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Generation of a secondary electromagnetic source with desired statistical properties

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ABSTRACT

Two known alternative techniques for generating a secondary electromagnetic Schell-model source with the prescribed statistical properties are analyzed and compared from the point of view of their capability to provide the given degree of polarization and the desired transverse coherence length.

1. Introduction

In many optical applications it is necessary to create a secondary electromagnetic source with the desired statistical properties (see, e.g. Refs. [1–3]). The following two opposite approaches can be used to solve this problem.

In the first approach one can start from completely incoherent and completely unpolarized primary source and try to get the desired result due to the so-called correlation-induced changes of an electromagnetic field on propagation [4]. This can be done on making use of the vector version of the van Cittert-Zernike theorem for the first time derived by Gori et al. [5] and later on reformulated in slightly different form in Ref. [6]. Such an approach has been employed for generating an electromagnetic Gaussian Schell-model source [7] and for generating the most general electromagnetic Schell-model source [8] (earlier it was used for synthesis of a scalar Collett-Wolf source [9,10]).

In the second approach the desired result can be obtained due to modulation of the electromagnetic radiation from completely coherent and completely polarized primary source by a random phase screen [11]. Such an approach has been also used for generating a genuine electromagnetic Gaussian Schell-model source [12] and for generating some kind of the electromagnetic Schell-model source [13].

In the present paper we analyze and compare two known alternative techniques, which realize the mentioned approaches, from the point of view of their capability for generating an electromagnetic Schell-model source with the given uniform degree of polarization and the desired transverse coherence length.

2. Background

As well known [14], the second-order statistical properties of a random planar electromagnetic source (primary or secondary one) can be completely characterized by the 2×2 cross-spectral density matrix $\mathbf{W}(\mathbf{x}_1,\mathbf{x}_2)$ (for brevity we omit the explicit dependence of the considered quantities on optical frequency ν). Using this matrix, the fundamental statistical characteristics of the source can be defined: the power spectrum

$$S(\mathrm{x}) = \mathrm{Tr}\,\mathbf{W}(\mathrm{x},\mathrm{x}), \tag{1}$$

the spectral degree of coherence

$$\eta(\mathrm{x}_1,\mathrm{x}_2) = \frac{\mathrm{Tr}\,\mathbf{W}(\mathrm{x}_1,\mathrm{x}_2)}{[\mathrm{Tr}\,\mathbf{W}(\mathrm{x}_1,\mathrm{x}_1)\mathrm{Tr}\,\mathbf{W}(\mathrm{x}_2,\mathrm{x}_2)]^{1/2}}, \tag{2}$$

and the spectral degree of polarization

$$P(\mathrm{x}) = \left\{1-\frac{4\,\mathrm{Det}\,\mathbf{W}(\mathrm{x},\mathrm{x})}{[\mathrm{Tr}\,\mathbf{W}(\mathrm{x},\mathrm{x})]^2}\right\}^{1/2}. \tag{3}$$

In Eqs. (1)–(3) Tr stands for the trace and Det denotes the determinant. If the modulus of the degree of coherence $\eta(\mathbf{x}_1,\mathbf{x}_2)$ depends on the distance between the points $\mathbf{x}_1$ and $\mathbf{x}_2$, and does not depend on their positions, the coherence of the source can be characterized numerically by the quantity $\triangle\xi$, known as the transverse coherence length and defined as [1,15]

$$\triangle\xi = 2\int_0^\infty |\eta(\xi)|^2 d\xi, \tag{4}$$

where $\xi = |\mathbf{x}_1 - \mathbf{x}_2|$. The larger $\triangle\xi$, the greater is the coherence of the source.

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3. Generation of secondary source using changes of coherence and polarization induced by free space propagation

Let the primary quasi-monochromatic source occupying a finite domain D in the plane $z=0$ be characterized by the cross-spectral density matrix

$$\mathbf{W}_{PS}(\mathbf{x}_1',\mathbf{x}_2') = \frac{\varkappa}{2}\left[S_{PS}(\mathbf{x}_1')S_{PS}(\mathbf{x}_2')\right]^{1/2}\delta(\mathbf{x}_1'-\mathbf{x}_2')\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \tag{5}$$

where $\delta(\mathbf{x})$ is the two-dimensional Dirac function and $\varkappa$ is a constant with dimensions of squared length. It may be readily verified that in this case $\eta_{PS}(\mathbf{x}_1,\mathbf{x}_2)=0$ for any $\mathbf{x}_1\neq\mathbf{x}_2$ and $P_{PS}(\mathbf{x})=0$ (see Appendix A), i.e. that such a source is completely incoherent and completely unpolarized. Let us assume that the radiation from this source passes through a modified Mach-Zehnder interferometer sketched schematically in Fig. 1.

The polarizing beam splitter PBS separates the orthogonal field components so that each of them can be independently modified by means of two optical attenuators A1 and A2, chosen to attenuate the power spectra of optical fields at the opposite arms of the interferometer in the ratio α:$(1-\alpha)$ with α being the desired coefficient of attenuation. The field components are superposed at the output of the second (non-polarizing) beam splitter BS and are the subject of the Fourier transform, realized by a thin converging lens, placed at its focal distance f from the primary and secondary source planes. Then, applying the paraxial approximation for beamlike wave propagation and using the vector version of the van Cittert-Zernike theorem [5], we find that the cross-spectral density matrix of the secondary source generated at the far-zone region is given by

$$\mathbf{W}_{SS}(\mathbf{x}_1,\mathbf{x}_2) = \frac{\varkappa}{2(\lambda f)^2}\int_{(z=0)} S_{PS}(\mathbf{x}')\exp\left[-i\frac{2\pi}{\lambda f}\mathbf{x}'\cdot(\mathbf{x}_2-\mathbf{x}_1)\right]d\mathbf{x}' \times\begin{bmatrix}\alpha & 0\\ 0 & 1-\alpha\end{bmatrix}, \tag{6}$$

where λ is the mean wavelength. On substituting from Eq. (6) into Eqs. (2) and (3), we obtain, respectively,

$$\eta_{SS}(\mathbf{x}_1,\mathbf{x}_2) = \frac{\int_{(z=0)} S_{PS}(\mathbf{x}')\exp\left[-i\frac{2\pi}{\lambda f}\mathbf{x}'\cdot(\mathbf{x}_2-\mathbf{x}_1)\right]d\mathbf{x}'}{\int_{(z=0)} S_{PS}(\mathbf{x}')d\mathbf{x}'}, \tag{7}$$

$$P_{SS}(\mathbf{x}) = |2\alpha-1|. \tag{8}$$

As can be seen from Eq. (8), when α varies from 0.5 to 1, the degree of polarization changes in the full range from 0 to 1. To examine the behavior of the degree of coherence given by Eq. (7), we consider a particular case, when the primary source has the Gaussian power spectrum

$$S_{PS}(\mathbf{x}') = S_0\exp\left(-\frac{\mathbf{x}'^2}{4\varepsilon^2}\right), \tag{9}$$

where S_0 and ε are positive constants. It may be readily shown that in this case Eq. (7) takes the form

$$\eta_{SS}(\mathbf{x}_1,\mathbf{x}_2) = \exp\left[-\left(\frac{2\pi\varepsilon}{\lambda f}\right)^2\xi^2\right], \tag{10}$$

where ξ has the same meaning as in Eq. (4). Then, substituting from Eq. (10) into Eq. (4), after straightforward calculations we obtain

$$\triangle\xi = 0.2\frac{\lambda f}{\varepsilon}. \tag{11}$$

Analyzing Eq. (11), we come to the conclusion that the considered technique allows to generate an electromagnetic source with a very small transverse coherence length. For example, for $\lambda=0.5\mu$m, $f=1$m and $\varepsilon=1$mm, the value of $\triangle\xi$ consists of 0.1mm that is surely much larger than the wavelength but is much less than the size of the primary source. Nevertheless, it may be useful in some practical applications, as in remote optical sensing and in communication optics.

4. Generation of secondary source using changes of coherence and polarization induced by a random phase screen

Now let the primary quasi-monochromatic source be characterized by the cross-spectral density matrix

$$\mathbf{W}_{PS}(\mathbf{x}_1,\mathbf{x}_2) = \frac{1}{2}\left[S_{PS}(\mathbf{x}_1)S_{PS}(\mathbf{x}_2)\right]^{1/2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}. \tag{12}$$

It may be readily verified that in this case $\eta_{PS}(\mathbf{x}_1,\mathbf{x}_2)=1$ and $P_{PS}(\mathbf{x})=1$, i.e. that such a source is completely coherent and completely polarized. Let us assume that the radiation from this source passes through a system which consists of two crossed zero-twisted (parallel aligned) nematic liquid-crystal displays controlled by computer (Fig. 2). It has

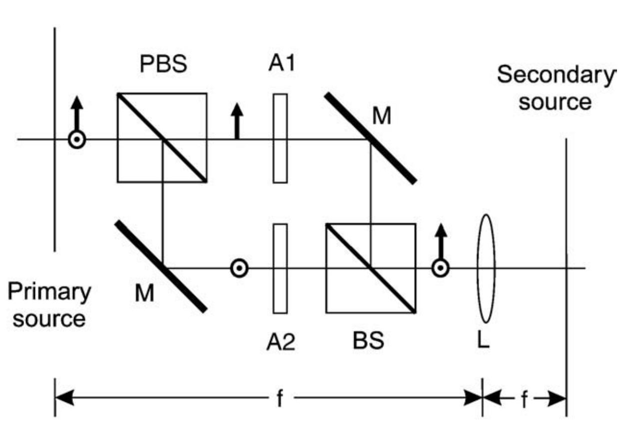

Fig. 1. Modified Mach-Zehnder interferometer: BS — beam splitter; PBS — polarizing beam splitter; M — mirror; A — attenuator; and bold-faced arrows denote polarization state.

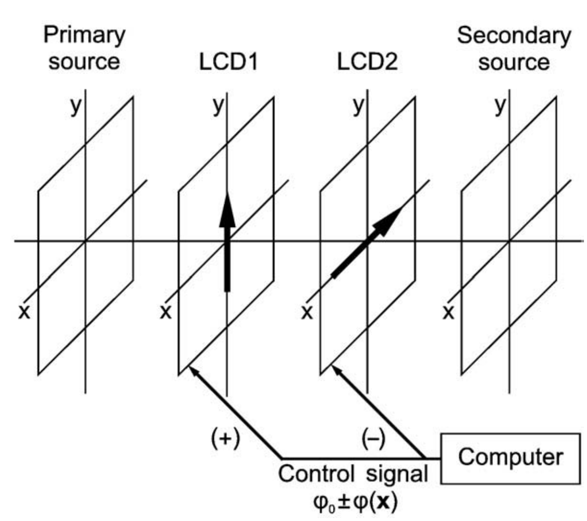

Fig. 2. System of two crossed zero-twisted liquid-crystal displays: LCD — liquid-crystal display; and bold-faced arrows denote the fast axis of liquid-crystal.

been shown [13] that under certain conditions the transmittance of such a system can be represented as

$$\mathbf{T}(\mathbf{x}) = \exp(\mathrm{i}\varphi_0)\begin{bmatrix} \exp[-\mathrm{i}\varphi(\mathbf{x})] & 0 \\ 0 & \exp[\mathrm{i}\varphi(\mathbf{x})] \end{bmatrix}, \tag{13}$$

where φ_0 is a constant and $\varphi(\mathbf{x})$ is a varying control signal generated by a computer. We assume also that the control signal $\varphi(\mathbf{x})$ represents a random variable which obeys the Gaussian statistics with zero mean, variance σ_φ, and Gaussian cross-correlation of width γ_φ for two points $\mathbf{x}_1$ and $\mathbf{x}_2$.

The cross-spectral density matrix of the secondary source formed at the output of liquid-crystal displays is given by

$$\mathbf{W}_{SS}(\mathbf{x}_1,\mathbf{x}_2) = \langle \mathbf{T}^\dagger(\mathbf{x}_1)\mathbf{W}_{PS}(\mathbf{x}_1,\mathbf{x}_2)\mathbf{T}(\mathbf{x}_2)\rangle, \tag{14}$$

where the angle brackets denote the ensemble average and the dagger denotes the Hermitian adjoint. On substituting from Eqs. (12) and (13) into Eq. (14) and making use of formula [13]

$$\begin{aligned}\langle \exp\{+\mathrm{i}[\varphi(\mathbf{x}_2)\pm\varphi(\mathbf{x}_1)]\}\rangle &= \langle \exp\{-\mathrm{i}[\varphi(\mathbf{x}_2)\pm\varphi(\mathbf{x}_1)]\}\rangle \\ &= \exp\left\{-\sigma_\varphi^2\left[1\pm\exp\left(-\frac{\xi^2}{2\gamma_\varphi^2}\right)\right]\right\},\end{aligned} \tag{15}$$

we obtain

$$\begin{aligned}\mathbf{W}_{SS}(\mathbf{x}_1,\mathbf{x}_2) &= \frac{1}{2}[S_{PS}(\mathbf{x}_1)S_{PS}(\mathbf{x}_2)]^{1/2}\exp\left(-\sigma_\varphi^2\right) \\ &\times \begin{bmatrix} \exp\left[\sigma_\varphi^2\exp\left(-\frac{\xi^2}{2\gamma_\varphi^2}\right)\right] & \exp\left[-\sigma_\varphi^2\exp\left(-\frac{\xi^2}{2\gamma_\varphi^2}\right)\right] \\ \exp\left[-\sigma_\varphi^2\exp\left(-\frac{\xi^2}{2\gamma_\varphi^2}\right)\right] & \exp\left[\sigma_\varphi^2\exp\left(-\frac{\xi^2}{2\gamma_\varphi^2}\right)\right] \end{bmatrix}.\end{aligned} \tag{16}$$

On substituting from Eq. (16) into Eqs. (2) and (3), we obtain, respectively,

$$\eta_{SS}(\mathbf{x}_1,\mathbf{x}_2) = \exp\left\{-\sigma_\varphi^2\left[1-\exp\left(-\frac{\xi^2}{2\gamma_\varphi^2}\right)\right]\right\}, \tag{17}$$

$$P_{SS}(\mathbf{x}) = \exp\left(-2\sigma_\varphi^2\right). \tag{18}$$

As can be seen from Eq. (18), varying the parameter σ_φ, one can change the degree of polarization in a wide range, practically from 0 to1. To examine the behavior of the degree of coherence, we note that for large values of σ_φ Eq. (17) can be well approximated by the expression [11]

$$\eta_{SS}(\mathbf{x}_1,\mathbf{x}_2) = \exp\left(-\frac{\sigma_\varphi^2\xi^2}{2\gamma_\varphi^2}\right). \tag{19}$$

Then, substituting from Eq. (19) into Eq. (4), after straightforward calculations we obtain

$$\triangle\xi = 1.8\frac{\gamma_\varphi}{\sigma_\varphi}. \tag{20}$$

Hence, varying the parameter γ_φ, one can change the transverse coherence length of the secondary source in a very large range independently on the wavelength and size of the primary source as it had place in the case of previous technique.

5. Discussion

We have examined the two alternative techniques for generating a secondary electromagnetic source with the desired statistical properties. The first technique is based on the correlation-induced changes of the electromagnetic field on propagation while the second one uses the modulation of the coherence and polarization by a random phase screen, such as the liquid-crystal display. Analyzing the corresponding results given by Eqs. (6) and (16), first of all, we come to the conclusion that both techniques allow to produce some secondary electromagnetic source of the Schell-model class, namely the source whose cross-spectral density matrix depends only on the distance between points $\mathbf{x}_1$ and $\mathbf{x}_2$. Secondly, the degree (uniform) of polarization of the generated source can be practically varied from 0 to 1. But, the transverse coherence length of the generated source can be made relatively larger only with the use of the second technique. Physically the last fact can be clearly understood, taking into account that in the first case we relies only upon the physics of the electromagnetic field propagation, while in the second case we make the specially forced changes of the statistical structure of the field. It seems that the results obtained here can be useful when one deals with the experimental synthesis of an electromagnetic source with the desired statistical properties and will serve to the development of modern coherence theory.

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Appendix A. Proof of the equality PPS(x) = 0 for source given by Eq. (5)

Integrating both sides of Eq. (5) in the source plane with respect to the variable $\mathbf{x}_1'$ and applying the filtering property of the Dirac function, we obtain

$$\int_{-\infty}^{\infty}\mathbf{W}_{PS}(\mathbf{x}_1',\mathbf{x}_2')\mathrm{d}\mathbf{x}_1' = \frac{\varkappa}{2}S_{PS}(\mathbf{x}_2')\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \tag{A1}$$

Since the matrix $\mathbf{W}_{PS}(\mathbf{x}_1',\mathbf{x}_2')$ vanishes outside finite domain D, Eq. (A1) can be rewritten as

$$\int_{(D)}\mathbf{W}_{PS}(\mathbf{x}_1',\mathbf{x}_2')\mathrm{d}\mathbf{x}_1' = \frac{\varkappa}{2}S_{PS}(\mathbf{x}_2')\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \tag{A2}$$

Equality (A2) holds for any pair $(\mathbf{x}_1',\mathbf{x}_2')$, and hence, for the pair $(\mathbf{x}_2',\mathbf{x}_2')$, i.e.

$$\mathbf{W}_{PS}(\mathbf{x}_2',\mathbf{x}_2')\int_{(D)}\mathrm{d}\mathbf{x}_1' = \frac{\varkappa}{2}S_{PS}(\mathbf{x}_2')\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \tag{A3}$$

From Eq. (A2) it follows that

$$\mathbf{W}_{PS}(\mathbf{x}',\mathbf{x}') = \frac{\varkappa}{2Q}S_{PS}(\mathbf{x}')\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \tag{A4}$$

where Q is the area of domain D. Then, substituting from Eq. (A3) into Eq. (3), we obtain the required result

$$P(\mathbf{x}') = 0. \tag{A5}$$

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