coherence (it is not a trivial mathematical problem). We will limit ourselves to only one of the alternative definitions, namely the definition given in Ref. 11.

According to Ref. 11, the degree of coherence of the electromagnetic field is defined as the normalized Frobenius norm of the cross-spectral density matrix $\mathbf{W}(\mathbf{x}_1, \mathbf{x}_2)$, i.e.,

$$\mu(\mathbf{x}_1, \mathbf{x}_2) = \left(\frac{\text{Tr}[\mathbf{W}^\dagger(\mathbf{x}_1, \mathbf{x}_2)\,\mathbf{W}(\mathbf{x}_1, \mathbf{x}_2)]}{\text{Tr}\mathbf{W}(\mathbf{x}_1, \mathbf{x}_1)\,\text{Tr}\mathbf{W}(\mathbf{x}_2, \mathbf{x}_2)}\right)^{1/2}. \tag{24}$$

#106643 - $15.00 USD Received 21 Jan 2009; revised 22 Feb 2009; accepted 12 Mar 2009; published 18 Mar 2009

Then, substituting Eq. (21) into Eq. (24), we obtain

$$\mu_{out}(\xi) = \exp(-\sigma_\varphi^2) \times \left\{ \exp\left[2\sigma_\varphi^2 \exp\left(-\frac{\xi^2}{2\gamma^2} \right) \right] (\sin^4\theta + \cos^4\theta) + 2\exp\left[-2\sigma_\varphi^2 \exp\left(-\frac{\xi^2}{2\gamma^2} \right) \right] \sin^2\theta \cos^2\theta \right\}^{\frac{1}{2}}, \tag{25}$$

In particular, for the choice $\theta = \pi/4$

$$\mu_{out}(\xi) = \frac{1}{\sqrt{2}} \exp(-\sigma_\varphi^2) \left\{ \exp\left[2\sigma_\varphi^2 \exp\left(-\frac{\xi^2}{2\gamma^2} \right) \right] + \exp\left[-2\sigma_\varphi^2 \exp\left(-\frac{\xi^2}{2\gamma^2} \right) \right] \right\}^{1/2}. \tag{26}$$

The behavior of the degree of coherence given by Eq. (26) in comparison with the one given by Eq. (22) is illustrated in Fig. 3. The difference between curves $\eta_{out}(\xi)$ and $\mu_{out}(\xi)$ is not so large to affect significantly the transverse coherence length, i.e. the main result of the previous section.

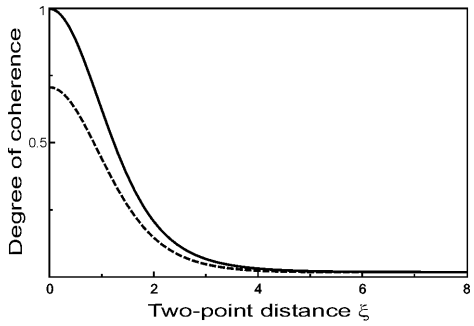

Fig. 3. Degree of coherence given by Eq. (22) (solid line) and degree of coherence given by Eq. (26) (dotted line), $\sigma_\varphi = 2$ and $\gamma = 2$.

5. Conclusion

We have shown that the LC-based technique analyzed in Ref. 5 does not provide the appropriate simultaneous modulation of coherence and polarization. At the same time the proposed technique solves successfully this problem. Of course this technique is more complicated than the former one, but this is the price we pay to obtain the desired result. We have also shown that our results, as well as the results obtained in Ref. 5, depend on the used definition of the degree of electromagnetic coherence. Nevertheless this dependence can not be considered as significant in practice. The proposed technique can be used in practical applications to generate the secondary electromagnetic source with the desired statistical properties.

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Appendix: To derivation of Eqs. (12) and (21)

Taking into account Eq. (10), one can state the relation

$$\left\langle \exp[\pm \mathrm{i}\,\varphi(\mathbf{x})] \right\rangle = \frac{1}{\sqrt{2\pi}\sigma_\varphi} \int_{-\infty}^{\infty} \exp[\pm \mathrm{i}\,\varphi(\mathbf{x})] \exp\left(-\frac{\varphi^2(\mathbf{x})}{2\sigma_\varphi^2} \right) \mathrm{d}\varphi \,. \tag{A1}$$

Then, making use of well known Fourier-transform relation

$$\int_{-\infty}^{\infty} \exp(-\pi a^2 \varphi^2) \exp(\pm \mathrm{i}\, 2\pi\varphi u)\, \mathrm{d}\varphi = \frac{1}{|a|} \exp\left(-\pi \frac{u^2}{a^2} \right), \tag{A2}$$

we find

$$\left\langle \exp[\pm \mathrm{i}\,\varphi(\mathbf{x})] \right\rangle = \exp\left(-\frac{\sigma_\varphi^2}{2} \right). \tag{A3}$$

Now we will introduce a random variable

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = \varphi(\mathbf{x}_2) \pm \varphi(\mathbf{x}_1) \,. \tag{A4}$$

Employing Eq. (10), one can write the probability distribution of this function as

$$p[\psi(\mathbf{x}_1, \mathbf{x}_2)] = \frac{1}{\sqrt{2\pi}\sigma_\psi} \exp\left(-\frac{\psi^2(\mathbf{x}_1, \mathbf{x}_2)}{2\sigma_\psi^2} \right), \tag{A5}$$

where

$$\sigma_\psi^2 = \left\langle \psi^2(\mathbf{x}_1, \mathbf{x}_2) \right\rangle = \left\langle [\varphi(\mathbf{x}_2) \pm \varphi(\mathbf{x}_1)]^2 \right\rangle, \tag{A6}$$

Calculating the square in Eq. (A6) and applying Eq. (11), we find

$$\sigma_\psi^2 = 2\sigma_\varphi^2 \left[1 \pm \exp\left(-\frac{\xi^2}{2\gamma^2} \right) \right]. \tag{A7}$$

Then, by analogy with derivation of Eq. (A3), but this time for argument ψ, we find

$$\left\langle \exp\left\{ +\mathrm{i}\,[\varphi(\mathbf{x}_2) \pm \varphi(\mathbf{x}_1)] \right\} \right\rangle = \left\langle \exp\left\{ -\mathrm{i}\,[\varphi(\mathbf{x}_2) \pm \varphi(\mathbf{x}_1)] \right\} \right\rangle = \exp\left\{ -\sigma_\varphi^2 \left[1 \pm \exp\left(-\frac{\xi^2}{2\gamma^2} \right) \right] \right\}. \tag{A8}$$

Equations (A3) and (A8) are used in the text for derivation of Eqs. (12) and (21).

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