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Exploring fabrication tolerances of optical systems by solving inequalities

Franciso-J. Renero-C *

Instituto Nacional de Astrofísica Óptica y Electrónica, Luis Enrique Erro No 1, Tonantzintla 72840, Puebla, México

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ABSTRACT

A scheme to study fabrication tolerances of optical components and systems is presented. By solving a set of non-linear inequalities an optimal set of lens parameters is found. Then, by selecting another, but strict, set of boundaries to the non-linear inequalities system, a new set of lens parameters is found. The fabrication tolerances are determined by the intersection domain of each lens parameters obtained by solving the set of non-linear equations, with a condition that one solution should come from a strict set on boundaries of the non-linear equation system. This scheme is applied to a classical triplet lens system, used as starting point, and is compacted in effective and back focuses lengths, showing the versatility to study the fabrication tolerances.

1. Introduction

A primary problem found in the fabrication of optical components and systems is related to tolerances for the lens parameters (radii of curvature, thickness, glass material, etc.). Several techniques are available; some of them are based on experience of the workshop, sometimes supported by commercial software. In some applications, tolerances can be given in general terms by means of percentage to the specific parameters. Also by means of the merit function, which is used to predict a permitted increase, the tolerances can be studied. Also, tolerances can be estimated by means of statistics [1–5].

On the other hand, several computation algorithms achieve automatic lens design. One of them is by solving inequalities [6]. The creators of this technique showed its potential for balancing optical aberrations. It had also been showed its potential to optimize optical lens systems with misalignment and optical lens systems for multiple images [7,8].

In this work, the algorithm of Automatic Lens design by Solving InEqualities (ALSIE) is used to study tolerances to the lens parameters. Several restrictions sets to a given inequalities system (it can be stated as a set of boundaries for the inequality system), which describes the optical system, can be solved to find several sets of lens parameters. The domain established by these sets of lens parameters settles the tolerances.

The scheme is applied to a conventional triplet lens system (*f*/♯ 8.3, *f*: 99.95 mm, B*f*: 84 mm), to be compacted in effective back focus lengths (*f*/♯ 4.0, *f*: 50 mm, *Bf*: 40 mm). Thus, all non-linear inequalities systems are composed of 33 non-linear functions and 12 variables (the performance functions and the lens parameters, respectively). Performance functions are optical aberrations and mechanical conditions. The lens parameters allowed to change are radii of curvature and thicknesses. Tolerances are different for each lens parameter; for radii of curvature they fluctuate between 3% and 16%, while for thickness between 0.3% and 25%.

2. Automatic Lens design by Solving InEqualities (ALSIE)

The problem of finding a set of lens parameters, achieving an optimal optical performance, is stated by

$$\alpha_i \le f_i(\vec{x}) \le \beta_i, \tag{1}$$

where α_i and β_i are the boundaries for each performance function, $f_i(\vec{x})$, which are chosen according to the application of the optical system; and $\vec{x}$ is a vector representing the lens parameters (radii of curvatures, thicknesses, etc.). Later, the inequalities system (1) is solved, for instance by the relaxation method (linearization is required). Then, vector $\vec{x}$ becomes the lens parameters that achieve the desired optical performance.

Thus, given an optical system or once its application is given, the performance functions and their sets of boundaries are chosen. Then, the non-linear system of inequalities (1) is solved, getting

$$\vec{x}^{(1)} : f_i(\vec{x}^{(1)}) \in [a_i^{(1)}, \beta_i^{(1)}. \tag{2}$$

By choosing a new set of boundaries such as

$$[a_i^{(2)}, \beta_i^{(2)}] \in [a_i^{(1)}, \beta_i^{(1)}]. \tag{3}$$

A different set of lens parameters $\vec{x}^{(2)}$ is found

$$\vec{x}^{(2)} : f_i(\vec{x}^{(2)}) \in [a_i^{(2)}, \beta_i^{(2)}]. \tag{4}$$

* Tel.: +52 222 266 3100x2209; fax: +52 222 247 2940.
E-mail address: paco@inaoep.mx

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Table 1.
Some performance function measured in ALSIE, and its dependence on ray parameters and surface number.

Performance function	*Corresponding number (CN)*	*Principal ray angle (θ)*	*High at entrance pupil (h)*	*Surface number (n)*
Effective focal length	1			
Back focus	2			
f/♯	3			
Total length of lens system	25			
Axial separation	26			√
Spherical aberration	4		√	
Marginal astigmatism	6	√		
Sagital astigmatism	7	√		
Marginal coma	10	√	√	
Sagital coma	11	√	√	
Distortion	14	√		

Table 2.
Lens parameters of a classical triplet lens ($\vec{x}^{(0)}$: f/#=8.25, f=99.95, all units are given in millimeter).

Surface ♯	Radii of curvature	Thickness	Ohara S-type glass
1[a]	∞	∞	Air
2	24.11	3.70	S-BSM4
3	215.09	4.66	Air
4	−94.81	1.60	S-TIM8
5	23.76	2.37	Air
6[b]	∞	6.76	Air
7	104.50	3.50	S-BSM4
8	−63.89	84.12[c]	Air

[a] Object.
[b] Stop.
[c] Back focus.

This process can be achieved as many times as possible (or allowed by the restrictions and solution space). Then, the space defined by vectors $\vec{x}^{(1)}$ and $\vec{x}^{(2)}$ is used to specify the tolerance for each lens parameter.

Table 1 shows some performance functions measured in ALSIE. They show the dependence on ray parameters for optical aberrations. Total length of lens system is taken from first to last physical surfaces.

3. Measurement of closeness as tolerances

Table 2 shows the lens parameters for a classical triplet lens system (represented by $\vec{x}^{(0)}$). It is a distinctive image forming lens system (*f*: 99.95 mm, *Bf*: 84.12 mm, *f*/♯ 8.25). Particularly, this lens system shows astigmatism of about −1 mm for the full field of view (22°); all other aberrations are smaller than 0.5 mm; distortion is about 0.6%. Table 3 shows the performance functions evaluated for this classical triplet lens system, with the scheme shown in Table 1.

Thus, the goal by using ALSIE, summarized in Table 4, is to obtain shorter effective and back focuses lengths, trying to maintain optical aberrations as small as possible (the *f*/# have to be reduced too), and to preserve the field of view; moreover, the total length of the lens system is also reduced.

Table 5 shows α's and β's of four sets of restrictions to the non-linear inequalities to be solved by ALSIE. The first set,$\alpha_i^{(1)}$ and$\beta_i^{(1)}$, is conceived so as to get the requirements stated in Table 4. So, optical aberrations are allowed to have a maximum variation of ± 1 mm. The three following sets of α_i's and β_i's are done in order

Table 3.
Performance functions evaluated for the classical triplet lens system (represented by vector $\vec{x}^{(0)}$ and is shown in Table 2). Units of performance functions are given in millimeter (except distortion, which is given in percentage).

CN	***θ***	***H***	**n**	$f(\vec{x}^{(0)})$
1				99.950
2				84.125
3				8.250
25				22.590
26			2	3.700
26			3	4.660
26			4	1.600
26			5	2.370
26			6	6.760
26			7	3.500
4		0.7		−0.068
4		1.0		−0.010
6	7			−0.056
6	15			−0.110
6	22			−0.083
7	7			−0.208
7	15			−0.790
7	22			−1.055
10	7	0.7		−0.007
10	7	1.0		−0.019
11	7	0.7		−0.002
11	7	1.0		−0.005
10	15	0.7		−0.008
10	15	1.0		−0.026
11	15	0.7		−0.003
11	15	1.0		−0.007
10	22	0.7		0.004
10	22	1.0		−0.002
11	22	0.7		0.002
11	22	1.0		−0.005
14	7			−0.016
14	15			−0.137
14	22			−0.534

Table 4.
Initial and required optical characteristics, to be found by ALSIE, for a classical triplet lens system. The field of view is being preserved unchanged.

Optical characteristics	**Initial**	**Required (α's and β's)**	
Effective focal length (mm)	100	40	50
Back focus (mm)	84	41	45
f/♯	8	4	5
Optical aberrations (mm)	≤	−1.0	1.0
Field of view	22°		

Table 5.
Performance functions (PF) and its dependence on ray parameters and surface number (θ, h, n). Four sets of boundaries ($\alpha^{(1)}$,$\beta^{(1)}$) used in ALSIE to find the four sets of optimal lens parameters. The shadowed numbers is used to show reduction on the restrictions sets.

PF	θ	h	n	$\alpha^{(1)}$	$\beta^{(1)}$	$\alpha^{(2)}$	$\beta^{((2)}$	$\alpha^{(3)}$	$\beta^{(3)}$	$\alpha^{(4)}$	$\beta^{(4)}$
1				40.00	50.00	40.00	50.00	40.00	50.00	40.00	50.00
2				41.00	45.00	41.00	45.00	41.00	45.00	41.00	45.00
3				4.00	5.00	4.00	5.00	4.00	5.00	4.00	5.00
25				12.00	18.00	12.00	18.00	12.00	18.00	12.00	18.00
26			2	2.00	3.00	2.00	3.00	2.00	3.00	2.00	3.00
26			3	4.00	5.00	4.00	5.00	4.00	5.00	4.00	5.00
26			4	0.70	1.50	0.70	1.50	0.70	1.50	0.70	1.50
26			5	0.70	1.50	0.70	1.50	0.70	1.50	0.70	1.50
26			6	2.50	3.00	2.50	3.00	2.50	3.00	2.50	3.00
26			7	2.50	3.50	2.50	3.50	2.50	3.50	2.50	3.50
4		0.7		−0.50	0.50	−0.50	0.50	−0.50	0.50	−0.50	0.50
4		1.0		−0.50	0.50	−0.50	0.50	−0.50	0.50	−0.50	0.50
6	7			−1.00	1.00	−0.50	0.50	−0.50	0.50	−0.50	0.50
6	15			−1.00	1.00	−1.00	1.00	−0.50	0.50	−0.50	0.50
6	22			−1.00	1.00	−1.00	1.00	−1.00	1.00	−0.50	0.50
7	7			−1.00	1.00	−0.50	0.50	−0.50	0.50	−0.50	0.50
7	15			−1.00	1.00	-1.00	1.00	−0.50	0.50	−0.50	0.50
7	22			−1.00	1.00	−1.00	1.00	−1.00	1.00	−0.50	0.50
10	7	0.7		−1.00	1.00	−0.50	0.50	−0.50	0.50	−0.50	0.50
10	7	1.0		−1.00	1.00	−0.50	0.50	−0.50	0.50	−0.50	0.50
11	7	0.7		−1.00	1.00	−0.50	0.50	−0.50	0.50	−0.50	0.50
11	7	1.0		−1.00	1.00	−0.50	0.50	−0.50	0.50	−0.50	0.50
10	15	0.7		−1.00	1.00	−1.00	1.00	−0.50	0.50	−0.50	0.50
10	15	1.0		−1.00	1.00	−1.00	1.00	−0.50	0.50	−0.50	0.50
11	15	0.7		−1.00	1.00	−1.00	1.00	−0.50	0.50	−0.50	0.50
11	15	1.0		−1.00	1.00	−1.00	1.00	−0.50	0.50	−0.50	0.50
10	22	0.7		−1.00	1.00	−1.00	1.00	−1.00	1.00	−0.50	0.50
10	22	1.0		−1.00	1.00	−1.00	1.00	−1.00	1.00	−0.50	0.50
11	22	0.7		−1.00	1.00	−1.00	1.00	−1.00	1.00	−0.50	0.50
11	22	1.0		−1.00	1.00	−1.00	1.00	−1.00	1.00	−0.50	0.50
14	7			−1.00	1.00	−0.50	0.50	−0.50	0.50	−0.50	0.50
14	15			−1.00	1.00	−1.00	1.00	−0.50	0.50	−0.50	0.50
14	22			−1.00	1.00	−1.00	1.00	−1.00	1.00	−0.50	0.50

Table 6.
Lens parameters triplet lens given by vector $\vec{x}^{(1)}$ ($f/\#=4.05$, $f=50$).

Surface #	Radii of curvature	Thickness	Ohara S-type glass
1[a]	∞	∞	Air
2	17.90	2.75	S-BSM4
3	129.00	4.33	Air
4	−34.70	1.18	S-TIM8
5	21.80	1.47	Air
6[b]	∞	2.94	Air
7	53.60	3.24	S-BSM4
8	−30.4	41.00[c]	Air

[a] Object.
[b] Stop.
[c] Back focus.

to reduce optical aberrations (at least up to ± 0.5 mm, as the optical performance of the initial lens system, $\vec{x}^{(0)}$).

Tables 6 and 7 show the lens parameters for the triplet system achieving the requirements given in Table 4. Lens parameters, in Table 6, came from solving non-linear inequalities system for restrictions given by $\alpha_i^{(1)}$ and$\beta_i^{(1)}$; while lens parameters, in Table 7, came from solving non-linear inequalities for$\alpha_i^{(4)}$ and $\beta_i^{(4)}$. Table 8 shows the performance functions achieved by the lens parameters represented by vectors $\vec{x}^{(0)}$, $\vec{x}^{(1)}$ and $\vec{x}^{(4)}$.

Table 9 summarizes the way of estimating the tolerances to radii of curvature and thicknesses. Let us choose the lens parameters, given by vector $\vec{x}^{(4)}$, to be fabricated. For this

Table 7.
Lens parameters of a compacted triplet lens given by vector $\vec{x}^{(4)}$ ($f/\#=4$, $f=50$).

Surface ♯	Radii of curvature	Thickness	Ohara S-type glass
1[a]	∞	∞	Air
2	17.10	2.74	S-BSM4
3	152.00	4.43	Air
4	−31.30	0.94	S-TIM8
5	18.60	1.48	Air
6[b]	∞	2.86	Air
7	55.90	3.23	S-BSM4
8	−26.00	41.13[c]	Air

[a] Object.
[b] Stop.
[c] Back focus.

Table 8.
Performance functions achieved by three triplet lens system represented by vectors $\vec{x}^{(0)}$, $\vec{x}^{(1)}$ and $\vec{x}^{(4)}$.

CN	θ	h	n	$f(\vec{x}^{(0)})$	$f(\vec{x}^{(1)})$	$f(\vec{x}^{(4)})$
1				99.950	50.000	50.004
2				84.125	41.000	41.133
3				8.250	4.047	4.000
25				22.590	15.900	15.690
26			2	3.700	2.745	2.740
26			3	4.660	4.330	4.433
26			4	1.600	1.181	0.940
26			5	2.370	1.468	1.481
26			6	6.760	2.940	2.864
26			7	3.500	3.240	3.230
4		0.7		−0.068	−0.287	−0.130
4		1.0		−0.010	−0.500	−0.075
6	7			−0.056	−0.063	0.021
6	15			−0.110	−0.193	0.196
6	22			−0.083	−0.271	0.500
7	7			−0.208	−0.144	−0.103
7	15			−0.790	−0.600	−0.386
7	22			−1.055	−1.000	−0.500
10	7	0.7		−0.007	−0.067	−0.047
10	7	1.0		−0.019	−0.156	−0.118
11	7	0.7		−0.002	−0.022	−0.014
11	7	1.0		−0.005	−0.048	−0.034
10	15	0.7		−0.008	−0.160	−0.112
10	15	1.0		−0.026	−0.370	−0.285
11	15	0.7		−0.003	−0.052	−0.035
11	15	1.0		−0.007	−0.114	−0.081
10	22	0.7		0.004	−0.270	−0196
10	22	1.0		−0.002	−0.630	−0.498
11	22	0.7		0.002	−0.088	−0.060
11	22	1.0		−0.005	−0.193	−0.140
14	7			−0.016	−0.010	−0.015
14	15			−0.137	−0.111	−0.130
14	22			−0.534	−0.485	−0.500

application, the worst lens parameters that can be fabricated are those represented by vector $\vec{x}^{(1)}$. Thus, these vectors define the tolerances for each lens parameter. Thus, computed tolerances for each lens parameter are: for radii of curvature tolerance fluctuates between 3% and 16%, while for thicknesses between 0.3% and 25%.

4. Conclusions

By using ALSIE, tolerances were estimated for a triplet lens system. It can easily fix a lens parameter, while varying the rest during solving of non-linear inequalities. This is useful when there are optical components already fabricated, but others remain to

Table 9.
Lens parameters and tolerances, for radii of curvatures and thicknesses, for a compacted triplet system. Tolerances are estimated considering that lens system given by vector $\vec{x}^{(4)}$ is to be constructed but it may become as worst as lens system represented by vector $\vec{x}^{(1)}$.

Radii of curvature	*Tolerance*	Thickness	*Tolerance*	Ohara S-Type glass
171.0	*+8.0*	2.74	*+0.01*	S-BSM4
152.0	*−23.00*	4.43	*−0.11*	Air
−31.3	*−3.40*	0.94	*+0.24*	S-TIM8
18.6	*+3.50*	1.48	*−0.01*	Air
∞	***	2.86	*+0.07*	Air
55.9	*−2.90*	3.23	*+0.01*	S-BSM4
−26.0	*−4.40*	***	***	Air

*** Tolerance not assigned to infinite radius of curvature; and distance to imaging plane and its tolerance not determined here.

be fabricated and its tolerances can be studied through this scheme.

All non-linear inequalities systems were solved starting from $\vec{x}^{(0)}$. However, it can be done from different starting vectors with the same restriction sets (α's and β's). Specific aberrations or mechanical conditions can be selected, allowed to increase or decrease, to estimate tolerances. Some lens parameters can be fixed, and solve the non-linear inequality system for different aberrations or mechanical conditions.

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