Digital Signal Processing 19 (2009) 827–837

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

Design of wideband CIC compensator filter for a digital IF receiver

Gordana Jovanovic Dolecek[a,b,*], fred harris[b]

[a] Department of Electronics, Institute INAOE, Puebla, Mexico
[b] Department of Electrical and Computer Engineering, San Diego State University, San Diego, CA 92182-1309, USA

ARTICLE INFO

Article history:
Available online 5 April 2009

Keywords:
CIC filter
Compensator
Sharpening
Software defined radio

ABSTRACT

This paper presents the multiplierless CIC compensation filter based on the 2M-order filter and the sharpening technique. This technique proposed by Kaiser and Hamming attempts to improve the pass band and the stop band of a symmetric nonrecursive filter using the multiple copies of the same filter. We have considered the simplest sharpening polynomial that improves the frequency characteristic with minimum increase in computational complexity. The proposed filter provides wideband compensation over the specified CIC main lobe bandwidth. The design parameter is a single integer b which depends on K, the number of cascade CIC stages and is independent on the decimation factor M. The values of b tabulated here were obtained from MATLAB simulations. A number of demonstrated characteristics make the proposed structure a good candidate for software defined radio (SDR) applications.

1. Introduction

Sampling rate conversion (SRC) is an important part of today's digital signal processing systems and is one of the most frequent and useful tasks in the field of communication [1–3]. SRC involves resampling which, in the absence of filtering, causes spectral aliasing and imaging. We eliminate the undesired spectral terms resulting from resampling by appropriate filtering. Software defined radio (SDR) systems must accommodate different communication standards with a wide range of bandwidth and sample rates. The cascaded-integrator-comb (CIC) filter easily facilitates the implementation of high order and high sample rate digital filters. The CIC filter proposed in [4] is widely used as the initial decimation filter due to its simplicity which requires no multiplication or coefficient storage. A K-stage version of this filter consists of two main sections, K-integrators followed by K-comb filters. Traditionally, since the filter has reduced the input signal bandwidth from f_S to f_S/M, a factor of M-to-1, we can follow the filter with a commensurate M-to-1 down sampler as shown in the upper segment of Fig. 1. The noble identity permits the reordering of a polynomial in z^M followed by an M-to-1 down sampler to become the M-to-1 resampler followed by a polynomial in z. When this reordering is invoked, the M-to-1 resampler resides between the input integrators and the now output derivatives. This modified form shown in the lower segment of Fig. 1 is the standard Hogenauer filter structure. Both forms of this filter are referred to as a CIC filter or a Hogenauer filter.

The transfer function of the K-stage prototype of an M-tap CIC decimation filter shown in the upper segment of Fig. 1 is shown in (1).

$$H(z) = \left(\frac{1 - z^{-M}}{1 - z^{-1}}\right)^K. \tag{1}$$

* Corresponding author at: Department of Electronics, Institute INAOE, Puebla, Mexico.
E-mail addresses: gordana@inaoep.mx (G.J. Dolecek), fred.harris@sdsu.edu (f. harris).

1051-2004/$ – see front matter
doi:10.1016/j.dsp.2009.03.013

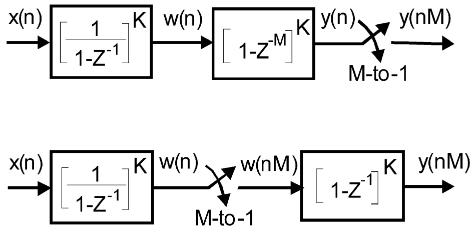


Fig. 1. K-stage, Mth order CIC filter with M-to-1 resampler and Hogenauer filter with embedded M-to-1 resampler.

This filter is also known as a K-stage M-tap boxcar filter or a K-stage recursive (length M) running sum (RRS) filter. It goes without saying that after the reordering of the resampler and output comb filters, as shown in the lower segment of Fig. 1, the filter with the embedded resampler no longer has a transfer function.

The magnitude response of the filter exhibits a linear phase, low-pass $\sin(Mx)/\sin(x)$ characteristic of the form shown in (2). Here we use the notation that digital frequency is θ, where $\theta = \omega T$, with units of radians/sample. We also note that, as with all filters, the equivalent boxcar filter has a processing gain of M for which the K-stage version has a gain of M^K. This gain is removed by an appropriate scale factor at the filter output.

$$\left|H(e^{j\theta})\right| = \left|\frac{\sin(\theta M/2)}{\sin(\theta/2)}\right|^K. \tag{2}$$

The primary spectral response of this filter is the main lobe sinc characteristic which has a DC gain normalized to unity with appropriate scaling and the quadratic main lobe droop in the desired pass band that is a function of the equivalent boxcar length M and the number of cascade sections K. We choose higher order CIC filters to obtain repeated stop band zeros with their associated reduced amplitude stop band side lobes. Higher-order filters are characterized by increased main lobe pass band droop. We compensate for this droop to make the filter suitable for SDR applications. A number of different methods have been proposed to compensate for the pass band droop of the CIC filter [5–8]. Many of the proposed methods can only compensate a narrow pass band segment of the CIC filter.

In this paper as was done in [6] we propose compensation for a wide bandwidth segment of the CIC main lobe. In the next section we introduce the standard compensating filter. Section 3 describes the sharpening technique. The proposed structure is given in Section 4 and illustrated with three examples. The discussion of the results is given in Section 5.

2. Traditional compensation filter

The spectral response of the multiple CIC stages has a main-lobe response with a Taylor series expansion about zero frequency dominated by its quadratic term. Traditional compensators select a spectral shape with a Taylor series expansion about zero frequency also dominated by its quadratic term. With proper scaling, when we perform the product of the two spectra, we can cancel the quadratic term and hence improve the flatness of the filter in cascade with its compensator. The simplest such spectrum to offer a compensating quadratic spectra is a cosine. We now illustrate how the spectral cosine is formed and properly scaled to achieve traditional compensation. We invite the reader to look at Fig. 2, described in detail shortly, to see an illustration of the two locally quadratic spectra and their interaction that achieves local spectral flattening.

Consider a compensation filter with the frequency response of (3) [8].

$$G(e^{j\theta M}) = 1 + \varepsilon \cdot \sin^2\left(\frac{M\theta}{2}\right). \tag{3}$$

Substituting the relationship of (4) in (3) we obtain (5).

$$\sin^2(\phi) = \frac{1}{2}\left[1 - \cos(2\phi)\right], \tag{4}$$

$$G(e^{j\theta M}) = 1 + \frac{\varepsilon}{2} \cdot \left[1 - \cos(M\theta)\right] = -\frac{\varepsilon}{4}e^{jM\theta} + \left(1 + \frac{\varepsilon}{2}\right) - \frac{\varepsilon}{4}e^{-jM\theta} = -\frac{\varepsilon}{4}\left[e^{jM\theta} - \frac{(1+\varepsilon/2)}{\varepsilon/4} + e^{-jM\theta}\right]. \tag{5}$$

Simplifying the center term of (5) we obtain (6) where the constants A and B are also defined for later use.

$$G(e^{j\theta M}) = -\frac{\varepsilon}{4}\left[e^{jM\theta} - \left(\frac{4}{\varepsilon} + 2\right) + e^{-jM\theta}\right] = B\left[e^{jM\theta} + A + e^{-jM\theta}\right], \tag{6a}$$

where

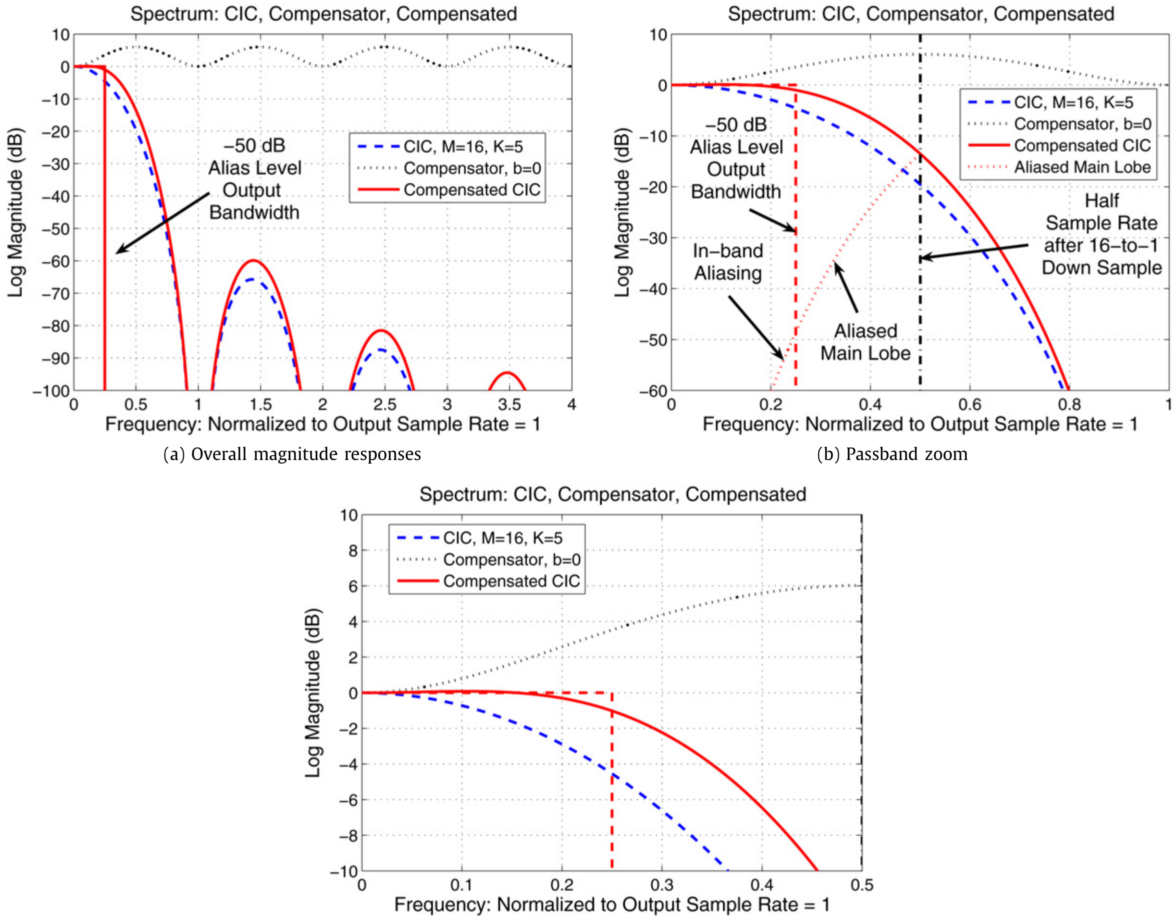
$$B = -\frac{\varepsilon}{4} \quad \text{and} \quad A = -\left(\frac{4}{\varepsilon} + 2\right). \tag{6b}$$

We recognize that the frequency response shown in (6) corresponds to the noncausal transfer function of (7).

$$G(z^M) = B\left[z^M + A + z^{-M}\right]. \tag{7}$$

Table 1
Typical values of parameter b.

K	1	2, 3	4, 5	6, 7
Parameter b	2	1	0	−1



(a) Overall magnitude responses

(b) Passband zoom

(c) Higher resolution passband zoom

Fig. 2. CIC, compensator, and compensated CIC filters.

The Taylor series expansion of (5) is shown in (8).

$$G(e^{j\theta M}) = 1 + \frac{\varepsilon}{2}\big[1 - \cos(M\theta)\big] = 1 + \frac{\varepsilon}{2}\left[1 - \left(1 - (M\theta)^2\frac{1}{2} + \cdots\right)\right] = 1 + \frac{\varepsilon}{4}(M\theta)^2 + \cdots. \tag{8}$$

We see the low frequency response of the compensator is quadratic with a positive second derivative which will compensate for the quadratic response of the CIC main lobe response with a negative second derivative. We can determine the values of ε to match the second derivatives of the two transfer functions which will give us maximally flat compensation at DC. However, we choose not to do this. Instead we restrict ε to be a power of 2, say 2^{-b}, so that the scale factor can be implemented as a binary shift and then choose the particular value of b to obtain merely a good compensation which will be converted to an excellent compensation by the sharpening filter structure. With this restriction the coefficients A and B of (6) have the form shown in (9).

$$B = -2^{-(b+2)}, \qquad A = -\big(2^{(b+2)} + 2\big), \tag{9}$$

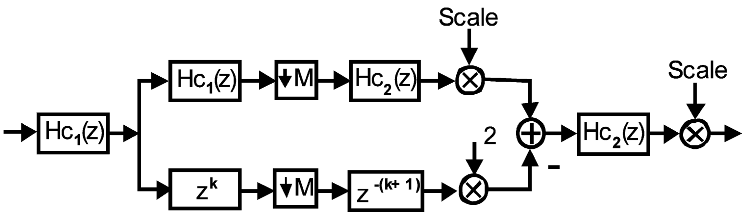
where b is integer and the values of b are in the interval $[-1, 2]$.

A MATLAB program based on (7) and (9) determined suggested values of b for a range of K. These values are shown in Table 1.

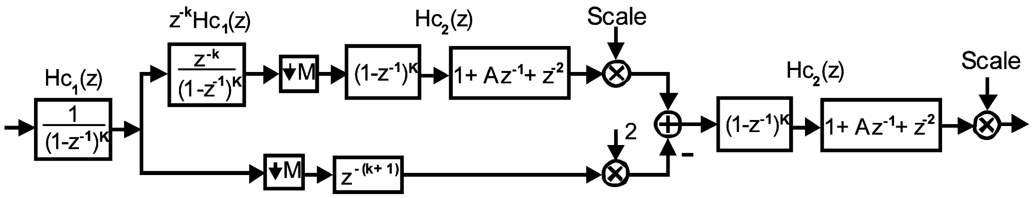
Fig. 2a presents the magnitude responses of a CIC filter for $K = 5$ and $M = 16$, along with the responses of compensation filter, the compensated CIC filter, and the desired low-pass band to be protected from the down sampling alias. Fig. 2b presents a spectral zoom to the main lobe of the CIC's spectral response. The figure shows the folding frequency at the

Table 2
ACF polynomials.

m	n	H_0
1	0	$2H - H^2$
1	1	$3H^2 - 2H^3$
2	0	$H^3 - 3H^2 + 3H$



(a)



(b)

Fig. 3. Proposed structure.

CIC output sample rate along with the alias of its main lobe response to illustrate its aliased intrusion into the pass band spectral interval at levels below −60 dB. Finally, Fig. 2c shows a more detailed zoom to the main lobe of the CIC, the compensator, and the compensated response in the desired pass band region. Here we note that the pass band droop has been significantly reduced but is still unacceptably high. In order to obtain more compensation in this wide band we propose to use the sharpening technique briefly introduced in next section.

3. Sharpening technique

To improve the compensated gain response characteristics, we propose the use of the sharpening technique which can be used to simultaneously improve the pass band and stop band characteristics of a linear-phase FIR digital filter [9]. The technique uses the amplitude change function (ACF) which is a polynomial relationship of the form $H_0 = f(H)$ between the amplitudes of the overall and the prototype filters, H_0 and H, respectively. The improvement in the gain response near the pass band edge $H = 1$, or near the stop band edge $H = 0$, depends on the order of tangencies m and n of the ACF at $H = 1$ and at $H = 0$.

The expressions for the mth and nth order of zero valued tangencies of the ACF at $H = 1$ and $H = 0$, respectively, are given in (10) as

$$H_0 = H^{n+1} \sum_{s=0}^{m} \frac{(n+s)!}{n!s!} (1-H)^s = H^{n+1} \sum_{s=0}^{m} C(n,s)(1-H)^s, \tag{10}$$

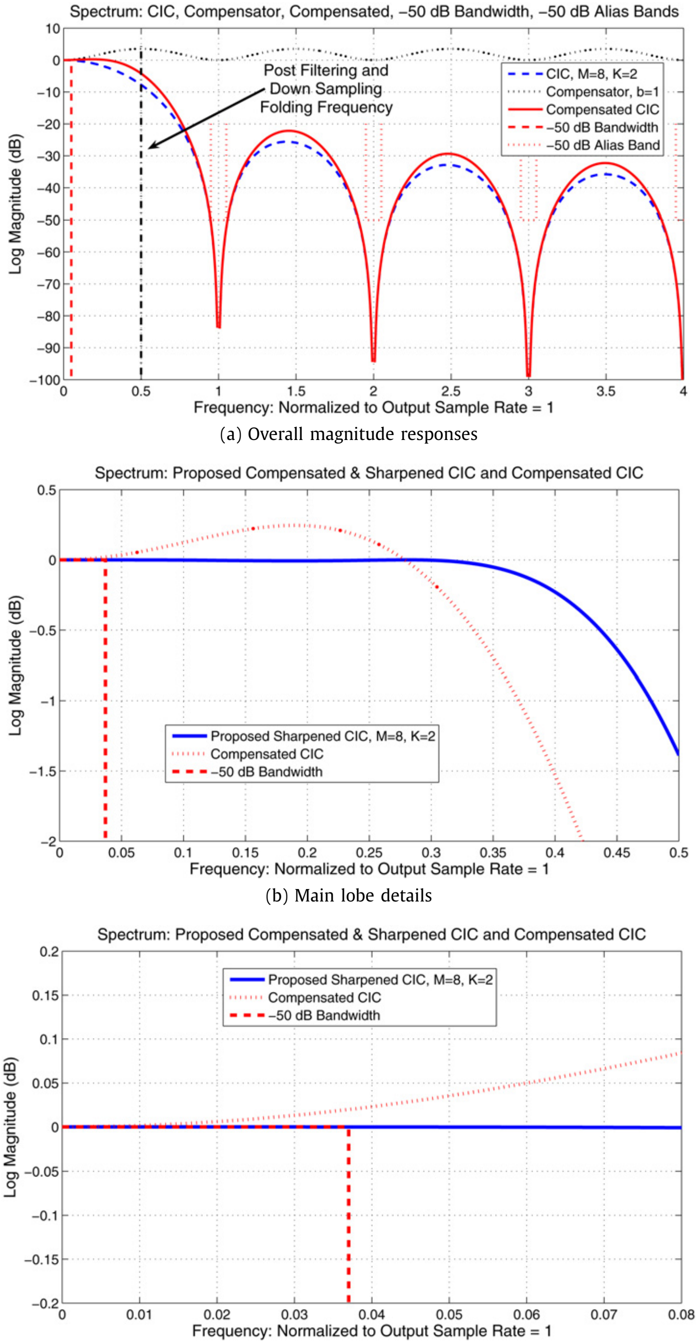
where $C(n,s)$ is the binomial coefficient. The ACF polynomials for some typical values of m and n are given in Table 2.

We obtain better improvement of the original magnitude response by using higher-order polynomials but of course with commensurately higher workload.

4. Proposed structure

Considering that we would like to compensate the CIC filter's pass band and to not increase to much the complexity of the compensated CIC filter we propose to use sharpening polynomial with $m = 1$ and $n = 0$ from Table 2.

For this paper we will use the proposed filter of (11) where the compensated filter $H_C(z)$ is shown in (12).



(a) Overall magnitude responses

(b) Main lobe details

(c) −50 dB pass band bandwidth details

Fig. 4. Example 1: $M = 8$, proposed: $K = 2$, $b = 1$, CIC: $K = 2$.

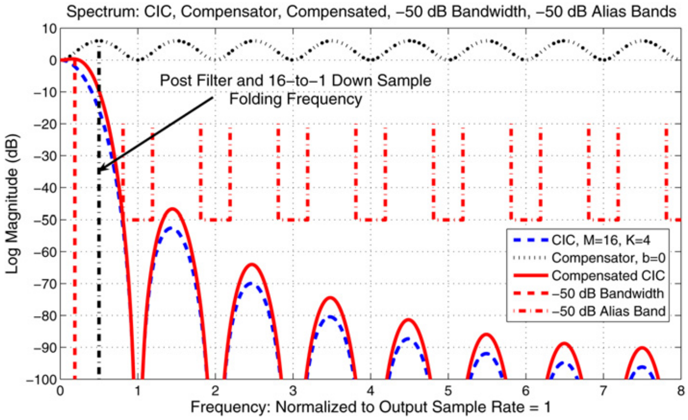
$$H_p(z) = 2H_c(z) - H_c^2(z) = H_c(z)\left[2z^{-\tau} - H_c(z)\right], \tag{11}$$

$$H_c(z) = H(z)G(z^M). \tag{12}$$

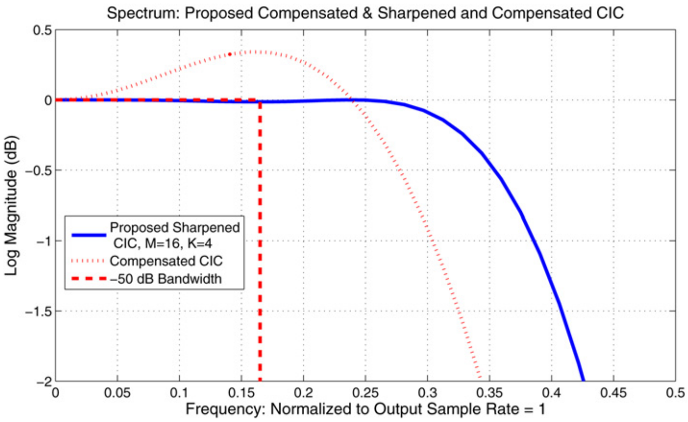
The delay τ in (11) is introduced to keep the linear phase of the filter and is defined in (13). Here the first term is half the delay of the CIC filter and the second term is half the delay of the compensator filter.

$$\tau = (M-1)K/2 + M. \tag{13}$$

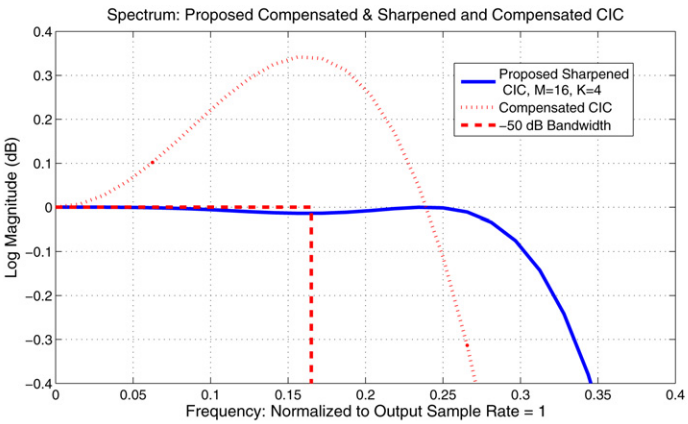
Note that K has to be even to avoid fractional delay, i.e. $K = 2k$, where k is an integer. Using (1), (6), (7), (9), (11) and (13) we rewrite Eq. (11) as follows



(a) Overall magnitude responses



(b) Main lobe details



(c) Pass band details

Fig. 5. Example 2: $M = 16$, proposed: $K = 4$, $b = 0$, CIC: $K = 4$.

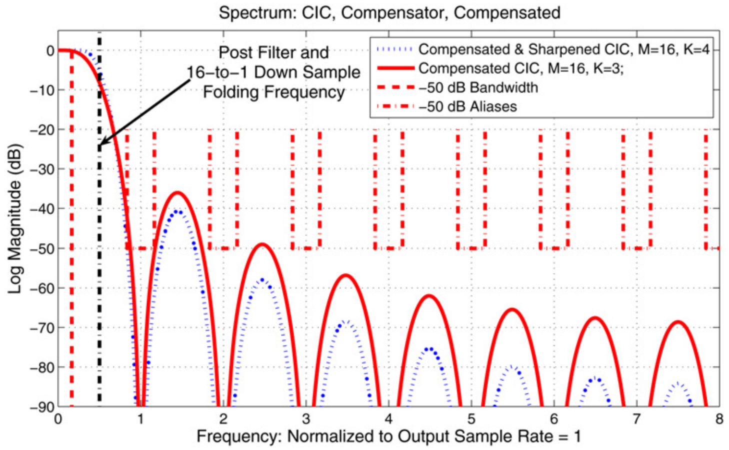
$$H_p(z) = \frac{B}{M^K} H_{c1}(z) H_{c2}(z^M) \cdot \left[2z^{-M(k+1)} z^k - \frac{B}{M^K} H_{c1}(z) H_{c2}(z^M) \right], \tag{14}$$

where

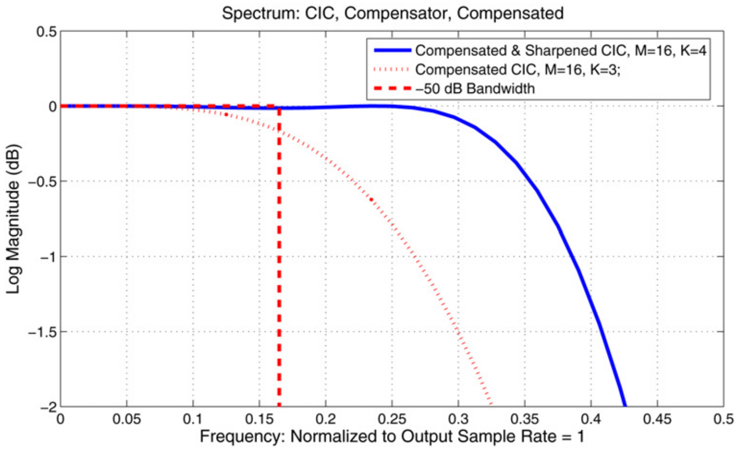
$$H_{c1}(z) = \left[\frac{1}{1 - z^{-1}} \right]^K, \tag{15}$$

$$H_{c2}(z^M) = (1 - z^{-M})^K (1 + Az^{-M} + z^{-2M}), \tag{16}$$

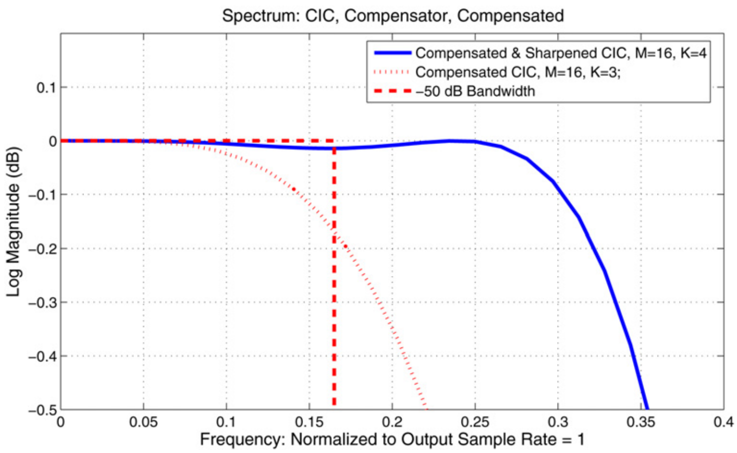
and A and B are defined in (9).



(a) Overall magnitude responses



(b) Pass band details



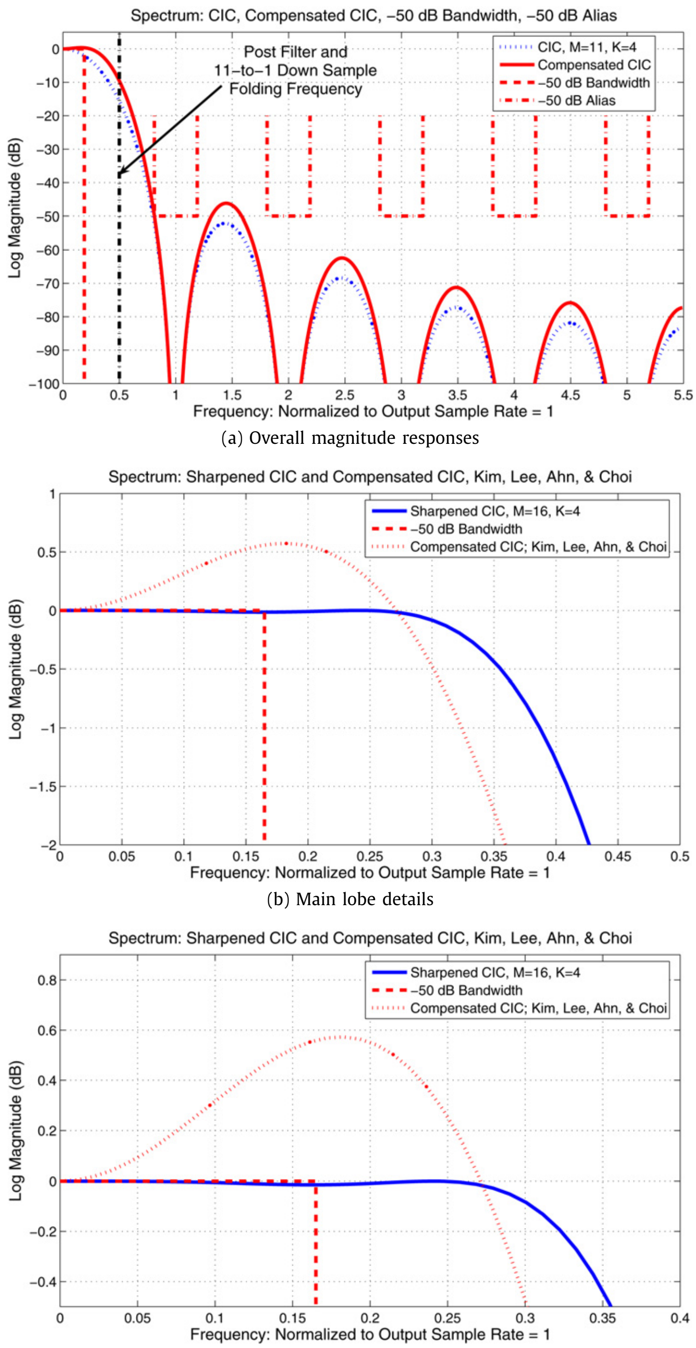
(c) Pass band details

Fig. 6. Example 3: $M = 16$, proposed: $K = 4$, $b = 0$, compensated CIC: $K = 3$.

From (14)–(16) we have structure shown in Fig. 3a. After using multirate identity [2,3] and applying the delay z^{-k} at the input to avoid advance z^k we obtain the more efficient structure shown in Fig. 3b.

From (2), (3) and (12) we have the corresponding magnitude response

$$\left|H_p(e^{j\theta})\right| = 2\left|H_c(e^{j\theta})\right| - \left|H_c(e^{j\theta})\right|^2, \tag{17}$$



(a) Overall magnitude responses

(b) Main lobe details

(c) Pass band details

Fig. 7. Comparison with method [6], $M = 11$.

where

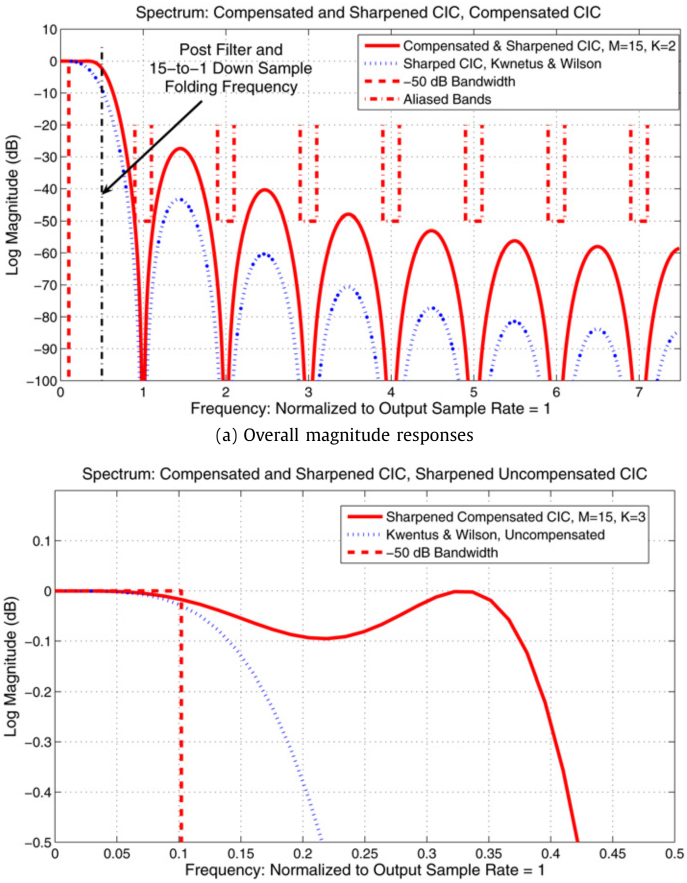
$$H_c(e^{j\theta}) = \left[\frac{\sin((M\theta)/2)}{M\sin(\theta/2)}\right]^K \left[1 + 2^{-b}\sin^2\left(\frac{\theta M}{2}\right)\right]. \tag{18}$$

Next examples illustrate the performances of the structure.

Example 1. We consider $M = 8$ and $K = 2$. From Table 1 we have $b = 1$. Fig. 4a shows the corresponding magnitude responses of the compensator and of the uncompensated and compensated CIC filters. The figure shows the bands centered at multiples of the output sample rate that alias to baseband when down sampled 8-to-1. We have selected the baseband

Table 3
Complexity comparisons.

Filter	Memory	APOS
Proposed	$(2K+k)M+2(K+2)$	$2(KM+K+3)+1$
CIC	$KM+K$	$KM+K$



(a) Overall magnitude responses

(b) Passband details

Fig. 8. Comparison with method [5].

bandwidth as that interval for which the aliasing levels are 50 dB below peak signal level. We see that the 50 dB low-pass bandwidth is approximately 4% of the output sample rate after decimation. The corresponding pass band details are given in Figs. 4b and 4c, respectively.

Example 2. Consider $M = 16$ and $K = 4$ yielding $b = 0$. The corresponding magnitude responses are given in Fig. 5a. Here, due to the greater number of CIC stages, we see the -50 dB bandwidth is approximately 17% of the output sample rate after the 16-to-1 down sampling. Pass band zooms of the main lobe response are presented in Figs. 5b and 5c.

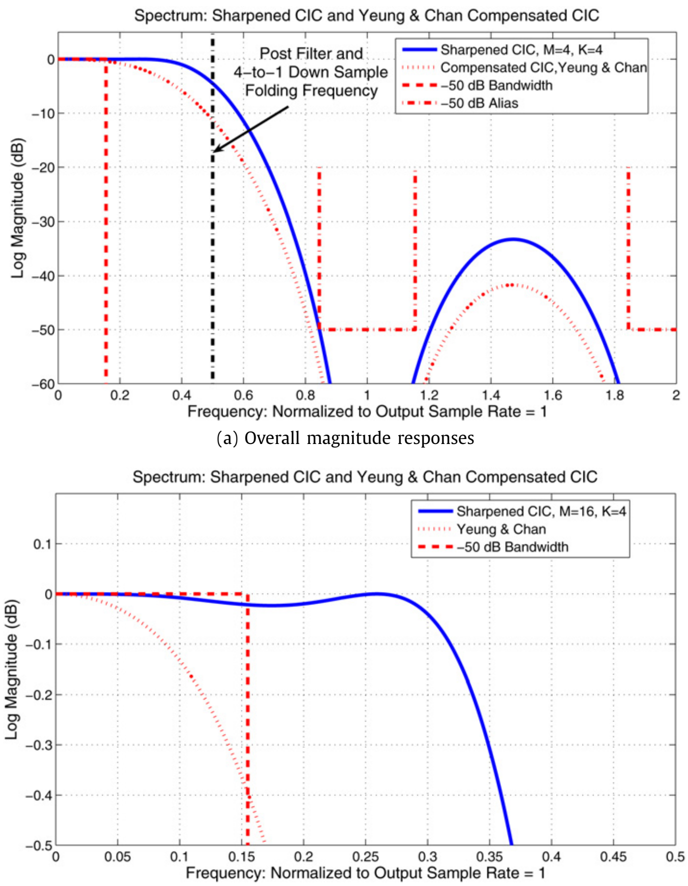
Example 3. We now compare the $K = 4$ proposed filter with a $K = 3$ CIC filter. The corresponding responses are given in Figs. 6a, 6b and 6c. In this case the proposed filter exhibits improved pass band characteristics as well as additional stop band side-lobe attenuation.

5. Discussion of the results

The complexities of the proposed filter and the conventional CIC can be compared in terms of their memory requirements and number of additions (or subtractions) per output sample (APOS) as shown in Table 3.

It follows that the proposed structure is approximately two times more complex in terms of APOS and required memory elements, than the corresponding CIC structure with the same K.

Next we compare the performance of the proposed method with that of some recent methods.



(a) Overall magnitude responses

(b) Passband details

Fig. 9. Comparison of proposed method with method [7].

Example 4 *(Method of Kim et al. [6]).* Here these authors [6] proposed the use of a CIC roll-off compensation filter in a W-CDMA digital IF receiver. The coefficients of the compensation filter are given by

$$\left[-a/(1-2a), 1/(1-2a), -a/(1-2a)\right]. \tag{19}$$

The performance of the compensation filter depends on the value of a, which is obtained by minimizing the corresponding error function. This filter provides compensation in the wideband. We compare the proposed result with the method of Kim for $M = 11$, where the second stage decimation is 2. For this configuration, the value of a is 0.1799 [6]. Fig. 7a shows the corresponding magnitude responses. Fig. 7b compares the compensated pass bands in the filter's main lobes. Note that the proposed filter exhibits much better compensation comparing with the filter of [6]. Fig. 7c is a high resolution zoom to the pass band region where we can compare the improved compensation of the proposed filter in the −50 dB pass band equal to approximately 17% of the output sample rate following the 11-to-1 down sample.

Example 5 *(Method of Kwentus and Willson [5]).* Kwentus and Willson [5] have proposed the application of a higher order sharpening filter to a conventional CIC filter, without the pre-compensation, to provide significantly improved performances of the CIC filter. The sharpening polynomial from Table 1 was $n = m = 1$. Fig. 8 compares this option [5] where $M = 15$ and K was 2 to the proposed method with $K = 3$. Note that the proposed method provides much better pass band compensation and has reduced amplitude stop-band side lobes.

Example 6 *(Method of Yeung and Chan [7]).* Yeung and Chan proposed use of a second-order CIC compensator $a + bz^{-1} + az^{-2}$ with two real-valued constants a and b. Given the frequency response of the CIC filter, the constants are determined using the Parks-McClellan algorithm. To reduce the implementation complexity, the constants are expressed as SOPOT (sum-of-powers-of-two) representation. The random search algorithm is employed to minimize the total number of SOPOT terms. The resulting SOPOT coefficients from [7] are $a = -2^{-3}$; $b = 2^0 + 2^{-2}$. Fig. 9 compares this option to the proposed filter for $M = 4$ and $K = 4$. Note that the proposed filter provides improved compensation in the −50 dB bandwidth of approximately 16% of the output sample rate after the 4-to-1 down sample. The filter designed by method [7] has slightly reduced level side lobes in the stop band region. Note here, as in previous example, it is the main lobe width, as opposed to the side lobe levels that determines the alias free bandwidth of the compensated and sharpened filter.

6. Conclusions

The simple method for compensation of CIC filters is presented here. The method is based on a simple multiplier less compensation filter embedded in the sharpening filter technique. The compensation filter has a single parameter b which depends on the number of cascaded CIC filters. The sharpening polynomial is chosen with the goal to improve only the pass band frequency interval with minor increase in computational complexity. Consequently we have selected the sharpening polynomial structure of Table 1 corresponding to $m = 1$, $n = 0$. The proposed filter provides good compensation in the selected alias level pass band.

Acknowledgments

This work is supported by CONACYT grant Nos. 49640 and 91013.

References

[1] T. Hentschel, G. Fettweis, Sample rate conversion for software radio, IEEE Commun. Mag. (2000) 142–150.
[2] f.j. harris, Multirate Signal Processing for Communication Systems, Prentice-Hall, 2006.
[3] E.B. Hogenauer, An economical class of digital filters for decimation and interpolation, IEEE Trans. Acoust. Speech Signal Process. ASSP-29 (2) (1981) 155–162.
[4] G. Jovanovic Dolecek (Ed.), Multirate Systems, Design and Applications, Idea Group Publishing, Hershey, 2002.
[5] A. Kwentus, A. Willson Jr., Application of filter sharpening to cascaded integrator–comb decimation filters, IEEE Trans. Signal Process. 45 (2) (1997) 457–467.
[6] S. Kim, W.C. Lee, S. Ahn, S. Choi, Design of CIC roll-off compensation filter in a W-CDMA digital receiver, Digit. Signal Process. 16 (2006) 846–854.
[7] K.S. Yeung, S.C. Chan, The design and multiplier-less realization of software radio receivers with reduced system delay, IEEE Trans. Circuits Syst.-I: Regul. Pap. 51 (12) (2004) 2444–2459.
[8] G. Jovanovic Dolecek, S.K. Mitra, Simple method for compensation of CIC decimation filter, Electron. Lett. 44 (19) (2008) 1162–1163.
[9] J.F. Kaiser, R.W. Hamming, Sharpening the response of a symmetric nonrecursive filter by multiple use of the same filter, IEEE Trans. Acoust. Speech Signal Process. 25 (5) (1977) 415–422.

Gordana Jovanovic Dolecek received a BS degree from the Department of Electrical Engineering, University of Sarajevo, an MSc degree from University of Belgrade, and a PhD degree from the Faculty of Electrical Engineering, University of Sarajevo. She was professor at the Faculty of Electrical Engineering, University of Sarajevo until 1993, and 1993–1995 she was with the Institute Mihailo Pupin, Belgrade. In 1995 she joined Institute INAOE, Department for Electronics, Puebla, Mexico, where she works as a professor and researcher. During 2001–2002 and 2006 she was with Department of Electrical & Computer Engineering, University of California, Santa Barbara, as visiting researcher. She is currently with San Diego State University as visiting researcher on a sabbatical leave. She is the author of three books, editor of one book, and author of more than 200 papers. Her research interests include digital signal processing and digital communications. She is a Senior member of IEEE, the member of Mexican Academy of Sciences, and the member of National Researcher System (SNI) Mexico.

fred harris is professor of Electrical and Computer Engineering at San Diego State University, where he holds the CUBIC Signal Processing Chair of the Communication Systems and Signal Processing Institute. He has extensive practical experience applying his skills to satellite and cable TV communication systems, wire-line and wireless modems, underwater acoustics, advanced radar and high performance laboratory instrumentation. He holds several patents on digital receiver and DSP technology, lectures on DSP worldwide, and consults for organizations requiring high performance DSP systems including the Navy Ocean Systems Center, Lockheed, ESL, Cubic, Hughes, Sylvania-GTE, Scientific Atlanta, Rockwell, Brooktree, Comstream, Hewlett Packard, Tektronix, TRW, GDE, and Motorola. He is well published and has contributed to a number of books on DSP. In 1990 and 1991 he was the Technical and then the General Chair of the Asilomar Conference on Signals, Systems, and Computers which meets annually in Pacific Grove, California. He is Editor of Digital Signal Processing, Journal of Elsevier.

He is the author of the book "Multirate Signal Processing for Communication Systems", published by Prentice Hall in 2004.