The Reduction of the Duration of the Transient Response in a Class of Continuous-Time LTV Filters

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***Abstract*—Linear time-varying (LTV) systems have found a niche of their own in the processing of continuous-time signals. In this brief, it is shown how the reduction of the duration of the transient response of a class of continuous-time LTV filters may be seen as the combined effect of the increased dampening of its amplitude response and the increase of the instantaneous frequency of its damped oscillations. For this aim, an LTV system whose damping factor and the damped frequency of its oscillations may be specified as functions of time is used as a vehicle of study. Time-varying eigenvalues are used to assess the behavior of the proposed system. Simulation results are used to verify the proposed mechanism behind the reduction of the duration of the transient response in the LTV filters under study.**

***Index Terms*—Continuous-time signal processing, linear time-varying (LTV) systems, LTV filters, time-varying eigenvalues, transient response duration.**

I. Introduction

In recent times, an increasing interest in continuous-time systems with time-varying parameters has arisen. Following this trend, a new class of filters whose describing differential equations are linear time-varying (LTV) and do not involve companding techniques has been developed as well in [1], [2]. These filters have found some applications in the field of biomedical instrumentation in the acquisition of brainstem auditory-evoked potentials [1] as well as in the compensation of the dynamic response of load cells [2].

The general form of the LTV filters proposed in [1], [2] is given by the following LTV scalar differential equation:

$$x''(t) + 2\xi(t)\omega_n(t)x'(t) + \omega_n^2(t)x(t) = \omega_n^2(t)u(t) \quad (1)$$

where $u(t)$ and $x(t)$ are the scalar functions that represent, respectively, the input and output of the filter, whereas $\xi(t)$ and $\omega_n(t)$ are the time-varying parameters. If an analogy is made with the linear time-invariant (LTI) differential equation

$$x''(t) + 2\xi\omega_n x'(t) + \omega_n^2 x(t) = 0 \quad (2)$$

then the time-varying parameters $\xi(t)$ and $\omega_n(t)$ may be considered to define the damping ratio and the undamped natural frequency of the homogeneous response of (1). In [1], the performance of the LTV filter described by (1) is improved compared to the response of a prototype LTI filter by shortening the duration of its transient response. This is achieved by tuning its parameters, particularly its undamped natural frequency, by making it larger [1].

In this brief, it will be shown that the modulation of the parameters of the LTV filter may be seen as a strategy to simultaneously reduce the amplitude of its transient response and to increase the frequency of its damped oscillations. The rest of this brief will be organized as follows. In Section II, the differential equation that describes an LTV system with adjustable homogeneous response will be derived. This equation will be used to show in principle how an increase of the coefficients of an LTV filter of the form given in (1) may influence the amplitude of its transient behavior as well as its damped oscillatory response. To obtain more information on the dynamical behavior of the proposed system, time-varying eigenvalues [3], [4] will be used. The basic theory behind the time-varying eigenvalues will be presented in Section III. In Section IV, the proposed mechanism for the reduction of the duration of the transient response will be presented. A formal proof of this mechanism based on the concept of the time-varying eigenvalues will be given. The simulation results that confirm the validity of the proposed mechanism will be presented in Section V. Finally, some closing remarks will be given in Section VI.

II. LTV System With Adjustable Homogeneous Response

To show how an increase of the coefficients of the LTV filter described by (1) may influence its dynamic behavior, consider a system modeled by the following second-order scalar LTV differential equation:

$$x''(t) + a_1(t)x'(t) + a_0(t)x(t) = u(t) \quad (3)$$

where $u(t)$ represents a known function of the time variable t, $x(t)$ is the unknown variable, and $a_1(t)$ and $a_0(t)$ are

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time-varying coefficients. It is required that the homogeneous solution $x_h(t)$ of (3) satisfies

$$x_h(t) = C_1 x_1(t) + C_2 x_2(t) \tag{4}$$

where C_1 and C_2 are arbitrary constants, whereas $x_1(t)$ and $x_2(t)$ are given by

$$x_1(t) = e^{-\int \sigma(t)\,dt + j\int \omega_d(t)\,dt} \tag{5}$$

$$x_2(t) = e^{-\int \sigma(t)\,dt - j\int \omega_d(t)\,dt} \tag{6}$$

where $\sigma(t)$ and $\omega_d(t)$ are continuous functions of t, which are assumed to be bounded and to take values greater than zero for all t. The function $\sigma(t)$ may be seen as a function that defines the decay rate of the solutions given in (5) and (6). If $\sigma(t)$ increases, then the homogeneous response of (3) will decrease in magnitude. On the other hand, $\omega_d(t)$ may be interpreted as the instantaneous frequency of the damped oscillations of the solutions of (3). If the magnitude of this term is increased or decreased, it will not have any effect on the magnitude of $e^{j\int \omega_d(t)\,dt}$ since it will always be equal to 1. However, if $\omega_d(t)$ changes in value, then the frequency of the damped oscillations associated to the homogeneous response of (3) will change.

To guarantee that the homogeneous solution $x_h(t)$ of the system modeled by (3) is equal to (4), $a_1(t)$ and $a_0(t)$ must satisfy the following set of linear algebraic equations:

$$\begin{bmatrix} x_1'(t) & x_1(t) \\ x_2'(t) & x_2(t) \end{bmatrix} \begin{bmatrix} a_1(t) \\ a_0(t) \end{bmatrix} = - \begin{bmatrix} x_1''(t) \\ x_2''(t) \end{bmatrix}. \tag{7}$$

These equations are obtained after substituting (5) and (6) in (3) with $u(t) = 0$. Given that $x_1(t)$ and $x_2(t)$ are linearly independent functions, it is guaranteed that system (7) will have an unique solution that is given by

$$a_1(t) = 2\sigma(t) - \frac{\omega_d'(t)}{\omega_d(t)} \tag{8}$$

$$a_0(t) = (\sigma(t))^2 + (\omega_d(t))^2 - \frac{\sigma(t)\omega_d'(t)}{\omega_d(t)} + \sigma'(t). \tag{9}$$

The function $\omega_d(t)$ cannot be equal to zero because otherwise $x_1(t)$ and $x_2(t)$ would not be linearly independent. Furthermore, if $\omega_d(t) \neq 0$ for all t, then the time-varying coefficients $a_1(t)$ and $a_0(t)$ will be defined for all t.

At this point, it is necessary to demonstrate that the solution of (3) is bounded when $u(t)$ is bounded. For this aim, the following lemma is presented.

Lemma 1: Consider a system that is described by the following system of differential equation:

$$\mathbf{x}'(t) = \mathbf{F}(t)\mathbf{x}(t) + \mathbf{G}(t)\mathbf{u}(t) \tag{10}$$

where $\mathbf{x}(t)$ is a vector that contains n state variables, $\mathbf{u}(t)$ is a vector that contains p inputs, and $\mathbf{F}(t)$ and $\mathbf{G}(t)$ are matrices of real continuous functions with appropriate dimensions. The system will be bounded-input–bounded-state (BIBS) if the following conditions are met.

1) The elements of $\mathbf{F}(t)$ and $\mathbf{G}(t)$ are bounded.
2) The homogeneous response of (10) shows an exponential asymptotic stability.

Proof: See [5] for more details. ■

Theorem 1: The response to a bounded input $u(t)$ of the system that is represented by (3) with the coefficients $a_1(t)$ and $a_0(t)$ defined as given in (8) and (9) with functions $\sigma(t)$ and $\omega_d(t)$ being positive for all t is also bounded. In other words, the system is BIBO stable.

Proof: To demonstrate this theorem, it suffices to verify if the state space representation of (3) using phase variables satisfies the constraints given in Lemma 1 for BIBS stability. Given that the output variable of the system under consideration is a state variable, the existence of the aforementioned stability will also imply BIBO stability. Equation (3) may be rewritten as

$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0(t) & -a_1(t) \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{11}$$

where $y_1(t)$ and $y_2(t)$ are equal to $x(t)$ and $x'(t)$, respectively. Given that the functions $\sigma(t)$ and $\omega_d(t)$ are bounded, all the entries of the system matrix of (11) are bounded. Moreover, the entries of the input matrix present in (11) are bounded as well.

The homogeneous response of (11) for an arbitrary initial condition is given by

$$y_{1,h}(t) = C_1 x_1(t) + C_2 x_2(t) \tag{12}$$

$$\begin{aligned} y_{2,h}(t) = {} & C_1 \left[-\sigma(t) + j\omega(t)\right] x_1(t) \\ & + C_2 \left[-\sigma(t) - j\omega(t)\right] x_2(t) \end{aligned} \tag{13}$$

where C_1 and C_2 are arbitrary constants, and $x_1(t)$ and $x_2(t)$ stand for the linearly independent solutions of (3) given in (5) and (6). It should be noticed that the exponential asymptotic stability of the homogeneous solutions of (3) when $a_1(t)$ and $a_0(t)$ are defined as given in (8) and (9) is guaranteed provided that $\sigma(t)$ is always positive. Given that $\sigma(t)$ and $\omega(t)$ are also bounded functions, and that the solutions $x_1(t)$ and $x_2(t)$ are exponentially asymptotically stable, the responses $y_{1,h}(t)$ and $y_{2,h}(t)$ will be exponentially asymptotically stable as well.

Finally, given that the conditions given in Lemma 1 are satisfied, it can be concluded that the system represented by (3) is BIBO stable. ■

III. Time-Varying Eigenvalues

To further understand the dynamics of the system modeled by (3), it is necessary to resort to the concept of time-varying eigenvalues. The term was coined in [3] to denote quantities that contain information on the stability of LTV systems. These quantities are determined from LTV systems through the free formulation of an eigenvalue-like problem from LTV systems. There are many mathematical frameworks proposed in the literature that consider the definition of quantities that match the description given in [3] for the time-varying eigenvalues. The interested reader may consult, for instance, the work of Zhu, van der Kloet, and Neerhoff [6]–[8] on this subject.

What follows now is a brief summary of how the time-varying eigenvalues may be computed for (3) according to the formulation given by Zhu and Johnson [6]. Using this framework, it is possible to assess in a convenient way the dynamic behavior of (3). An arbitrary nth-order scalar LTV differential equation of the form

$$x^{(n)}(t)+a_{n-1}(t)x^{(n-1)}(t)+\cdots+a_1(t)x'(t)+a_0(t)x(t)=0 \tag{14}$$

where $x(t)$ is the unknown function, and $a_i(t)$, $i = 0, 1, \ldots, n-1$, are time-varying coefficients has an associated nth-order differential operator $\mathcal{D}_a$ of the form

$$\mathcal{D}_a = \delta^n + a_{n-1}(t)\cdot\delta^{n-1}+\cdots+a_1(t)\cdot\delta+a_0(t) \tag{15}$$

where δ and $\cdot$ represent the operations of differentiation with respect to the variable t and the scalar multiplication by the preceding scalar function, respectively. In this expression, δ^i, $i = 2, 3, \ldots, n$, represents the ith-order derivative with respect to t. With the aid of operator (15), (14) may be rewritten as

$$\mathcal{D}_a\{x(t)\} = 0. \tag{16}$$

Operator (15) may be rewritten as a composition of first-order differential operators as [6]

$$\mathcal{D}_a = [\delta - \lambda_n(t)\cdot]\circ[\delta-\lambda_{n-1}(t)\cdot]\circ\cdots\circ[\delta-\lambda_1(t)\cdot]. \tag{17}$$

In this expression, each of the first-order differential operators is enclosed in square brackets, and the symbol $\circ$ is used to denote the composition of two operators. Furthermore, the operator on the left of $\circ$ is applied to the result obtained from the application of the operator on the right of $\circ$ to a given function. The quantities $\lambda_i(t)$, $i = 1, \ldots, n$, are known as the time-varying eigenvalues associated to the differential operator (15).

According to [6], if a set of n linearly independent solutions $x_i(t)$, $i = 1, \ldots, n$, is known for (14), then it is possible to determine each of the time-varying eigenvalues associated to (15) as

$$\lambda_i(t) = \frac{d}{dt}\ln\frac{\Omega_i(t)}{\Omega_{i-1}(t)} \tag{18}$$

where $\Omega_i(t)$ and $\Omega_0(t)$ are defined as

$$\Omega_i(t) = \det \mathbf{W}_i(t) \tag{19}$$

$$\Omega_0(t) = 1 \tag{20}$$

and $\mathbf{W}_i(t)$ is given by

$$\mathbf{W}_i(t) = \begin{bmatrix} x_1(t) & x_2(t) & \cdots & x_i(t) \\ x_1'(t) & x_2'(t) & \cdots & x_i'(t) \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{(i-1)}(t) & x_2^{(i-1)}(t) & \cdots & x_i^{(i-1)}(t) \end{bmatrix}. \tag{21}$$

From the previous expressions, it should be noted that the time-varying eigenvalues defined in this way are not unique since any set of linearly independent homogeneous solutions of (3) may be used to formulate a valid set of time-varying eigenvalues. For a given scalar LTI system, the solutions of its associated characteristic equation always define a valid set of time-varying eigenvalues [4].

IV. Reduction of the Duration of the Transient Response in the Proposed System

In this section, time-varying eigenvalues will be used to determine under which conditions the transient response of the system proposed in (3) may be reduced in duration. Using the formulas given in the previous section, a set of time-varying eigenvalues for the second-order differential operator associated to (3) may be computed by using the set of homogeneous solutions given in (5) and (6). Substituting (5) and (6) in (18)–(21), the following time-varying eigenvalues $\lambda_1(t)$ and $\lambda_2(t)$ are obtained:

$$\lambda_1(t) = -\sigma(t) + j\omega_d(t) \tag{22}$$

$$\lambda_2(t) = -\sigma(t) + \frac{\omega_d'(t)}{\omega_d(t)} - j\omega_d(t). \tag{23}$$

Before presenting the main result of this brief, it must be demonstrated that the time-varying eigenvalues given in (22) and (23) represent the rates of instantaneous exponential growth of the homogeneous solutions $x(t)$ and $x'(t)$ of (3). For this purpose, the following definition is introduced.

Definition 1: The instantaneous exponential growth rate of a nonnegative function $f(t)$ for a given time instant t_0 is equal to the derivative of $\ln f(t)$ that is evaluated at $t = t_0$. For the function $f(t) = e^{\alpha t}$, for instance, its instantaneous exponential growth rate is equal to the constant α for any time instant t_0. If $\alpha > 0$, then $f(t)$ will increase. However, if $\alpha < 0$, then $f(t)$ will decay to zero.

Lemma 2: The time-varying eigenvalues $\lambda_1(t)$ and $\lambda_2(t)$ as given in (22) and (23) determine, respectively, the instantaneous exponential growth rates for $x(t)$ and $x'(t)$ in (3) when $a_1(t)$ and $a_0(t)$ are given as in (8) and (9).

Proof: From (18)–(21), the following relation may be obtained for $\lambda_1(t)$:

$$\lambda_1(t) = \frac{d}{dt}\ln x_1(t). \tag{24}$$

Given that the right side of (24) matches Definition 1 for the instantaneous exponential growth rate of $x_1(t)$, $\lambda_1(t)$ defines the instantaneous rate of exponential growth of an arbitrary homogeneous solution of (3) and, therefore, is correlated to the growth of $x(t)$.

$\lambda_2(t)$ is defined as

$$\lambda_2(t) = \frac{d}{dt}\ln\frac{\det \mathbf{W}_2(t)}{x_1(t)} \tag{25}$$

where $\mathbf{W}_2(t)$ is given by

$$\mathbf{W}_2(t) = \begin{bmatrix} x_1(t) & x_2(t) \\ x_1'(t) & x_2'(t) \end{bmatrix}. \tag{26}$$

The right part of (25) also satisfies Definition 1 for the instantaneous exponential growth rate of the scalar expression $\det \mathbf{W}_2(t)/x_1(t)$. In this expression, the matrix $\mathbf{W}_2(t)$ contains in its columns a complete set of solutions for (11) when $u(t) = 0$. The determinant present in (25) may be interpreted as the oriented area of the parallelogram defined by the homogenous solutions of (11) in the phase plane of the state variables $y_1(t)$ and $y_2(t)$. These variables are equal to $x(t)$ and $x'(t)$, respectively, in (3). The area covered by the solution vectors in the phase plane will increase or decrease in time, and its growth rate will depend on the growth rate of each of these state variables. Using (24) and (25), the instantaneous exponential growth rate of the area covered by the solution vectors may be expressed as

$$\frac{d}{dt}\ln\det \mathbf{W}_2(t) = \lambda_1(t) + \lambda_2(t). \qquad (27)$$

If the determinant of $\mathbf{W}_2(t)$ is divided by $x_1(t)$, as done on the right side of (25), then the following expression is obtained:

$$\begin{aligned}\frac{\det \mathbf{W}_2(t)}{x_1(t)} &= -2j\omega_d(t)e^{-\int \sigma(t)\,dt - j\int \omega_d(t)\,dt}\\ &= -2j\omega_d(t)x_2(t). \qquad (28)\end{aligned}$$

The right side of (28) is similar to the last term of the derivative of the solution $x_2(t)$ of (3) as

$$x_2'(t) = -\sigma(t)x_2(t) - j\omega_d(t)x_2(t). \qquad (29)$$

This means that the contribution done by $y_1(t)$ to the area spanned in the phase plane by the homogeneous solutions of (11) is "eliminated" since $x_1(t)$ is also a component of the homogeneous solution of $y_1(t)$ in (11). Moreover, if the instantaneous exponential growth rate is determined for (28) and for $-j\omega_d(t)x_2(t)$, the result will be the same since both expressions differ by a scalar factor that does not influence their instantaneous exponential rate of change. Therefore, the influence of $y_2(t)$ in the growth of the area spanned in the phase plane by the homogeneous solutions of (11) is obtained. Given that $y_2(t)$ is equivalent to $x'(t)$ in (3), it may be concluded that $\lambda_2(t)$ is related to the instantaneous exponential growth rate of $x'(t)$. ■

It should be noticed that the time-varying eigenvalues calculated from the solutions given in (5) and (6) are complex quantities since they were calculated from complex-valued solutions. In this particular case, however, it is easy to demonstrate that the real parts of $\lambda_1(t)$ and $\lambda_2(t)$ define the exponential rate of growth of the magnitudes of $x(t)$ and $x'(t)$.

Lemma 3: The real parts of $\lambda_1(t)$ and $\lambda_2(t)$, as given in (22) and (23), define the exponential rate of growth of the magnitudes of $x(t)$ and $x'(t)$, respectively, in (3) when $a_1(t)$ and $a_0(t)$ are given as in (8) and (9).

Proof: The magnitude of $x_1(t)$ is given by

$$|x_1(t)| = e^{-\int \sigma(t)\,dt}. \qquad (30)$$

From this expression, the following relation holds true:

$$\operatorname{Re}\lambda_1(t) = \frac{d}{dt}\ln|x_1(t)|\,. \qquad (31)$$

As mentioned in Lemma 2, the determinant of $\mathbf{W}_2(t)$ in (25) may be interpreted as the oriented area spanned by the homogeneous solutions of (11) in the phase plane formed by $y_1(t)$ and $y_2(t)$. If the magnitude of the oriented area spanned by the homogeneous solutions of (11) is now "normalized" (or divided) by the magnitude of $x_1(t)$, then the following expression may be formulated:

$$\operatorname{Re}\lambda_2(t) = \frac{d}{dt}\ln\frac{|\det \mathbf{W}_2(t)|}{|x_1(t)|}. \qquad (32)$$

■

Lemmas 2 and 3 may now be used to prove the main result of this brief.

Theorem 2: Assuming that $\sigma(t)$ and $\omega_d(t)$ adopt the following form:

$$\sigma(t) = \sigma_r + f(t)U(t-t_0) \qquad (33)$$

$$\omega_d(t) = \omega_{d_r} + \alpha e^{-\beta t}U(t-t_0) \qquad (34)$$

where $f(t)$ is a monotonically decreasing nonnegative function for $t \geq t_0$ that tends to zero in a finite time, $U(t)$ stands for the unit step function, and σ_r, ω_{d_r}, α, and β are positive constants, the following statements are true for $t \geq t_0$.

1) The coefficients $a_1(t)$ and $a_0(t)$ given in (8) and (9) will satisfy the following relations:

$$a_1(t) > 2\sigma_r \qquad (35)$$

$$a_0(t) > \sigma_r^2 + \omega_{d_r}^2. \qquad (36)$$

2) The real part of the time-varying eigenvalues $\lambda_1(t)$ and $\lambda_2(t)$ associated to (3) will satisfy the following relations:

$$\operatorname{Re}\lambda_1(t) < -\sigma_r \qquad (37)$$

$$\operatorname{Re}\lambda_2(t) < -\sigma_r. \qquad (38)$$

3) The homogeneous response of the system represented by (3) will decay faster compared to the homogeneous response of the system described by the following equation:

$$z''(t) + 2\sigma_r z'(t) + \left(\sigma_r^2 + \omega_{d_r}^2\right)z(t) = u(t). \qquad (39)$$

Proof: To prove Statements 1 and 2, the direct substitution of $\sigma(t)$ and $\omega_d(t)$, as given in (33) and (34), in (8), (9), (22) and (23) leads to functions that are greater for $t \geq t_0$ compared to the constants indicated in (35)–(38).

To prove Statement 3, it should be noticed that a set of time-varying eigenvalues for the system described by (39) is given by $-\sigma_r \pm j\omega_{d_r}$. These eigenvalues were calculated from the solutions of the characteristic equation associated to (39). Given that $\sigma_r > 0$, it can be demonstrated that the real parts of the eigenvalues of (39) define the exponential decay rates for $z(t)$ and $z'(t)$. The homogeneous response of the system described by (3) will also exponentially decay to zero since the real parts of $\lambda_1(t)$ and $\lambda_2(t)$ as given in (22) and (23) are negative for all t. According to Statement 2, the real parts of $\lambda_1(t)$ and $\lambda_2(t)$ are smaller than $-\sigma_r$ for $t \geq t_0$. Therefore, the homogeneous response of the system represented by (3) will decay faster compared to the homogeneous response of (39) for $t \geq t_0$. ■

V. Verification of the Proposed Mechanism for the Reduction of the Duration of the Transient Response

The proposed mechanism in Theorem 2 for the reduction of the duration of the transient response will be validated for an example. It will be assumed that it is desired to improve in a given time interval the transient response of the following dynamical system:

$$z''(t) + 10z'(t) + 386z(t) = u(t). \quad (40)$$

For (40), the solutions of its characteristic equation are equal to $-5 \pm 19j$.

According to the strategy proposed in [1] for the improvement of the transient response in filters of the form given in (1), the magnitude of the coefficients of (40) has to be increased in time to shorten the duration of its transient response. Assuming that the functions $\sigma(t)$ and $\omega(t)$ implicit in the definition of system (3) take the following form:

$$\sigma(t) = 5 + 4U(t)e^{-15t} \quad (41)$$
$$\omega_d(t) = 19 + 18U(t)e^{-15t} \quad (42)$$

where $U(t)$ is the input step function, system (3) should have an improved transient behavior compared to the response of (40). For these functions, the coefficients $a_1(t)$ and $a_0(t)$ given in (8) and (9) for $t \geq 0$ are equal to

$$a_1(t) = 10 + 8e^{-15t} + \frac{270}{18 + 19e^{15t}} \quad (43)$$
$$a_0(t) = 386 + 724e^{-15t} + 340e^{-30t} + \frac{210}{18 + 19e^{15t}}. \quad (44)$$

From these expressions, it is clear that the coefficients of the system given in (3) will be greater than the coefficients of the reference LTI system given in (40) for $t \geq 0$.

The responses of the systems represented by (3) and (40), respectively, were simulated in Simulink assuming that the input $u(t)$ is a sequence of pulses, as shown in Fig. 1. The results of the simulation are depicted in Fig. 2. As can be seen, the output of the system modeled by (40) shows a large overshoot for any transition of the input signal, whereas the response of the modeled equation by (3) has no overshoot for the first pulse. However, once $\sigma(t)$ and $\omega_d(t)$ are approximately equal to 5 and 19, respectively, for $t > 4/15$, there is no difference in the behavior of these systems.

VI. Conclusion

In this brief, it has been shown that the shortening of the duration of the transient response for a particular class of LTV filters may be understood as the consequence of the increased dampening of their transient response and the increase of the instantaneous frequency of their damped oscillations. For that aim, an LTV system whose transient response may be specified in terms of functions that define the magnitude of the damping factor and the instantaneous frequency of its damped oscillations has been proposed. The results obtained by means of computer simulations confirmed that the mechanism suggested in this brief may be responsible for the reduction of the duration of the transient response of the time-varying filter modeled by (1) when its coefficients are temporarily increased.

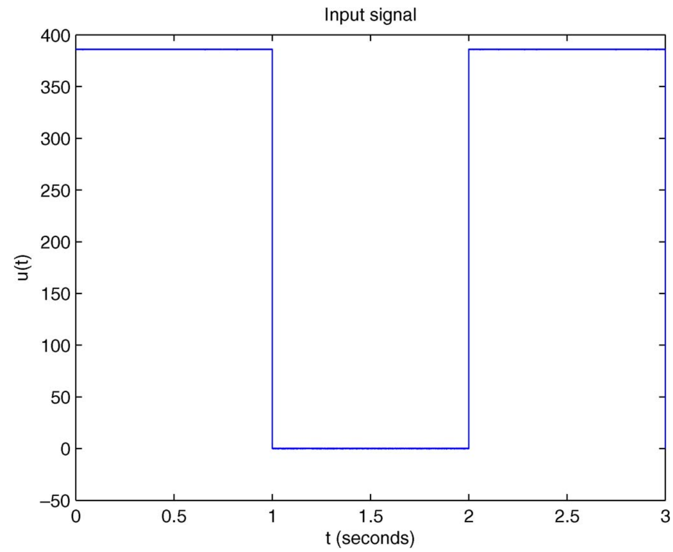

Fig. 1. Input $u(t)$.

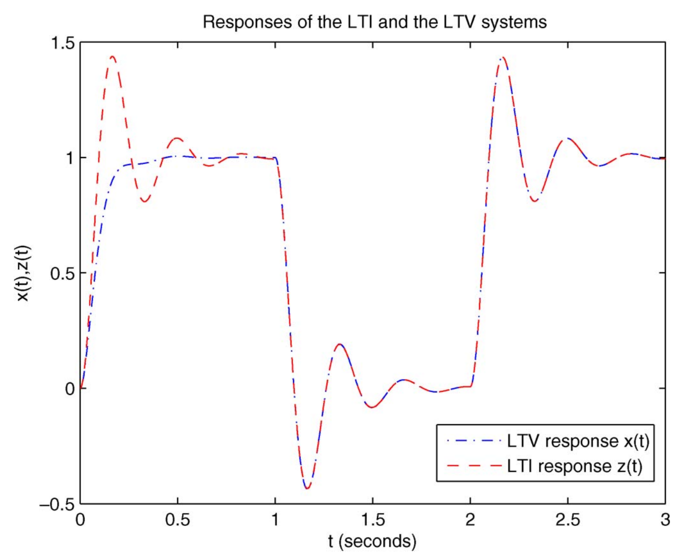

Fig. 2. Output $x(t)$ of system (3) and $z(t)$ of system (40).

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