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Research Article

Approximation for Transient of Nonlinear Circuits Using RHPM and BPES Methods

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The microelectronics area constantly demands better and improved circuit simulation tools. Therefore, in this paper, rational homotopy perturbation method and Boubaker Polynomials Expansion Scheme are applied to a differential equation from a nonlinear circuit. Comparing the results obtained by both techniques revealed that they are effective and convenient.

1. Introduction

Industrial competition constantly pushes the area of electronic circuit design to the limits of technology. This has caused a rapid growth in the levels of integration for integrated circuits and the emergence of novel devices such as single-electron transistors and memristors. Because of this, the development and improvement of mathematical and numerical tools, applied to circuit simulation for the transient domain, are important. In the dynamic domain (transient), the circuit analysis is carried out only numerically because the resulting differential equations are highly nonlinear. Nevertheless, several methods are focused to find approximate solutions to nonlinear differential equations like homotopy perturbation method (HPM) [1–12], rational homotopy perturbation method (RHPM) [5, 6], variational iteration method (VIM) [13–16], and Boubaker Polynomials Expansion Scheme (BPES) [17–36], among many others. Therefore, we propose the comparison between RHPM and BPES methods by solving the nonlinear differential equation that represents the dynamics of a nonlinear circuit. The results should be a meaningful supply for monitoring complex nonlinear circuits behaviours and responses. In fact, the used protocols try to embed boundary conditions instead of direct solving, as preceded in spectral or limit-cycle bifurcations approaches.

This paper is arranged as follows. In Section 2, we present the differential equation of a nonlinear circuit. Sections 3 and 4 present the fundamentals of RHPM and BPES methods, respectively. The solutions obtained using both methods are explained in Section 5. Comparisons between the two methods and some other results presented in the recent literature have been illustrated in Section 6. Conclusions will be discussed in Section 7.

2. Nonlinear Circuit

The rapid increase in the number of transistors by integrated circuit and the increase of complexity for the models (a result of lowering the dimension of the components) results in a complex calculation for the transient. Furthermore, the task of tracing the transient for nonlinear circuits is a critical and difficult task. In fact, commercial circuit simulators do not provide any symbolic/analytic solution for the transient of any given circuit. Instead, the simulator provides only

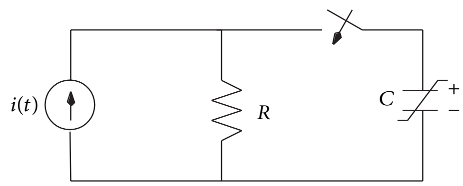

Figure 1: Nonlinear RC circuit.

numerical data that allows circuit designers to explore a limited range of dynamics of nonlinear circuits.

Consider the analysis of the nonlinear circuit depicted in Figure 1 as a case study [37]. Let the branch relationship of the nonlinear capacitor to be

$$v(q) = \alpha q^3, \tag{1}$$

where v is the voltage, q is the charge of the capacitor, and α is a parameter of the capacitor.

By applying Kirchhoff Laws, we obtain the equation for the transient

$$\frac{dq(t)}{dt} + \frac{1}{R}v(q) = i(t). \tag{2}$$

Therefore,

$$\frac{dq(t)}{dt} + \frac{\alpha}{R}q^3(t) = i(t). \tag{3}$$

If we consider the case for DC excitation, then $i(t) = I$, resulting in

$$\frac{dq(t)}{dt} + \frac{\alpha}{R}q^3(t) = I, \qquad q(0) = 0. \tag{4}$$

3. Fundamentals of the Rational Homotopy Perturbation Method

The rational homotopy perturbation method RHPM [5, 6] can be considered as a combination of the classical perturbation technique [38, 39] and the homotopy (whose origin is in the topology) [40–42] but not restricted to a small parameter like traditional perturbation methods. For example, RHPM requires neither small parameter nor linearization, but only few iterations to obtain accurate solutions.

To figure out how RHPM method works, consider a general nonlinear equation in the form:

$$A(u) - f(r) = 0, \quad r \in \Omega \tag{5}$$

with the following boundary conditions:

$$B\left(u, \frac{\partial u}{\partial u}\right) = 0, \quad r \in \Gamma, \tag{6}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytical function, and Γ is the domain boundary for Ω. A can be divided into two operators L and N, where L is linear and N nonlinear; from this last statement, (5) can be rewritten as

$$L(u) + N(u) - f(r) = 0. \tag{7}$$

Generally, a homotopy can be constructed in the form [1–3]:

$$\begin{aligned} H(v,p) = {} & (1-p)\left[L(v) - L(u_0)\right] \\ & + p\left[L(v) + N(v) - f(r)\right] = 0, \\ & p \in [0,1], \; r \in \Omega, \end{aligned} \tag{8}$$

where p is a homotopy parameter, whose values are within the range of 0 and 1, u_0 is the first approximation for the solution of (6) that satisfies the boundary conditions.

When $p \to 0$, (8) is reduced to

$$L(v) - L(u_0) = 0, \tag{9}$$

where operator L possesses trivial solution.

For $p \to 1$, (8) is reduced to the original problem

$$N(v) + L(v) - f(r) = 0. \tag{10}$$

Assuming that the solution for (8) can be written as a power series of p:

$$v = \frac{v_0 + pv_1 + p^2 v_2 + \cdots}{w_0 + pw_1 + p^2 w_2 + \cdots}, \tag{11}$$

where $v_0, v_1, v_2, \ldots$ are unknown functions to be determined by the RHPM, and $w_0, w_1, w_2, \ldots$ are known analytic functions of the independent variable.

Substituting (11) into (8) and equating identical powers of p terms, it is possible to obtain values for the sequence $v_0, v_1, v_2, \ldots$.

When $p \to 1$ in (11), it yields in the approximate solution for (5) in the form:

$$u = \lim_{p \to 1}(v) = \frac{v_0 + v_1 + v_2 + \cdots}{w_0 + w_1 + w_2 + \cdots}. \tag{12}$$

Convergence of RHPM method is studied in [5, 6].

4. Fundamentals of the Boubaker Polynomials Expansion Scheme BPES

The Boubaker Polynomials Expansion Scheme BPES [17–36] is a resolution protocol, which has been successfully applied to several applied-physics and mathematical problems. The BPES protocol ensures the validity of the related boundary conditions regardless of main equation features. The Boubaker Polynomials Expansion Scheme BPES is based on the Boubaker polynomials first derivatives properties:

$$\begin{aligned} \sum_{q=1}^{N} B_{4q}(x)\Big|_{x=0} &= -2N \neq 0, \\ \sum_{q=1}^{N} B_{4q}(x)\Big|_{x=r_q} &= 0, \\ \sum_{q=1}^{N} \frac{dB_{4q}(x)}{dx}\Big|_{x=0} &= 0, \\ \sum_{q=1}^{N} \frac{dB_{4q}(x)}{dx}\Big|_{x=r_q} &= \sum_{q=1}^{N} H_q, \end{aligned} \tag{13}$$

with

$$H_n = B'_{4n}(r_n) = \left(\frac{4r_n \left[2 - r_n^2\right] \times \sum_{q=1}^{n} B_{4q}^2(r_n)}{B_{4(n+1)}(r_n)} + 4r_n^3 \right). \tag{14}$$

Several solutions have been proposed through the BPES in many fields like numerical analysis [17–20], theoretical physics [21–24], mathematical algorithms [25], heat transfer [26], homodynamic [27, 28], material characterization [29], fuzzy systems modelling [30–34], and biology [35, 36].

5. Application of RHPM and BPES

5.1. Solution Using RHPM Method. Using (8), we establish the following RHPM homotopy map:

$$(1-p)\left(v' - u'_0\right) + p\left(v' + \frac{\alpha}{R}v^3 - I\right) = 0, \tag{15}$$

where the trial function $u_0 = 0$.

Using (11), we propose the following rational solution:

$$\begin{aligned} v = &\left(v_0 + pv_1 + p^2v_2 + p^3v_3 + p^4v_4 \right.\\ &\left. + p^5v_5 + p^6v_6 + p^7v_7\right) \\ &\times \left(1 + pk_1x^3 + p^2k_2x^6\right)^{-1}, \end{aligned} \tag{16}$$

where $w_0 = 1$, $w_1 = k_1x^3$, and $w_2 = k_2x^6$.

We substitute (16) into (15), regroup, and equate terms with identical powers of p. In order to fulfil boundary condition of (16), it follows that $v_0(0) = 0, v_1(0), \ldots$ for the homotopy map.

The results are recast in the following systems of differential equations:

$$\begin{array}{lll} p^0: & v'_0 = 0, & v_0(0) = 0, \\ p^1: & v'_1 - I + \frac{\alpha}{R}v^3 + 2v'_0K_1X^3 - 3X^2v_0K_1 = 0, & v_1(0) = 0, \\ \vdots & \vdots & \vdots \end{array} \tag{17}$$

Solving (17) yields

$$\begin{aligned} v_0 &= u_0 = 0, \\ v_1 &= Ix, \\ v_2 &= Ik_1x^4 \\ &\vdots \end{aligned} \tag{18}$$

Substituting (18) into (16) and calculating the limit when $p \to 1$, we obtain the seventh-order approximation:

$$\begin{aligned} q(t) &= \lim_{p\to 1}(v) \\ &= \left(Ix + \left(Ik_1 - (1/4)\left(\alpha I^3/R\right)\right)x^4 \right.\\ &\quad + \left(Ik_2 - (1/4)\left(\alpha k_1 I^3/R\right) + (3/28)\left(\alpha^2 I^5/R^2\right)\right)x^7 \\ &\quad \left. - (1/4)\left(\alpha k_2 I^3/R\right)x^{10}\right) \\ &\quad \times \left(1 + k_1x^3 + k_2x^6\right)^{-1}. \end{aligned} \tag{19}$$

If we consider $R = 1/20$, $I = 10$, and $\alpha = 40$ as reported in [37], it is possible to obtain the adjustment parameters using the procedure reported in [6, 12], resulting in $k_1 = 36289$ and $k_2 = 4471843$.

5.2. Solution Using the Boubaker Polynomials Expansion Scheme BPES. The Boubaker Polynomials Expansion Scheme BPES is applied to (4) using the setting expression:

$$q(t) = \frac{1}{2N_0}\sum_{k=1}^{N_0}\lambda_k \times \frac{dB_{4k}(r_k t)}{dt}. \tag{20}$$

Using the properties provided by (13), boundary conditions are verified in advance of the resolution process. The system in (16) is reduced to

$$\frac{1}{2N_0}\sum_{k=1}^{N_0}\lambda_k r_k \frac{d^2B_{4k}(r_k t)}{d^2t} + \frac{\alpha}{8N_0^3R}\left[\sum_{k=1}^{N_0}\lambda_k\frac{dB_{4k}(r_k t)}{dt}\right]^3 = I. \tag{21}$$

Boundary conditions become redundant since they are already verified by the proposed expansion, consecutively, and thus, majoring and integrating along the given interval for the time variable t transform the problem in a linear system with unknown real variables: $\lambda_k|_{k=1\ldots N_0}$. Calculations are reduced to approximately $(8N_0)^3$arithmetical operations. Solutions are obtained by using the Householder [39, 40] algorithm detailed elsewhere and are denoted by $\lambda_k^{(\mathrm{sol.})}|_{k=1\ldots N_0}$. The final solution is given as

$$q(t) = \frac{1}{2N_0}\sum_{k=1}^{N_0}\lambda_k^{(\mathrm{sol.})} \times \frac{dB_{4k}(r_k t)}{dt}. \tag{22}$$

6. Results and Discussion

From Table 1, we can observe that RHPM solution (19) and BPES solution (22) are in good agreement with the numerical results obtained using Fehlberg fourth-fifth-order Runge-Kutta method with degree four interpolant (RKF45) [43, 44] (built-in function of Maple software). In order to guarantee a good numerical reference, RKF45 is configured using an absolute error of $10^{-7}$and a relative error of 10^{-6}. The power

Table 1: Numerical comparison of proposed solutions and RKF45 solution of (4).

t	$q(t)$ (RKF45)	RHPM	BPES		
			$N_0 = 37$	$N_0 = 107$	$N_0 = 113$
0.00	0.00000	0.00000	0.00000	0.00000	0.00000
0.01	0.098066	0.098070	0.09436	0.09456	0.09827
0.02	.17475	0.17481	0.17123	0.17395	0.17513
0.03	.21314	0.21388	0.21051	0.21266	0.21394
0.04	0.22656	0.22802	0.22669	0.22557	0.22775
0.05	0.23054	0.23005	0.22678	0.23009	0.23076
0.06	0.23165	0.23235	0.22346	0.23056	0.23246

of RHPM method is based on the capability of rational expressions containing a huge amount of information of dynamics from asymptotic problems.

Moreover, convergence of the BPES algorithm has been obtained for moderate values of N_0 ($N_0 < 120$), since, as mentioned above, boundary conditions were verified in advance of the resolution process. Both methods generated analytical expressions useful for other analysis like circuit power consumption; such analytical expressions can provide more information about the nature and behaviour of circuits than numerical integration schemes with variable step size [41, 44–46]. Nonetheless, semianalytical techniques like RHPM and BPES may be combined with numerical methods [43–46] to improve the simulation tools of VLSI circuits.

7. Conclusion

In this paper, powerful analytical methods like rational homotopy perturbation method (RHPM) and Boubaker Polynomials Expansion Scheme (BPES) are presented to construct semianalytical solutions for the transient of a nonlinear circuit. The results exhibited that both techniques are powerful, obtaining highly accurate analytical expressions for the transient of a simple test circuit. While RHPM yielded accurate and reliable results, BPES exhibited the advantage of ensuring the validity of boundary conditions regardless of main equation features. This feature made the protocol yielding faster and provided more convergent solutions than many numerical integration schemes with variable step size. Further work is necessary to extend the use of both methods for larger circuits.

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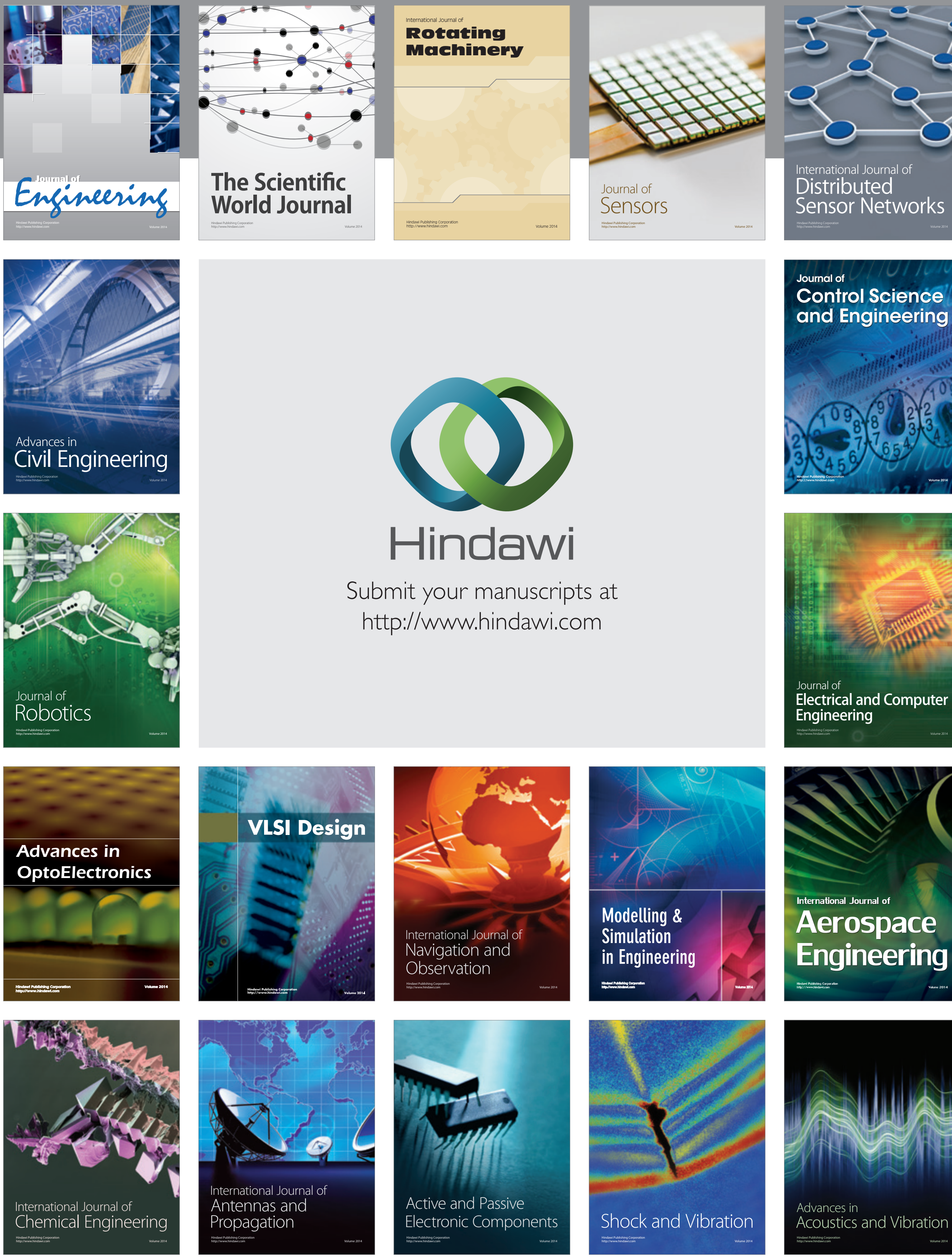
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