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Generalized admittance matrix models of OTRAs and COAs

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ABSTRACT

This paper proposes new admittance matrix models to approach the behavior of fully-differential Operational Transresistance Amplifiers (OTRAs) and Current Operational Amplifiers (COAs). The infinity-variables method is used in order to derive the new generalized models. As a consequence, standard nodal analysis being improved to compute fully-symbolic small-signal characteristics of fully-differential analog circuits.

1. Introduction

Symbolic analysis is a powerful tool to the analysis of electronic circuits, where all or part of the circuit elements are considered as symbols [1–5]. Several methods to formulate the system of network equations of linear and time-invariant circuits with lumped parameters have been proposed and they can be classified in topological and algebraic methods [1–4]. In order to apply an algebraic method, such as the nodal analysis (NA) method, each active device must be first modeled using controlled sources. For instance, the behavior of the transistor is well modeled by using a voltage-controlled current source (VCCS) [1–4]. However, only passive elements along with the VCCS, have an admittance matrix representation and others key circuit elements such as: voltage-controlled voltage source (VCVS), current-controlled voltage source (CCVS) and current-controlled current source (CCCS) cannot be directly introduced into the admittance matrix [1–4]. This disadvantage has been overcome by using the modified nodal analysis (MNA) technique, where additional columns and rows are added to the original admittance matrix and the non-NA compatible elements are readily included by using the stamp method [1,2,4]. In fact, not only the voltage-mode amplifiers but current-mode and hybrid operational amplifiers can also be modeled by using controlled sources [6]. Also, all the versions of current conveyors can be modeled with controlled sources [6]. However, the dimensions of the admittance matrix depend on the number of node voltages and branch currents associated with the type of elements considered in the circuit [1–4].

Recently, a symbolic framework for analysis and synthesis of active devices by using the admittance matrix and infinity-variables has been presented in [7–9], in which, the behavior of active devices that have not an admittance matrix description, such as: the nullor, Opamps and impedance converter, are modeled as a VCCS by using infinity-variables. In the same way, new stamps associated to the VCVS, CCVS and CCCS have been deduced by considering their input and output ports grounded and nongrounded [8,9]. Particularly, three circuit models along with their stamps have been proposed for the CCVS and it has been demonstrated that they are identical for the ideal case. However, the stamps associated to the CCVS cannot model completely the behavior of a fully-differential OTRA [10,11], since they are only considering one input current, limiting their application to specific circuits, where the input current is flowing in one direction. A similar case happens with the fully-differential COA modeled as CCCS [12–14].

Therefore, as an extension of the work reported in [8,9], generalized models for the OTRA and COA by using infinity-variables are introduced and they are validated by generating symbolic transfer function of analog circuits. The originality of the proposed models is that they can be used to compute

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fully-symbolic transfer functions of fully-differential analog circuits, by applying only standard NA. The paper is organized as follows: In Section 2, the concept on admittance matrix and infinity-variables is briefly revised. In Section 3, the derivation of the admittance model for the OTRA is introduced. Section 4 introduces the new generalized stamp for the COA. Conclusions are given in Section 5.

2. Infinity-variables and admittance matrix

The nullor, shown in Fig. 1, consists of a pair of elements called nullator, where the voltage and current are both zero and the norator, where the voltage and current are arbitrary [5,15]. The nullor has widely been used to model not only the behavior of active devices at the transistor and circuit level of abstraction [1,3,6,15,16], both focused to compute fully-symbolic transfer functions of analog circuits, but also to generate the symbolic noise behavioral model at the transistor level of abstraction [17,18]. For instance, the nullator has been used to approximate the signal-path in voltage-mode analog circuits [19] and recently, the norator properties have also been employed to approximate the signal-path in current-mode analog circuits [20]. Both techniques oriented to generate the behavioral model of analog circuits at the transistor level of abstraction. On one hand, the admittance matrix stamp of the nullor considered as a VCCS and transconductance Gm_i, is given as [8,9,21]

$$\begin{array}{cc} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} c \\ d \end{array} & \begin{bmatrix} Gm_i & -Gm_i \\ -Gm_i & Gm_i \end{bmatrix} \end{array} \tag{1}$$

where Gm_i is taken to a limit of infinity. It has been shown in [8,9,21] that by using infinity-variables, (1) can be written as

$$\begin{array}{cc} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} c \\ d \end{array} & \begin{bmatrix} \infty_i & -\infty_i \\ -\infty_i & \infty_i \end{bmatrix} \end{array} \tag{2}$$

As a consequence, the nullor can be derived as a limit of any of the controlled sources when its gain tends to infinity. On the other hand, a linear(ized) circuit is composed by passive elements and active devices. Each active device can be modeled with controlled sources and in the same time by using nullors [6,15,16]. Thus, the nullor stamp given by (2) could be used to fill the admittance matrix. Therefore, rows c and d of the admittance matrix will have the form:

$$\begin{bmatrix} I_c \\ I_d \end{bmatrix} = \begin{bmatrix} Gm_i & -Gm_i \\ -Gm_i & Gm_i \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} + \begin{bmatrix} \textit{finite terms} \\ \textit{finite terms} \end{bmatrix} \tag{3}$$

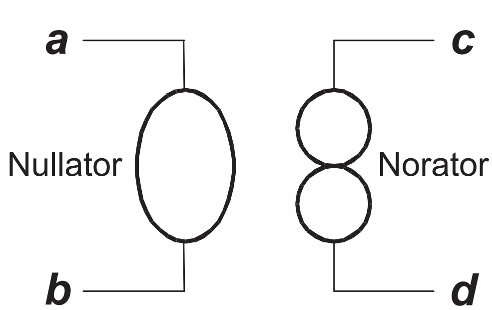

Fig. 1. Nullor element.

Dividing both sides by Gm_i and applying the limit:

$$\lim_{Gm_i \to \infty} \left\{ \begin{bmatrix} \frac{I_c}{Gm_i} \\ \frac{I_d}{Gm_i} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} + \begin{bmatrix} \frac{\textit{finite terms}}{Gm_i} \\ \frac{\textit{finite terms}}{Gm_i} \end{bmatrix} \right\} \tag{4}$$

Then, (4) is reduced to

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{5}$$

From (5) one can see that the rows corresponding to the norator nodes into the admittance matrix and the columns corresponding to the nullator nodes:

$$V_a = V_b \tag{6}$$

Since no matrix entries for rows a and b, then

$$I_a = I_b = 0 \tag{7}$$

Hence, (2) imposes finite relationships between the nodal voltages and currents of the nullator. Also, the rows imposes the constraint that the current entering at node c is equal to that leaving the node d. Otherwise, the values of the terminal voltages and currents for the norator are unconstrained.

3. General admittance matrix stamp for the OTRA

The OTRA shown in Fig. 2, is a four-terminal analog building block where its input-output terminals are characterized by low impedances and its ideal behavior can be described by the matrix [10,11]:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ R_m & -R_m & 0 & 0 \\ -R_m & R_m & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} \tag{8}$$

where R_m is the transresistance gain. The admittance matrix stamps for CCVSs have been introduced in [8,9]. However, the proposed stamps have been deduced with the constraint that the input current is flowing in one direction. From (8) and Fig. 2, the OTRA can be modeled by using CCVS-based nonideal models whose terminals are connected as shown in Fig. 3, with the transresistance gain of every CCVS as R_m and for the ideal case, it is taken to a limit of infinity. Analyzing each terminal in Fig. 3, the

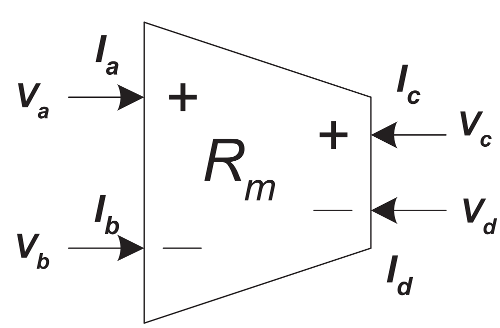

Fig. 2. OTRA symbol.

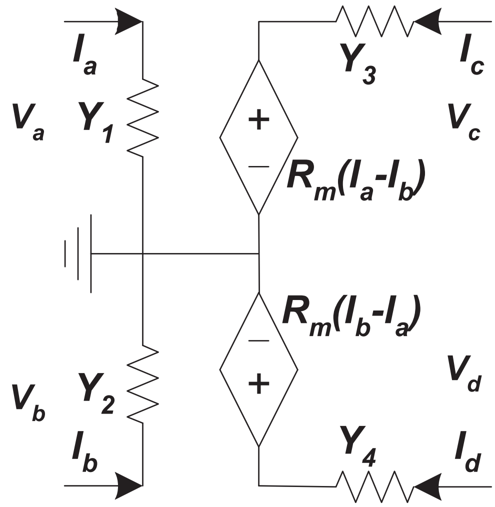

Fig. 3. Equivalent circuit for the OTRA.

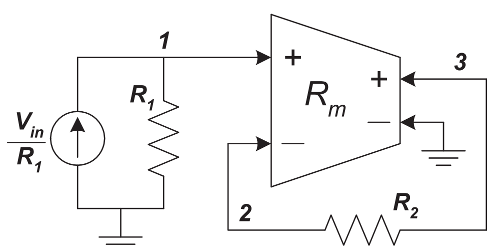

Fig. 4. Noninverting voltage amplifier.

generalized admittance matrix is obtained as

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} Y_1 & 0 & 0 & 0 \\ 0 & Y_2 & 0 & 0 \\ -R_mY_1Y_3 & R_mY_2Y_3 & Y_3 & 0 \\ R_mY_1Y_4 & -R_mY_2Y_4 & 0 & Y_4 \end{bmatrix} \end{array} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} \tag{9}$$

Here, Y_1,Y_2,Y_3 and Y_4 can be interpreted as finite input and output admittances of OTRAs. Note that if b and d terminals are grounded, (9) is reduced to the admittance matrix given in [8,9]. Also, by considering $Y_1,Y_2,Y_3,Y_4\to\infty$, the ideal representation given by (8) is obtained. On the other hand, by applying (4) the relationships between the independent voltages and currents are obtained as: $V_a=V_b=0$ and $I_a=I_b=arbitrary$.

3.1. Voltage amplifier

To show the potentiality of the proposed model, lets us consider the voltage amplifier circuit shown in Fig. 4 [10]. The voltage source has been transformed to current source by using R_1. So that, the nodal admittance matrix is given by

$$\begin{bmatrix} g_1+Y_1 & 0 & 0 \\ 0 & g_2+Y_2 & -g_2 \\ -R_mY_1Y_3 & -g_2+R_mY_2Y_3 & g_2+Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} g_1V_{in} \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

where $g_1=1/R_1$, $g_2=1/R_2$ and the stamp given by (9) has been used. By solving (10) to V_3, the fully-symbolic transfer function is given by

$$\frac{V_3}{V_{in}} = \frac{R_mY_1Y_3(g_2+Y_2)g_1}{(g_1+Y_1)(g_2Y_3+g_2Y_2+Y_2Y_3+g_2R_mY_2Y_3)} \tag{11}$$

which can be written as

$$\frac{V_3}{V_{in}} = \frac{g_1}{\frac{g_1+Y_1}{g_2+Y_2}\left(\frac{g_2}{R_mY_1}+\frac{g_2Y_2}{R_mY_1Y_3}+\frac{Y_2}{R_mY_1}+\frac{g_2Y_2}{Y_1}\right)} \tag{12}$$

By applying $Y_1,Y_2,Y_3\to\infty_i$, (12) is reduced to

$$\frac{V_3}{V_{in}} = \frac{g_1}{\frac{g_1+\infty_1}{g_2+\infty_2}\left(\frac{g_2}{R_m\infty_1}+\frac{g_2\infty_2}{R_m\infty_1\infty_3}+\frac{\infty_2}{R_m\infty_1}+\frac{g_2\infty_2}{\infty_1}\right)}$$
$$\approx \frac{g_1}{g_2+\frac{1}{R_m}} = \frac{\frac{R_2}{R_1}}{1+\frac{R_2}{R_m}} \tag{13}$$

4. General admittance matrix stamp for the COA

The circuit symbol of the COA, is illustrated in Fig. 5, with the output currents flowing toward the c and d nodes. It is also a four-terminal analog building block and its operation can be characterized by the following matrix form [12–14]:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -A_i & A_i \\ A_i & -A_i \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ I_c \\ I_d \end{bmatrix} \tag{14}$$

For ideal operation, the current gain A_i approaches to infinite, forcing to the input-currents to be equal. Both input-terminals are characterized by low-impedance and are internally grounded. On the contrary, the output-terminals are characterized by high-impedances. Although the admittance matrix stamp of the grounded and ungrounded-CCCS using infinity-variables has been deduced in [8,9], still the input current is flowing in one direction. An equivalent circuit of Fig. 5 using two nonideal CCCS, is shown in Fig. 6. Analyzing each terminal of Fig. 6, the system of equations is obtained and can be written as a general admittance matrix:

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} Y_1 & 0 \\ 0 & Y_2 \\ -A_iY_1 & A_iY_2 \\ A_iY_1 & -A_iY_2 \end{bmatrix} \end{array} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} \tag{15}$$

By considering $Y_1,Y_2\to\infty$ in (15), the ideal representation given by (14) is obtained. Note that if b and d terminals are open-circuited, (15) is reduced to the admittance matrix associated to the grounded CCCS given in [8,9]. Also by using (4), the relation between the independent voltages and current are given as: $V_a=V_b=0$ and $I_a=I_b=arbitrary$, respectively.

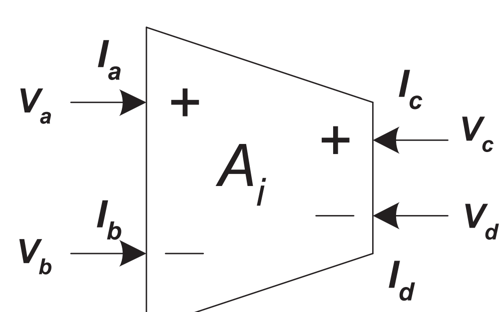

Fig. 5. COA symbol.

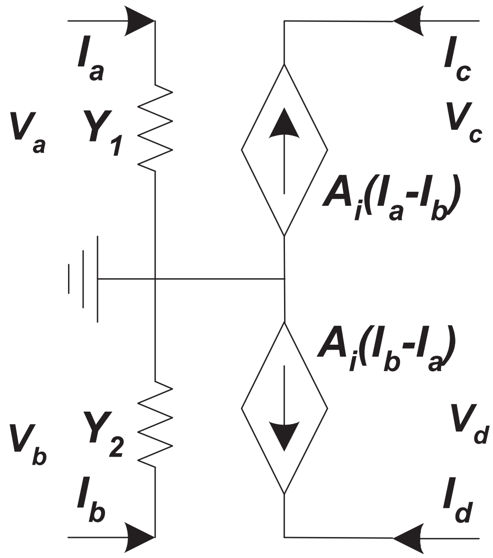

Fig. 6. Equivalent circuit for COAs.

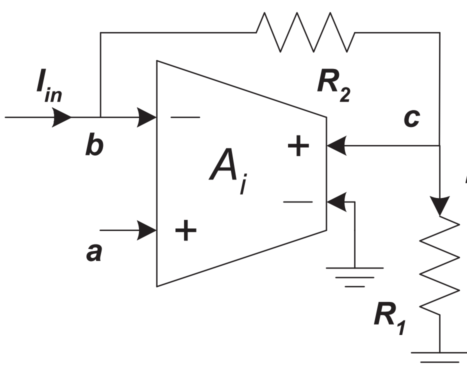

Fig. 7. Inverting current amplifier.

4.1. Current amplifier

Let us consider the current amplifier taken from [14], which is shown in Fig. 7. By applying standard NA and the stamp given by (15), the system of equations is given by

$$\begin{bmatrix} Y_1 & 0 & 0 \\ 0 & g_2+Y_2 & -g_2 \\ -A_iY_1 & -g_2+A_iY_2 & g_1+g_2 \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}=\begin{bmatrix} 0 \\ I_{in} \\ 0 \end{bmatrix} \tag{16}$$

where $g_1=1/R_1$ and $g_2=1/R_2$. By resolving (16) to V_c and since I_0=g_1V_c, the fully-symbolic transfer function is given by

$$\frac{I_0}{I_{in}}=\frac{g_1(g_2-A_iY_2)}{g_1g_2+g_1Y_2+g_2Y_2+g_2A_iY_2} \tag{17}$$

which can be written as

$$\frac{I_0}{I_{in}}=\frac{g_1}{\dfrac{g_1g_2}{g_2-A_iY_2}+\dfrac{Y_2(g_1+g_2)}{g_2-A_iY_2}+\dfrac{g_2A_iY_2}{g_2-A_iY_2}} \tag{18}$$

By applying $Y_2\to\infty_2$, (18) is reduced to

$$\frac{I_0}{I_{in}}=\frac{g_1}{\dfrac{g_1g_2}{g_2-A_i\infty_2}+\dfrac{\infty_2(g_1+g_2)}{g_2-A_i\infty_2}+\dfrac{g_2A_i\infty_2}{g_2-A_i\infty_2}}$$

$$\approx\frac{-g_1}{g_1+g_2+g_2A_i}=\frac{-R_2}{R_1+\dfrac{R_1+R_2}{A_i}} \tag{19}$$

5. Conclusions

Admittance matrix models of fully-differential OTRAs and COAs have been proposed. The new generalized models have been derived by using the infinity-variables method. This allows to compute accurate full-symbolic small-signal characteristics of analog circuits by using only standard NA and stamps. Therefore, the new stamps can be included in CAD tools.

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