The van Cittert-Zernike theorem for electromagnetic fields

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Abstract: The van Cittert-Zernike theorem, well known for the scalar optical fields, is generalized for the case of vector electromagnetic fields. The deduced theorem shows that the degree of coherence of the electromagnetic field produced by the completely incoherent vector source increases on propagation whereas the degree of polarization remains unchanged. The possible application of the deduced theorem is illustrated by an example of optical simulation of partially coherent and partially polarized secondary source with the controlled statistical properties.

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References and links

1. P.H. van Cittert, "Die Wahrscheinliche Schwingungsverteilung in Einer von Einer Lichtquelle Direkt Oder Mittels Einer Linse Beleuchteten Ebene," Physica **1**, 201-210 (1934).
2. F. Zernike, "The concept of degree of coherence and its application to optical problems," Physica **5**, 785-795 (1938).
3. M. Born and E. Wolf, *Principles of Optics* (Cambridge University Press, Cambridge, UK, 1997).
4. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, UK, 1995).
5. E. Wolf, *Introduction to the Theory of Coherence and Polarization of Light* (Cambridge University Press, Cambridge, UK, 2007).
6. A. S. Ostrovsky, P. Martínez-Vara, M. A. Olvera-Santamaría, and G. Martínez-Niconoff, "Vector coherence theory: An overview of Basic concepts and definitions," in *Recent Research Developments in Optics*, S.G. Pandalai, ed. (Research Signpost, Trivandrum, Kerala, India, to be published).
7. J. Tervo, T. Setälä, and A. T. Friberg, "Degree of coherence for electromagnetic fields," Opt. Express **11**, 1137-1143 (2003).
8. J. Tervo, T. Setälä, and A. T. Friberg, "Theory of partially coherent electromagnetic fields in the space-frequency domain," J. Opt. Soc. Am. A **21**, 2205-2215 (2004).
9. E. Wolf, "Correlation-induced changes in the degree of polarization, the degree of coherence, and the spectrum of random electromagnetic beams on propagation," Opt. Lett. **28**, 1078-1080 (2003).

1. Introduction

One of the central results of optical coherence theory is the van Cittert-Zernike theorem which shows that even completely incoherent source can generate a partially coherent optical field in the process of its propagation [1-5]. This theorem is formulated for a scalar optical field and hence does not take into account the vector nature of the light. At the same time, in the past few years an ever-growing attention is attracted to the theory of coherence of vector electromagnetic fields or, for brevity, vector coherence theory [6]. That is why it seems to be quite natural to try to generalize the van Cittert-Zernike theorem for the vectorial case. Such a generalization is just an object of the present paper.

The paper is organized as follows. In Section 2 we shortly review the basic concepts of the vector coherence theory. When doing so, we give preference to the definition of vector coherence proposed in Refs. [7,8] as the most consistent one (see the corresponding justification in Ref. [6]). In Section 3 we deduce the vector version of the van Cittert-Zernike theorem. An example of possible application of the deduced theorem is given in Section 4. Finally, the main results are summarized in Section 5.

2. Background

We consider a stochastic stationary electromagnetic field propagating within a narrow solid angle around the z axis. According to the coherence theory in the space-frequency domain [4,5] such a field at any typical point $\mathbf{x} = (x, y)$ in some plane $z = \text{const}$ and at any frequency ν can be represented by the statistical ensemble

$$\{\mathbf{E}(\mathbf{x}, \nu)\} = \{E_x(\mathbf{x}, \nu) \quad E_y(\mathbf{x}, \nu)\}, \tag{1}$$

where E_x and E_y are the Cartesian components of the electric field vector $\mathbf{E}$, written here as a row matrix. For the sake of simplicity later on we will omit the explicit dependence of the considered quantities on frequency ν. The second order correlation properties of the electromagnetic field (1) can be completely characterized by the so-called cross-spectral density matrix [5]

$$\mathbf{W}(\mathbf{x}_1, \mathbf{x}_2) = \left[W_{ij}(\mathbf{x}_1, \mathbf{x}_2)\right] = \left[\left\langle E_i^*(\mathbf{x}_1) E_j(\mathbf{x}_2)\right\rangle\right] \qquad (i, j = x, y), \tag{2}$$

where the asterisk and the angle brackets denote the complex conjugate and the ensemble average, respectively. For $\mathbf{x}_1 = \mathbf{x}_2 = \mathbf{x}$, the diagonal element W_{ii} becomes the spectral density, or power spectrum, of field component

$$S_i(\mathbf{x}) = W_{ii}(\mathbf{x}, \mathbf{x}) = \left\langle \left| E_i(\mathbf{x}) \right|^2 \right\rangle, \tag{3}$$

so that the spectral density of the hole field may be expressed as

$$S(\mathbf{x}) = \left\langle \mathbf{E}(\mathbf{x})\mathbf{E}^\dagger(\mathbf{x}) \right\rangle = S_i(\mathbf{x}) + S_j(\mathbf{x}) = \operatorname{Tr}\mathbf{W}(\mathbf{x}, \mathbf{x}), \tag{4}$$

where the dagger denotes the Hermitian conjugate and Tr stands for matrix trace.

It may be shown [5] that

$$\left|W_{ij}(\mathbf{x}_1, \mathbf{x}_2)\right| \le \sqrt{S_i(\mathbf{x}_1)}\sqrt{S_j(\mathbf{x}_2)}\ . \tag{5}$$

Hence, one can define the quantitative measure of the correlation between the vector components E_i and E_j as the normalized absolute value of the cross-correlation function $W_{ij}(\mathbf{x}_1, \mathbf{x}_2)$, i.e.,

$$\left|\mu_{ij}(\mathbf{x}_1, \mathbf{x}_2)\right| = \frac{\left|W_{ij}(\mathbf{x}_1, \mathbf{x}_2)\right|}{\sqrt{S_i(\mathbf{x}_1)}\sqrt{S_j(\mathbf{x}_2)}}\ , \tag{6}$$

which will be referred to as the coefficient of correlation. It is obvious that

$$0 \le \left|\mu_{ij}(\mathbf{x}_1, \mathbf{x}_2)\right| \le 1\ . \tag{7}$$

The vector components E_i and E_j are fully correlated when $\left|\mu_{ij}(\mathbf{x}_1, \mathbf{x}_2)\right| = 1$ and they are fully uncorrelated when $\left|\mu_{ij}(\mathbf{x}_1, \mathbf{x}_2)\right| = 0$. In classical coherence theory the coefficient of correlation given by Eq. (6), for $j = i$, is known as the spectral degree of coherence of a scalar wave field. This fact can serve as a key point for defining the degree of coherence of the vector electromagnetic field.

In Ref. [8] the degree of coherence for the vectorial case is defined as the normalized Frobenius (or Euclidean) norm of the cross-spectral density matrix $\mathbf{W}(\mathbf{x}_1, \mathbf{x}_2)$, i.e.,

$$\tilde{\mu}(\mathbf{x}_1, \mathbf{x}_2) = \left(\frac{\sum_{i,j} \left| W_{ij}(\mathbf{x}_1, \mathbf{x}_2) \right|^2}{\sum_{i,j} S_i(\mathbf{x}_1) S_j(\mathbf{x}_2)} \right)^{1/2} = \left(\frac{\mathrm{Tr}\left[\mathbf{W}^{\dagger}(\mathbf{x}_1, \mathbf{x}_2) \mathbf{W}(\mathbf{x}_1, \mathbf{x}_2) \right]}{\mathrm{Tr}\, \mathbf{W}(\mathbf{x}_1, \mathbf{x}_1) \mathrm{Tr}\, \mathbf{W}(\mathbf{x}_2, \mathbf{x}_2)} \right)^{1/2}. \tag{8}$$

On making use of definition (6), one can rewrite Eq. (8) as follows:

$$\tilde{\mu}(\mathbf{x}_1, \mathbf{x}_2) = \left(\frac{\sum_{i,j} \left| \mu_{ij}(\mathbf{x}_1, \mathbf{x}_2) \right|^2 S_i(\mathbf{x}_1) S_j(\mathbf{x}_2)}{\sum_{i,j} S_i(\mathbf{x}_1) S_j(\mathbf{x}_2)} \right)^{1/2}. \tag{9}$$

As can be seen from Eq. (9), $\tilde{\mu}(\mathbf{x}_1, \mathbf{x}_2)$ represents the power-weighted root-mean-square value of the correlation coefficients for all pairs (*i, j*). Equation (9), together with Eq. (7), states that $\tilde{\mu}(\mathbf{x}_1, \mathbf{x}_2) = 1$ if, and only if, $\left| \mu_{ij}(\mathbf{x}_1, \mathbf{x}_2) \right|^2 = 1$ for any pair (*i, j*), and $\tilde{\mu}(\mathbf{x}_1, \mathbf{x}_2) = 0$ if, and only if, $\left| \mu_{ij}(\mathbf{x}_1, \mathbf{x}_2) \right|^2 = 0$ for any pair (*i, j*). In other words, the vector electromagnetic field is completely coherent when its components are fully self- and cross-correlated and it is completely incoherent when its components are fully self- and cross-uncorrelated.

As well known, the polarization is another manifestation of the correlation properties of a stochastic electromagnetic field that can be characterized by the cross-spectral density matrix taken for $\mathbf{x}_1 = \mathbf{x}_2 = \mathbf{x}$. When

$$\mathbf{W}(\mathbf{x}, \mathbf{x}) = W_{xx}(\mathbf{x}, \mathbf{x}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{10}$$

the field is said to be completely unpolarized, and when

$$\mathrm{Det}\, \mathbf{W}(\mathbf{x}, \mathbf{x}) = 0, \tag{11}$$

where Det stands for determinant, the field is said to be completely polarized. The polarization can be evaluated quantitatively by the degree of polarization defined as [3-5]

$$P(\mathbf{x}) = \left(1 - \frac{4 \mathrm{Det}\, \mathbf{W}(\mathbf{x}, \mathbf{x})}{\left[\mathrm{Tr}\, \mathbf{W}(\mathbf{x}, \mathbf{x}) \right]^2} \right)^{1/2}. \tag{12}$$

Of course, for the completely polarized field $P(\mathbf{x}) = 1$ and for completely unpolarized field $P(\mathbf{x}) = 0$. It may be shown that the polarization properties of a stochastic electromagnetic field can be characterized by the degree of coherence $\tilde{\mu}(\mathbf{x}_1, \mathbf{x}_2)$ as well [7]. On making use of the relation

$$2 \mathrm{Det}\, \mathbf{W}(\mathbf{x}, \mathbf{x}) = \left[\mathrm{Tr}\, \mathbf{W}(\mathbf{x}, \mathbf{x}) \right]^2 - \mathrm{Tr}\left[\mathbf{W}^{\dagger}(\mathbf{x}, \mathbf{x}) \mathbf{W}(\mathbf{x}, \mathbf{x}) \right], \tag{13}$$

one finds

$$P(\mathbf{x}) = \sqrt{2 \tilde{\mu}^2(\mathbf{x}, \mathbf{x}) - 1}. \tag{14}$$

Equation (14) reveals an important fact: the completely coherent electromagnetic field is necessarily completely polarized. Indeed, for completely coherent field the equality $\tilde{\mu}(\mathbf{x}_1, \mathbf{x}_2) = 1$ holds for each pair $(\mathbf{x}_1, \mathbf{x}_2)$, including the case $\mathbf{x}_1 = \mathbf{x}_2 = \mathbf{x}$, and hence $P(\mathbf{x}) = 1$. Equation (14) shows also that the minimum possible value of $\tilde{\mu}(\mathbf{x}, \mathbf{x})$ is $1/\sqrt{2}$.

3. Vector generalization of the van Cittert-Zernike theorem

Now we will consider the propagation of the electromagnetic field given by the statistical ensemble (1) in free space between two parallel planes normal to the z axis. Let $E_i^{(0)}(\mathbf{x}')$ and $E_i^{(z)}(\mathbf{x})$ be the fluctuating orthogonal field components ($i = x, y$) in the source plane $z = 0$ and any transverse plane $z = \text{constant} > 0$, respectively. Applying the paraxial approximation, it may be shown (see [4]) that these components are related by the following formula:

$$E_i^{(z)}(\mathbf{x}) = \frac{1}{\mathrm{i}\lambda z}\exp\left(\mathrm{i}\frac{2\pi}{\lambda}z\right)\iint_{(z=0)} E_i^{(0)}(\mathbf{x}')\exp\left[\mathrm{i}\frac{\pi}{\lambda z}(\mathbf{x}-\mathbf{x}')^2\right]\mathrm{d}\mathbf{x}', \tag{15}$$

where λ is the wave length. On substituting for $E_i^{(z)}(\mathbf{x})$ from Eq. (15) into Eq. (2), one finds the following expression that describes the propagation of the cross-spectral density matrix $\mathbf{W}$ in free space:

$$\mathbf{W}^{(z)}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{(\lambda z)^2}\iint_{(z=0)} \mathbf{W}^{(0)}(\mathbf{x}_1', \mathbf{x}_2')\exp\left\{\mathrm{i}\frac{\pi}{\lambda z}\left[(\mathbf{x}_2 - \mathbf{x}_2')^2 - (\mathbf{x}_1 - \mathbf{x}_1')^2\right]\right\}\mathrm{d}\mathbf{x}_1'\mathrm{d}\mathbf{x}_2'. \tag{16}$$

On making use of Eq. (16) and definitions (8) and (12), one can determine the degree of coherence and the degree of polarization of the field in the plane $z = \text{constant}$. It is obvious that these quantities will, in general, change on propagation. Such changes are known as correlation-induced spectral changes [9]. Below we will examine the correlation-induced spectral changes of the electromagnetic field generated by the completely incoherent source.

Let us consider that the field in the source plane $z = 0$ is completely incoherent. The orthogonal components of such a field are statistically independent and self-uncorrelated. Hence, the completely incoherent source can be characterized by the cross-spectral density matrix $\mathbf{W}^{(0)}(\mathbf{x}_1', \mathbf{x}_2')$ with the diagonal elements

$$W_{ii}^{(0)}(\mathbf{x}_1', \mathbf{x}_2') = \left\langle E_i^{(0)*}(\mathbf{x}_1')E_i^{(0)}(\mathbf{x}_2')\right\rangle = \eta\sqrt{S_i^{(0)}(\mathbf{x}_1')}\sqrt{S_i^{(0)}(\mathbf{x}_2')}\,\delta(\mathbf{x}_1' - \mathbf{x}_2') \tag{17}$$

and off-diagonal elements

$$W_{ij}^{(0)}(\mathbf{x}_1', \mathbf{x}_2') = \left\langle E_i^{(0)*}(\mathbf{x}_1')E_j^{(0)}(\mathbf{x}_2')\right\rangle = \left\langle E_i^{(0)*}(\mathbf{x}_1')\right\rangle\left\langle E_j^{(0)}(\mathbf{x}_2')\right\rangle = 0, \tag{18}$$

where η is a positive constant, $\delta(.)$ is the two-dimensional Dirac delta function, and the components $E_i^{(0)}$ are assumed, without loss of generality, to be the fluctuations with zero mean. On substituting for $\mathbf{W}^{(0)}(\mathbf{x}_1', \mathbf{x}_2')$ from Eqs. (17) and (18) into Eq. (16) and making use of the filtering property of delta function, we find

$$W_{ii}^{(z)}(\mathbf{x}_1', \mathbf{x}_2') = \frac{\eta}{(\lambda z)^2}\exp\left[-\mathrm{i}\frac{\pi}{\lambda z}\left(\mathbf{x}_1^2 - \mathbf{x}_2^2\right)\right]\int_{(z=0)} S_i^{(0)}(\mathbf{x}')\exp\left(-\mathrm{i}\frac{2\pi}{\lambda z}\mathbf{x}'\cdot\Delta\mathbf{x}\right)\mathrm{d}\mathbf{x}' \tag{19}$$

and

$$W_{ij}^{(z)}(\mathbf{x}_1', \mathbf{x}_2') = 0, \tag{20}$$

where $\Delta\mathbf{x} = \mathbf{x}_2 - \mathbf{x}_1$. As can be seen from Eq. (19), each field component reaching the plane $z = 0$ is not already self-uncorrelated. If we recall the definition (6) and use Eq. (19), we obtain the following formula for the spectral degree of coherence of the field component:

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$$\left|\mu_{ii}^{(z)}(\mathbf{x}_1,\mathbf{x}_2)\right| = \frac{\left|\int_{(z=0)} S_i^{(0)}(\mathbf{x}')\exp\left(-\mathrm{i}\frac{2\pi}{\lambda z}\mathbf{x}'\cdot\Delta\mathbf{x}\right)\mathrm{d}\mathbf{x}'\right|}{\int_{(z=0)} S_i^{(0)}(\mathbf{x}')\mathrm{d}\mathbf{x}'}. \tag{21}$$

Taking into account that the integral in the numerator of Eq. (21) represents the Fourier transform operator with the spatial frequency $\Delta\mathbf{x}/\lambda z$, we come to the following conclusion: *the spectral degree of coherence of the field component in the far zone of the completely incoherent source is equal to the normalized Fourier transform of power spectrum of corresponding field component across the source.* In the scalar coherence theory this result is known as the van Cittert-Zernike theorem.

To generalize the van Cittert-Zernike theorem for the vectorial case, we substitute first from Eqs. (19) and (20) into definition of the degree of coherence of the electromagnetic field Eq. (8), i.e.,

$$\tilde{\mu}^{(z)}(\mathbf{x}_1,\mathbf{x}_2) = \left(\frac{\sum_i \left|\int_{(z=0)} S_i^{(0)}(\mathbf{x}')\exp\left(-\mathrm{i}\frac{2\pi}{\lambda z}\mathbf{x}'\cdot\Delta\mathbf{x}\right)\mathrm{d}\mathbf{x}'\right|^2}{\sum_{i,j}\int_{(z=0)} S_i^{(0)}(\mathbf{x}')\mathrm{d}\mathbf{x}'\int_{(z=0)} S_j^{(0)}(\mathbf{x}')\mathrm{d}\mathbf{x}'}\right)^{1/2}. \tag{22}$$

It may be readily shown that

$$\sum_{i,j}\int_{(z=0)} S_i^{(0)}(\mathbf{x}')\mathrm{d}\mathbf{x}'\int_{(z=0)} S_j^{(0)}(\mathbf{x}')\mathrm{d}\mathbf{x}' = \left[\int_{(z=0)} S^{(0)}(\mathbf{x}')\mathrm{d}\mathbf{x}'\right]^2, \tag{23}$$

where $S^{(0)}(\mathbf{x}')$ is the power spectrum of the source defined by Eq. (4). On substituting from Eq. (23) into Eq. (22), we obtain finally

$$\tilde{\mu}^{(z)}(\mathbf{x}_1,\mathbf{x}_2) = \frac{\left(\sum_i \left|\int_{(z=0)} S_i^{(0)}(\mathbf{x}')\exp\left(-\mathrm{i}\frac{2\pi}{\lambda z}\mathbf{x}'\cdot\Delta\mathbf{x}\right)\mathrm{d}\mathbf{x}'\right|^2\right)^{1/2}}{\int_{(z=0)} S^{(0)}(\mathbf{x}')\,\mathrm{d}\mathbf{x}'}. \tag{24}$$

Equation (24) represents the vector generalization of the van Cittert-Zernike theorem, which can be formulated as follows: *the spectral degree of coherence of the electromagnetic field in the far zone of the completely incoherent source is equal to the normalized root-mean-square absolute value of the Fourier transforms of power spectra of field components across the source.*

Now we will examine the polarization properties of the electromagnetic field in the far zone of the incoherent source. On making use of definition (12), or alternatively Eq. (14), we obtain the following formula for the degree of polarization of the field in the far zone of the incoherent source:

$$P^{(z)}(\mathbf{x}) = \sqrt{4\alpha^2 - 4\alpha + 1}, \tag{25}$$

where

$$\alpha = \frac{\int_{(z=0)} S_x^{(0)}(\mathbf{x}')\mathrm{d}\mathbf{x}'}{\int_{(z=0)} S^{(0)}(\mathbf{x}')\mathrm{d}\mathbf{x}'}. \tag{26}$$

Equations (25) and (26) show that the degree of polarization of the completely incoherent electromagnetic field does not change on propagation and depends only on the energy

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distribution between the field components in the source plane. In particular, for completely unpolarized source, when $S_x^{(0)}(\mathbf{x}') = S_y^{(0)}(\mathbf{x}')$, the equality $P^{(z)}(\mathbf{x}) = 0$ takes place, i.e., the generated field remains completely unpolarized on propagation. On the contrary, when $S_x^{(0)}(\mathbf{x}') = 0$ or $S_y^{(0)}(\mathbf{x}') = 0$, it takes place the equality $P^{(z)}(\mathbf{x}) = 1$, i.e., the generated field remains completely polarized on propagation. In general case the vector version of the van Cittert-Zernike theorem given by Eq. (24) must necessarily be supplemented with Eq. (25).

4. Example of theorem application

As an example of possible applications of the deduced theorem we consider the problem of optical simulation of partially coherent and partially polarized secondary source with the controlled statistical properties. Such a simulation may be realized by means of the modified Mach-Zehnder interferometer sketched schematically in Fig. 1.

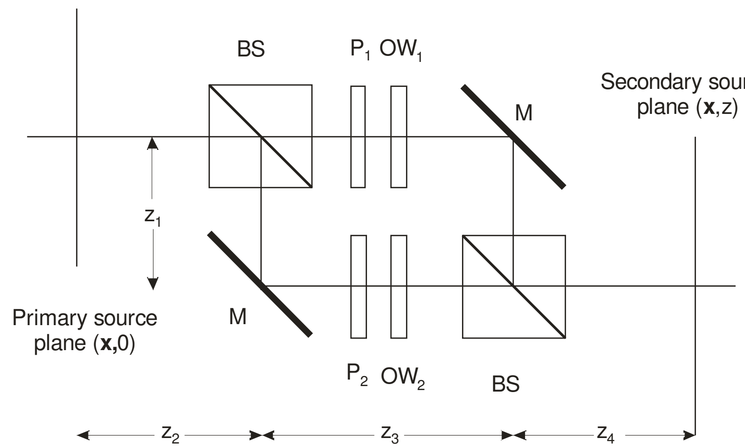

Fig. 1 Modified Mach-Zehnder interferometer: BS – beam splitter; M – mirror; P – polarizer; OW – optical wedge.

Let us consider that the completely incoherent circular source of radius R and of uniform power spectrum $S_0^{(0)}$ is placed in the input plane (primary source plane) of the interferometer. Let polarizers P_1 and P_2 be chosen to transmit only x component and only y component of the incident field, respectively, and let optical wedges OW_1 and OW_2 be chosen to attenuate the power spectra of the optical fields at the opposite arms of the interferometer in the ratio $\alpha : (1-\alpha)$ with α being the desired coefficient of attenuation. Then, assuming that the distance $z = z_1 + z_2 + z_3 + z_4$ between the primary source plane and the secondary source plane is large enough, the degree of coherence of the generated secondary source can be calculated using Eq. (24), i.e.,

$$\tilde{\mu}^{(z)}(\mathbf{x}_1, \mathbf{x}_2) = \frac{\sqrt{\alpha^2 + (1-\alpha)^2}}{\pi R^2} \left| \int_{(|\mathbf{x}'| \le R)} \exp\left(-\mathrm{i} \frac{2\pi}{\lambda z} \mathbf{x}' \cdot \Delta\mathbf{x} \right) \mathrm{d}\mathbf{x}' \right|. \tag{27}$$

Calculating the integral in Eq. (27) (see, e.g. [3]), we find

$$\tilde{\mu}^{(z)}(\mathbf{x}_1, \mathbf{x}_2) = \sqrt{2\alpha^2 - 2\alpha + 1} \left| \frac{2J_1\left(\frac{2\pi R}{\lambda z} |\Delta\mathbf{x}| \right)}{\left(\frac{2\pi R}{\lambda z} |\Delta\mathbf{x}| \right)} \right|, \tag{28}$$

where $J_1(.)$ is the Bessel function of the first kind and the first order.

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The function $2J_1(\xi)/\xi$ decreases steadily from the value unity when $\xi = 0$ to the value zero when $\xi = 0.61\lambda z/R$. Correspondingly, the radius of the area where the degree of coherence (28) exceeds 0.88 of its maximum value is approximately given by

$$r_{\mathrm{coh}} = 0.16\frac{\lambda z}{2R}. \tag{29}$$

As well known, this radius can serves as the numerical measure of coherence. Hence, changing the distance z (really z_4), one can control the coherence of the produced vector field over a rather wide range. On the other hand, varying the attenuation coefficient α, one can control the degree of polarization given by Eq. (25) within the range from zero to unity.

5. Conclusion

We have generalized the well known van Cittert-Zernike theorem for the case of vector electromagnetic fields. The deduced theorem shows that the degree of coherence of the electromagnetic field produced by the completely incoherent vector source increases on propagation whereas the degree of polarization remains unchanged. The possible application of the deduced theorem has been illustrated by an example of optical simulation of partially coherent and partially polarized secondary source with the controlled statistical properties.

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