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DIAGNOSTICS FOR REGIONS OF FORMATION OF ATOMIC LINES IN STELLAR ATMOSPHERES

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We present a procedure to make direct diagnostics of the physical conditions of the regions of formation of the atomic spectral lines in stellar atmospheres using the atomic line widths at half maximum and the number of lines visible of a given atomic series in the observed stellar spectra. This is accomplished using the theoretical widths at half maximum of the atomic lines induced by the broadening produced by thermal energy fluctuations and considering the maximum number of levels that exist in those atoms under the physical conditions of the given system. The procedure is easy to use in any application. As an example we apply the procedure to the observed Lyman lines of hydrogen in the ultraviolet of some stars.

Keywords: atomic spectra:line widths:plasmas:diagnostics - stars: stellar atmospheres

1. Introduction

The physical conditions in stellar atmospheres are in general obtained using model stellar atmospheres comparing the observed continuum flux and profiles of the lines with the calculated results of the models. A line broadening theory is used to generate the line profiles in a grid of models. In principle the profiles can be computed for many spectral lines. The first step in this method is to select the model atmosphere that most closely resembles the stellar atmosphere to be analyzed. This choice is made by comparing observed and computed values of certain key features in the spectrum. Typically the comparison is made for the continuum features such as the overall energy distribution, the Balmer jumps or colors; or the line profiles of the hydrogen lines, which are density sensitive; or the ratio of line-strengths for lines of two ionization stages of a given element [1-3]. Having selected the model atmosphere that best fits the observations, one generates the numerical results necessary for the analysis of the

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evolution of the main variables of the model with depth. The computed models are given in tables that contain the changes of the principal physical variables with respect to depth in the atmosphere where one can localize the region of formation of the lines to analyze, and from the selected regions one obtains the temperatures and pressures or total number density of particles. To complement the above procedures with some initial values of the main variables, we propose a new procedure for obtaining the temperatures and number densities using the line broadening produced by the energy fluctuations in the system to compare directly with the observations of the atomic line widths. This is described below. In this procedure the maximum level that an atom can attain under the given physical conditions is used to obtain an upper bound for the total number density of particles and consequently the temperatures.

In what follows in Section 2 the line broadening theory is described. The maximum number of levels that under certain physical conditions exist in the atoms is presented in Section 3. The procedures to obtain the temperatures and number densities is developed in Section 4. Lastly, in Section 5, some comments and conclusions about the procedure and results are given.

2. Line broadening

The line broadening theory applied in this analysis is the atomic line broadening by thermal energy fluctuations [4] that reproduces the width at half maximum (FWHM) of the lines of any type of atom through the formula

$$w = \frac{2\pi a_0}{3\alpha Z_{eff}} kT \sqrt[3]{N} \left(n_{1eff}^2 + n_{2eff}^2 \right) (\text{ergs}), \tag{1}$$

where T is the temperature, N the total number density of particles of all types in the medium, k is the Boltzmann constant, a_0 is the Bohr radius, Z_{eff} is the effective charge of the atom, n_{1eff} and n_{2eff} are the effective principal atomic quantum number of the two levels that participate in the transition that produces the line, and α is a numerical constant, which takes into account the width of the levels and of the lines and the normalization constant for the Voigt function [5] and is given by

$$\alpha = \pi\sqrt{\pi}\left(2\sqrt{2\ln 2}\right)^2 . \tag{2}$$

The effective quantum numbers are obtained from the tables of experimental or theoretical energy levels of the atoms [6,7]. These expressions were obtained considering that the atomic energy states are perturbed by the energy fluctuations in the system and hence produce the broadening of the atomic lines. A comparison of the model of line broadening produced by energy fluctuations with experimental line widths show good agreement even though the abundance of the constituents of the experimental mixtures is not given [4]. The full width at half maximum (FWHM) of the spectral lines is representative of the region of formation of the lines, because the FWHM is the main characterization of the distribution function of the lines, the Voigt function, and is an average value of the continuum and the tip of the line for the absorption coefficient of the lines. Given the FWHM of the lines, one can reconstruct the profiles from the Voigt function. The equivalent width of the lines, the integrated line strength, is given by $W = pcw$, where w is the full width at half maximum of the spectral line, c is the central ordinate of the line, and p is a numerical factor [8,5], the last two factors together are of the order of unity for lines in stellar atmospheres where the damping constant is small [9]. Thus the FWHM is representative of the integrated line strengths and therefore of the region of formation of the lines in the stellar atmosphere. The FWHM is easy to measure for any type of lines and spectra. For those reasons it is used in stellar atmospheres and laboratory work for making diagnostics of the physical conditions of plasmas of all types [10-13].

3. Number of levels in the atoms

The maximum number of levels n^* that an atom can attain in a system in local thermodynamic equilibrium (LTE) [14] is a function of the total number density of particles and is given by

$$n^* = \frac{q}{2}\left[1 + \sqrt{1 + \frac{4}{q}}\right] \tag{3}$$

with

$$q = \sqrt{\frac{Z_{eff}}{2\pi a_0}}\, N^{-1/6}\ , \tag{4}$$

and were derived by relating the thermodynamic state of a gaseous system and the energy separation of the upper energy states of the atoms making the number of levels finite. The number of levels in a given atom obtained using the results of [14] show that they represent very well the experimental results for any density [15]. There have been a great number of articles dealing with the number of lines in an observed atomic spectral series, among which is the Inglis-Teller formula [16]. In general, these results use the perturbation by charged particles for broadening the lines, which are a function of the number of electrons, e.g. [17,18] and of the number of electrons and ions, as in [16,19,20]. Those results are not suitable for our purpose because we need to use the main variables in the equation of state and in the construction of stellar atmosphere models, which are the temperature and the total number density of particles or the pressure.

4. Temperature and total number density of particles

As in other methods for analyzing the observational line profiles, it is necessary to take into account the broadening produced by rotation, when the light is collected over the whole surface of the star, through Doppler broadening. The observational line profiles should be corrected using the techniques of Fourier analysis such as those employed by Gray [3]. This broadening is not intrinsic to the atomic structure and therefore depends on the type of object under study, for example, inclination of the rotation axis, the rotational velocity, limb darkening, and macroturbulence, which are out of the scope of this article. This type of broadening should be taken into account when the rotational velocity is greater than, say, 15 km/s. The profile produced by rotation is not of the Gaussian type, and one can notice this when a Gaussian is fitted to the observational profile; therefore, one should select stars that show a good fit or otherwise go through the whole process proposed by Gray. We have chosen the stars that do not show a noticeable rotational broadening. In any case, one has to correct for rotational broadening before using our procedure when necessary. Clearly, the method can be applied to any procedure for calculating stellar model atmospheres because the formula depends on the main variables used in the calculations of the structure of the atmospheres through the hydrostatic equilibrium condition, among others. Equation (1) is expressed in ergs and in order to be able to compare the observational widths with the theoretical ones, one should convert it to frequency units (Hz) and then to angstroms (Å), using the relation

$$\Delta\lambda = \frac{\lambda^2}{c}\Delta\nu \,, \tag{5}$$

where c is the speed of light, λ is the wavelength of the line, $\Delta\lambda$ is the width in wavelength units, and $\Delta\nu$ is the width in frequency units. Given the effective temperature T_{eff} of the star under study we calculate the widths of the lines using Eq. (1) together with the relation (5) for different temperatures T_{eff} and draw the results as a function of the total number density of particles. The temperature in the region around the Rosseland optical depth $\tau_R = 2/3$ is equal to the effective temperature and is the region of formation of the continuum in the stellar atmospheres. Strong lines form farther out in the stellar atmosphere than the regions of formation of the continuum, hence in regions of

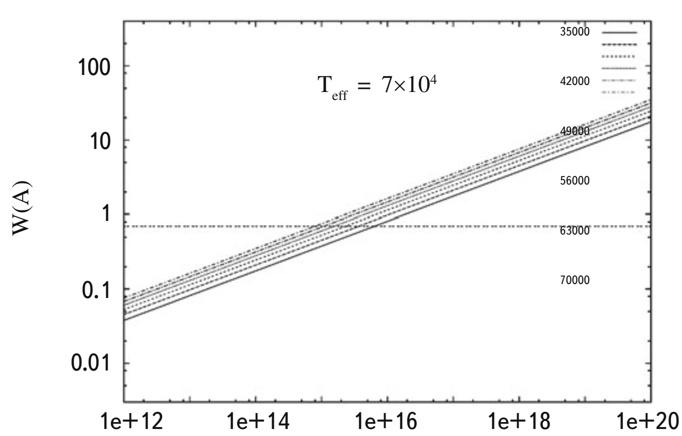

Fig.1. Calculated width at half maximum using formula (1) for the Lyman-ς line of hydrogen for different temperatures given in the upper right corner that correspond to 0.5, 0.6, 0.7, 0.8, 0.9, and 1 times the effective temperature T_{eff} = 70000 K. The horizontal line corresponds to the observed width of the line at half maximum.

lower temperature. We must also recognize that at any given frequency some layers do not contribute to the formation of the spectrum. They do not take an effective part in the radiative transfer process because either they are exceedingly transparent or they correspond to optically very deep regions. The layers intermediate between the above two groups constitute the specific spectral formation region. Figure 1 displays the results obtained with Eqs. (1) using T_{eff} = 70000 K as example. A horizontal line is drawn in order to show the intersections of the FWHM of an observed line with the calculated curves for the different temperatures, given in the upper right corner of Fig.1. That produces a region of number densities where the line under study can be formed, from the most transparent to the most opaque regions in the stellar atmosphere. From a spectroscopic series of observed atomic lines, one can deduce the conditions around the region of formation of that series in the stellar atmosphere also. If one can obtain the temperature and density of even one point in a stellar atmosphere, it is a great result that offers the opportunity to derive other thermodynamic variables at that point in the stellar atmosphere. The HWHM of the lines is representative of the physical conditions of the region of formation of the lines in the stellar atmosphere, as is mentioned above. In general, strong lines are formed at some fraction of the effective temperature; therefore, the upper bound of the number density is found that way. If one does not know anything about the regions of formation of the lines, one can use the observed lines series of a given element when visible to obtain the total number of lines *nl* of the series, and thus the maximum number of levels in the atom is given by

$$n^* = nl + 1 ; \tag{6}$$

using Eq. (3) and solving for *N*, we have

$$N = \left[\frac{Z_{eff}}{2\pi a_0 n^{*4}} \left(1 + n^*\right)^2 \right]^3 . \tag{7}$$

In this way improved values of the total number density can be derived together with the values obtained with the line widths given in this procedure. If the effective temperature of the star is not known with certainty, a series of figures are drawn for different effective temperatures as the one drawn before, and in each of them one draws the horizontal line through the observed line width, choosing the one that gives the width through the different temperatures as the ones given above and using the maximum number of levels to find the *N* that best fits the results. In this way one finds the effective temperature of the star in question. Then we can follow the procedure to obtain the other parameters.

5. Comparison with observations

The procedure for finding the main variables in stellar atmospheres using the line width of the observed hydrogen lines together with a theory of the line broadening by thermal energy fluctuations is presented. As was mentioned before, the maximum level that an atom can attain under the prevailing physical conditions is used to

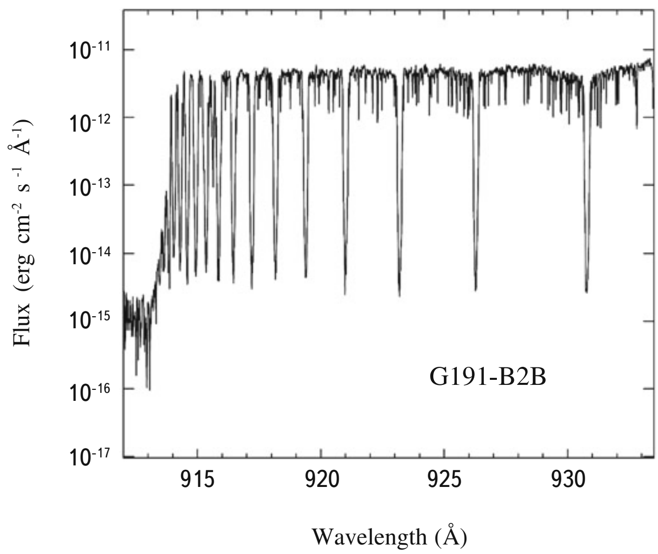

Fig. 2. The observed spectrum in the ultraviolet by FUSE of the white dwarf star G191-B2B with T_{eff} = 62300 K [23], which shows part of the Lyman line series of hydrogen.

obtain an upper bound for the total number density of particles. An example of the use of the procedure is when carrying out an analysis of some stars in the ultraviolet interval observed by FUSE [21, 22]. We consider two stars observed by FUSE, the white dwarf star G191-B2B of the WDA type with T_{eff} = 62300 K [23], the spectrum of which is shown in Figure 2, and the variable star V803-Cen of the RCrB type, to apply our analysis using the line widths of selected lines that show clearly the width at half maximum and which are free from contaminations by other processes. These selected stars show all the lines of the Lyman series necessary for our purpose.

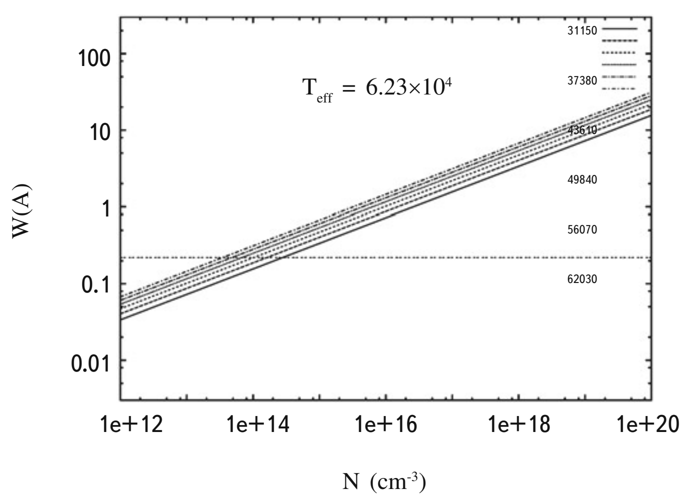

Fig.3. Calculated width for the same conditions of Fig.1 except that the effective temperature is T_{eff} = 62300 K for the white dwarf star G191-B2B [23]. The measured width at half maximum is 0.22 and is shown by the horizontal line, and the total number density is 1.5×10^{14} cm^{-3}.

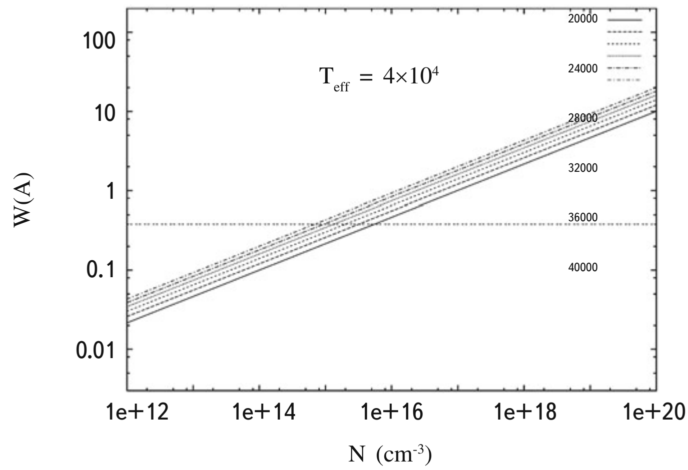

Fig.4. Calculated width for the same conditions as Fig.1 for the effective temperature T_{eff} = 40000 K found with the procedure mentioned in the text for the variable star V803-Cen. The measured width at half maximum is 0.38 and is shown by the horizontal line, and the total number density is 1.5 $\times10^{15}$ cm^{-3}.

Figures 3 and 4 show the calculated values of the width of the Lyman-ς line of hydrogen for the temperatures given in the upper right corner of the figures together with the measured line width at half maximum from the UV spectra by FUSE (dotted line). These temperatures correspond to the values of 0.5, 0.6, 0.7, 0.8, 0.9, and 1 times the effective temperature T_{eff}. In general, as we mentioned before, T_{eff} corresponds to the temperature of the models for $\tau_R \approx 2/3$, around the formation of the continuum. The lines form in outer layers above this optical depth with lower temperatures. The widths of the Lyman-ς line of hydrogen measured in the spectrum are 0.22 and 0.38 for the white dwarf and for the variable stars, respectively. We do not know the effective temperature of the variable star; therefore we follow the procedure mentioned before and we find that the effective temperature of the variable star is of the order of 4×10^4 K. The number of Lyman lines in the spectrum is 17 for both stars producing an upper bound of the densities of around 1.5×10^{14} cm^{-3} and 1.5×10^{15} cm^{-3}, which gives the temperatures of 56070 and 28000 K, respectively. In this way one finds the most important variables in the stellar atmospheres, T and N, for the region of formation of the Lyman lines, and the rest of the thermodynamic variables are derived from these using the standard procedures in LTE [1,24]. Figure 5 shows one of the Lyman lines of the Dwarf Star treated with the analysis software provided by FUSE, where the FWHM is 0.22 with a σ of 0.007. The spectra used to measure the FWHM of the hydrogen Lyman lines were calculated averaging all the good-quality spectra available in the Multimission Archive at the Space Telescope Science Institute (MAST). Because the uncertainties of the data points of each spectrum are different, we determined the mean spectrum weighting by the errors. Close to the Lyman break the errors in the spectrum of G191-B2B are two orders of magnitude lower than the values of the flux; the flux is, say, ~10^{-11}, whereas the error is ~10^{-13} in ergs/s cm^2 Å . The method developed gives results in accordance with the observational results; the smaller the errors, i.e., smaller than the errors given above, the better the determination of the physical conditions in the atmospheres. The results depend on the precision of the observed spectrum, not on the method. In general, for the calculation of the model atmospheres, the width of the lines is given for some temperature inside the atmosphere, and they are considered without any variation in the rest of the atmosphere. The lines are formed at a depth different from the depth assigned to the width of the profile, as can be seen in the figures. Therefore, it is possible to compare our results with those given by a model atmosphere in an approximate way. Nevertheless, in Figure 6 we show where the lines are formed inside the atmosphere, that is, the Rosseland optical depth τ_R, the geometrical or physical depth inside the atmosphere, for $\tau_\nu = 1$, for the formation of each frequency of the features in the spectrum for a star with effective temperature T_{eff} = 40000 K and $g = 3.16\times10^4$ and normal abundance of the elements [25-28]. For example, using Fig.6 we find $\tau_R = 0.8$ for the half height of the Lyman-ε line and from the model calculated we find the temperature and the pressure in the table of results, as mentioned above, giving for this case the temperature $T = 4.3\times10^4$ and the pressure $P = 1.7\times10^4$ or $N = 2.9\times10^{15}$. Also we did not find any results about how to determine directly the physical conditions of the regions of formation of the lines in stars in order to compare with our results. Therefore we have developed a method that offers a direct diagnostics of the physical conditions in the region of formation of the lines that one cannot find in any other method. This method is easy to use for any spectral region.

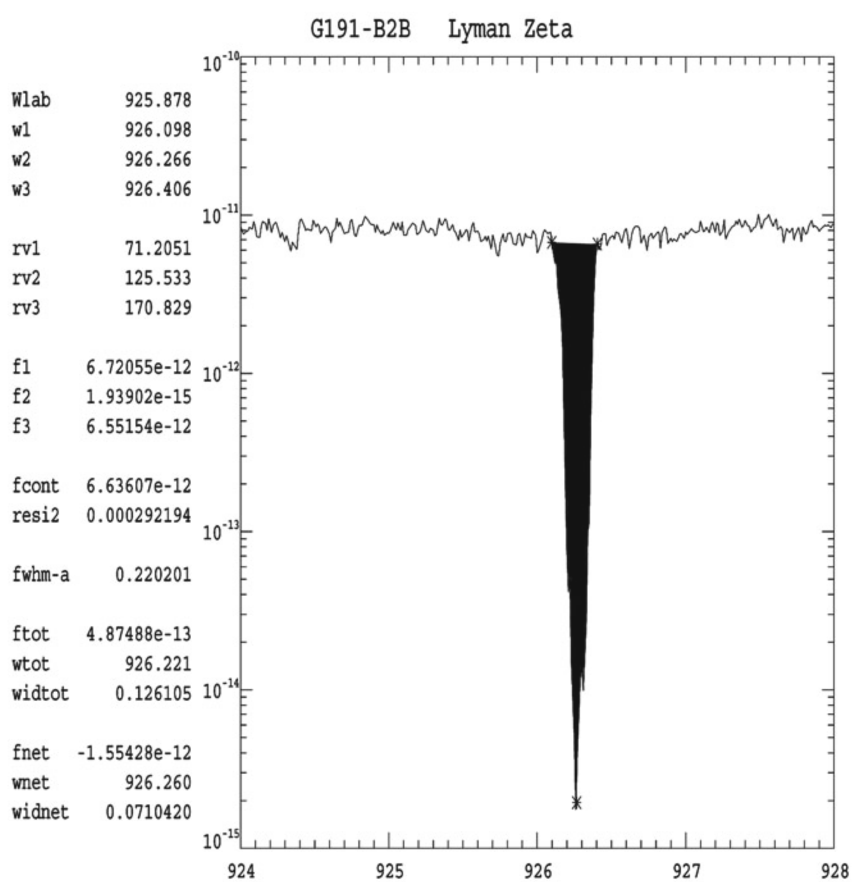

Fig.5. Width (fwhm-a) of Lyman Zeta of the star G191-B2B observed by FUSE.

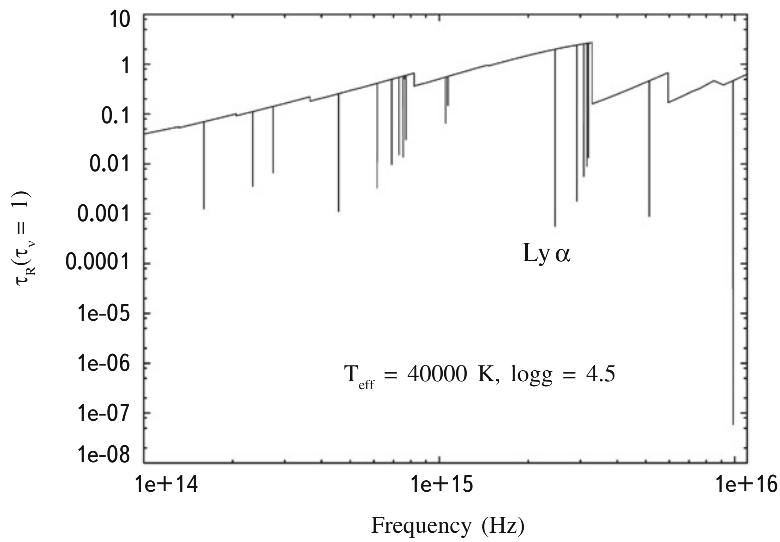

Fig.6. The Rosseland optical depth τ_R, the geometrical depth inside the atmosphere, for the formation of each frequency of the lines in the spectrum from a model atmosphere with $T_{eff} = 4\times10^4$ and superficial gravity of 3.1623×10^4 cm/s^2.

6. Conclusions and commentaries

A new procedure for finding directly the physical properties of the regions of formation of the atomic lines in stellar atmospheres is developed in this work using a theory of line broadening by energy fluctuations that produces the theoretical line width together with the results for the maximum number of levels in the atoms for a given total number density of particles in the atmosphere. The observations yield the line width at half maximum of the atomic lines and the number of lines in the spectra of a given atomic series. From the number of lines in the series an upper bound for the total number density of particles in the atmosphere is derived. The line broadening theory is applied to the ultraviolet Lyman lines of hydrogen to compare with the observations in the ultraviolet made by FUSE. Sometimes it is difficult to measure the width of the lines due to contamination by neighboring lines or by material surrounding the star under study. One way around that is to measure the lines that are less contaminated and can show clearly the width of the line at half maximum, for example in our case, Lyman-ς. The number of lines of the Lyman series that appear in the observed spectrum depends on the resolution of the spectra but is not difficult to count directly. This procedure can be used in other spectral regions. The procedure is simple and easy to use in any spectral region, and produces reliable results for the temperatures and total number densities for the regions of formation of the lines in stellar atmospheres.

This research has made use of the FUSE database, operated at the IAP, Paris, France and some/all of the data presented in this paper were obtained from the Multimission Archive at the Space Telescope Science Institute (MAST), Baltimore, MD, USA.

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