Efficient generation of an arbitrary nondiffracting Bessel beam employing its phase modulation

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We report a highly efficient method for generation of any high-order nondiffracting Bessel beam employing a phase hologram whose transmittance coincides with the phase modulation of such a beam. It is remarkable that the Bessel beam generated by this hologram, at the plane of this device, has peak amplitude higher than the amplitude of the beam employed to illuminate it. © 2009 Optical Society of America

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The generation of complex fields with diffractive techniques has been investigated for several decades. These techniques employ a spatial light modulator (SLM), in which a complex field distribution is codified. Unfortunately, nowadays no SLM exists in which the phase and the amplitude of a light wave can be modulated independently. Therefore, all techniques developed so far codify a complex field employing an amplitude modulation or a phase modulation exclusively. Since the invention of nondiffracting beams [1], their generation through diffractive and holographic methods has drawn the attention of several researchers [2–11]. Here we report a highly efficient method for generation of an arbitrary high-order nondiffracting Bessel beam employing a phase hologram (PH) whose transmittance coincides with the phase modulation of such a beam. The Bessel beam is generated by a simple spatial filtering operation at the Fourier domain of the PH. In general, if a complex beam is generated with a conventional PH, its peak amplitude at the plane of this device is smaller than the amplitude of the light beam employed to illuminate it. Thus, conventional holograms generate complex beams with relatively low efficiencies. In contrast, the proposed hologram generates nondiffracting Bessel beams, of arbitrary order, with peak amplitudes that are larger than the amplitude of the input beam.

The complex amplitude of the Bessel beam that we desire to generate is $J_q(2\pi\alpha r)\exp(iq\theta)$, where J_q is the qth order Bessel function of the first kind, r and θ are polar coordinates, and α is the beam radial frequency. We assume that this function will be generated within a circular pupil of radius R. For convenience we employ the normalized radial coordinate $\xi = r/R$ to express the complex amplitude of the Bessel beam as

$$B_q(\xi,\theta) = J_q(2\pi\alpha R\xi)\exp(iq\theta) \tag{1}$$

that must be limited by the circular pupil of unitary radius $p(\xi)=\mathrm{circ}(\xi)$. The PH proposed to generate the beam $B_q(\xi,\theta)$ is given by

$$h_q(\xi,\theta) = \mathrm{sgn}[J_q(2\pi\alpha R\xi)]\exp(iq\theta), \tag{2}$$

where sgn denotes the signum function. Since the factor $\mathrm{sgn}[J_q(2\pi\alpha R\xi)]$ at the right side of Eq. (2) is a binary phase function with possible phase delays 0 and π, the PH transmittance $h_q(\xi,\theta)$ is also a phase function. The function $f(\xi)\equiv\mathrm{sgn}[J_q(2\pi\alpha R\xi)]$ can be expressed, in the domain (0,1) of the coordinate ξ, by the superposition of functions $f(\xi)=\Sigma_n b_n J_q(\lambda_n\xi)$, performed for integer numbers $n\geq 1$ [12], in which λ_n is the nth positive root of $J_q(x)$, q is an arbitrary but fixed integer number, and b_n is the weighting factor,

$$b_n = \frac{2}{J_{q+1}^2(\lambda_n)}\int_0^1 x f(x) J_q(\lambda_n x)\mathrm{d}x. \tag{3}$$

Considering the above representation of $f(\xi)$, the PH transmittance [Eq. (2)] can be expressed as

$$h_q(\xi,\theta) = \sum_{n=1}^{\infty} b_n J_q(\lambda_n\xi)\exp(iq\theta). \tag{4}$$

Now, let us assume that the relation

$$2\pi\alpha R = \lambda_m \tag{5}$$

is satisfied for a chosen root λ_m. Thus the mth term of the series in Eq. (4), $J_q(\lambda_m\xi)\exp(iq\theta)$, corresponds to the encoded Bessel beam defined in Eq. (1), with a weighting factor b_m. It is noted that the mth root of this encoded beam appears at the edge of the pupil of radius R in such a way that this pupil contains m rings of the beam. The encoded beam can be recovered alone by employing an annular spatial filter (SF) of radius α in the Fourier domain of the PH. Considering that Eq. (5) is satisfied, it is found that the product $f(x)J_q(\lambda_m x)$ is positive everywhere, except at its zeros. Therefore, the coefficient b_m is much larger than any other coefficient of order $n\neq m$. In particular, it is shown that the coefficients b_m present values higher than unity. This fact is illustrated in Fig. 1, where the values of coefficient b_m for the beam orders $q=0$, 2, and 4 and several values of m are displayed. In connection with this last result it is found that the encoded beam (of order m) in Eq. (4) presents peak amplitude larger than the amplitude of the hologram itself (that by definition is equal to 1). This remarkable result does not contradict energy conservation, because the addition of complex ampli-

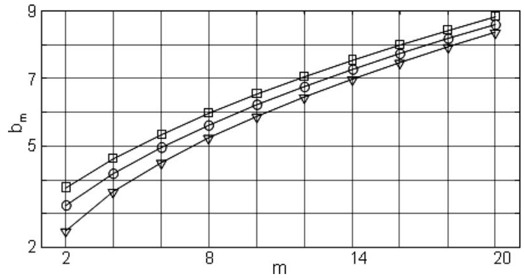

Fig. 1. Coefficients b_m for several values of index m. The order q of the codified Bessel beams are equal to 0 (triangles), 2 (circles), and 4 (squares).

tudes of all the beams that compose the PH [Eq. (4)] has a unitary modulus at every point of the PH. As a consequence of this feature of the PH, the encoded beam is reconstructed with a remarkable high efficiency. This efficiency is computed as the power P_m, of the encoded beam $b_m J_q(\lambda_m \xi)\exp(iq\theta)$, normalized by the power P_h, of the hologram field $h_q(\xi,\theta)$. Both powers are computed in the circular domain of unitary radius at the hologram plane (ξ,θ). Expressed in polar coordinates, this efficiency is obtained as

$$\eta_{qm} = P_h^{-1}\int_0^{2\pi}\int_0^1 \xi b_m^2 J_q^2(\lambda_m\xi)\mathrm{d}\xi\mathrm{d}\theta = 2b_m^2\int_0^1 \xi J_q^2(\lambda_m\xi)\mathrm{d}\xi. \tag{6}$$

The double integral in Eq. (6) represents the power of the encoded beam. The result in Eq. (6) is obtained by integrating the double integral in the coordinate θ and considering that $P_h=\pi$. The values of η_{qm}, for several indices q, and m, are depicted in Fig. 2. Similar efficiency values, greater than 0.7, are obtained for different combinations of indices q and m.

Now, we illustrate the procedure required to generate the encoded beams, with the proposed hologram, by means of numerical simulations. As example we consider the PHs that encode the beams $B_1(\xi,\theta) = J_1(\lambda_{10}\xi)\exp(i\theta)$ and $B_2(\xi,\theta)=J_2(\lambda_{10}\xi)\exp(i2\theta)$, where λ_{10} denotes the tenth root of either $J_1(x)$ or $J_2(x)$, for the respective cases. For numerical computations, the encoded beams and their PHs $h_1(\xi,\theta)$ and $h_2(\xi,\theta)$ are sampled with resolution $\delta\xi=1/128$. The phases of the encoded beams, which correspond to the phases of the PHs, are depicted in Fig. 3. In Fig. 4 we show the normalized Fourier spectra modules of the PHs $h_1(\xi,\theta)$ and $h_2(\xi,\theta)$, respectively. For each one of these spectra, the different rings with increasing radii correspond to the spectra of the different terms in Eq. (4), with increasing radial frequencies. The most brilliant rings in these spectra correspond to the tenth-order terms in the series expansions. This result confirms the dominant role of the mth term, corresponding to the encoded beam. Continuing the numerical simulation, each PH Fourier spectrum is spatially filtered by an annular pupil that transmits only the light of its most brilliant ring. The modules of the fields reconstructed by Fourier transforming the light transmitted through the annular pupil, in the two cases, are shown in Fig. 5.

We implemented experimentally the PHs that encode the Bessel beams $B_1(\xi,\theta)=J_1(\lambda_{10}\xi)\exp(i\theta)$ and $B_2(\xi,\theta)=J_2(\lambda_{10}\xi)\exp(i2\theta)$, described in the numerical simulations, employing the reflective phase SLM 1080P of Holoeye Photonics AG. The Bessel beams are generated using the setup depicted in Fig. 6, where the SLM is illuminated by a collimated He–Ne laser beam ($\lambda=633$ nm) that is conditioned by a beam expander (BE). For this illumination, the SLM provides phase modulation of 2π radians with 160 nonuniform steps. We employ a quasi-normal incidence of this input beam to the SLM to avoid the inconveniences of using a cube beam splitter. The PHs that encode the complex fields $B_1(\xi,\theta)$ and $B_2(\xi,\theta)$ are implemented within a circular pupil whose radius is

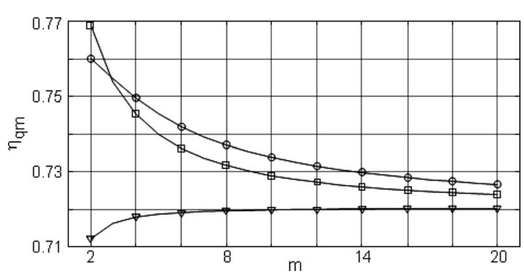

Fig. 2. Efficiencies η_{qm} of the PHs that encode the Bessel beams $J_q(\lambda_m\xi)\exp(iq\theta)$, for beams orders $q=0$ (triangles), $q=2$ (squares), and $q=4$ (circles).

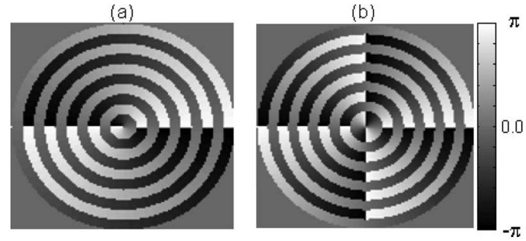

Fig. 3. Phases of the holograms that encode the beams (a) $B_1(\xi,\theta)=J_1(\lambda_{10}\xi)\exp(i\theta)$ and (b) $B_2(\xi,\theta)=J_2(\lambda_{10}\xi)\exp(i2\theta)$.

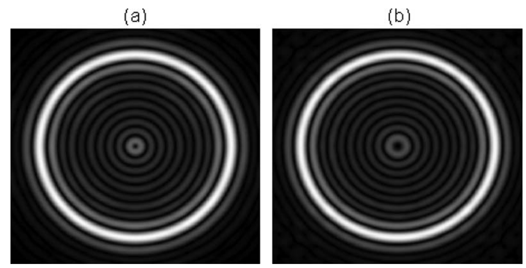

Fig. 4. Fourier transform amplitudes of the PHs shown in (a) Fig. 3(a) and (b) Fig. 3(b).

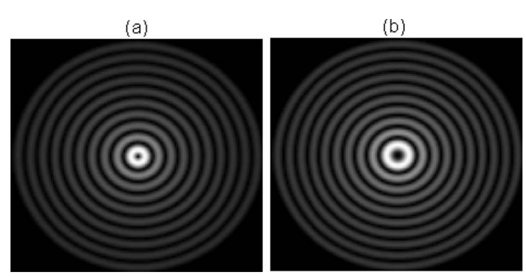

Fig. 5. Amplitudes of the reconstructed fields obtained by filtering the most brilliant rings in the Fourier spectra of (a) Fig. 4(a) and (b) Fig. 4(b).

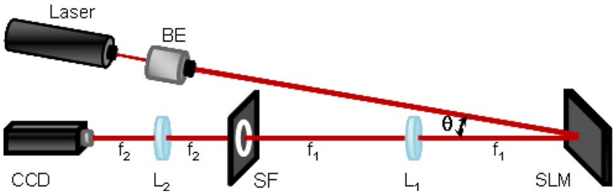

Fig. 6. (Color online) Experimental setup employed for the generation of nondiffracting Bessel beams.

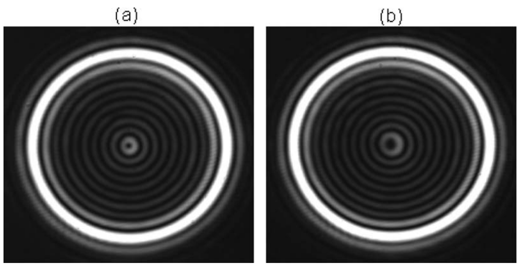

Fig. 7. Experimental Fourier transform amplitudes of the PHs that encode the beams (a) $B_1(\xi,\theta)=J_1(\lambda_{10}\xi)\exp(i\theta)$ and (b) $B_2(\xi,\theta)=J_2(\lambda_{10}\xi)\exp(i2\theta)$.

equal to 220 pixels of the SLM. To avoid a strong zero-order peak in the PHs' Fourier spectra, owing to the first surface reflection in the SLM and other imperfections of this device, we implemented off-axis versions of the PHs, by modulating their transmittances with a linear phase factor. The amplitudes of the experimentally generated Fourier spectra of the PHs are shown in Fig. 7. These spectra are generated by the Fourier transforming lens L_1. We recorded a slightly overexposed image of these spectra, to enhance the inner rings. With the same purpose, we have displayed the amplitudes instead of the intensities of these Fourier spectra. It is noted that the most brilliant ring in each one of these spectra is the tenth ring, which corresponds to the encoded beam. An annular SF is employed to transmit only the most brilliant rings of the PHs' spectra. The field transmitted by the SF is Fourier transformed by the lens L_2 that generates the field encoded by each PH. The intensities of the experimentally generated fields $B_1(\xi,\theta)$ and $B_2(\xi,\theta)$ are recorded at the back focal plane of lens L_2 by a CCD camera. The amplitudes of the experimentally generated fields, obtained from the digitized intensities, are shown in Fig. 8.

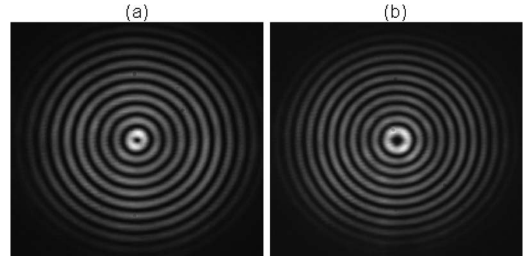

Fig. 8. Amplitudes of the experimentally generated fields (a) $B_1(\xi,\theta)=J_1(\lambda_{10}\xi)\exp(i\theta)$, and (b) $B_2(\xi,\theta)=J_2(\lambda_{10}\xi)\times\exp(i2\theta)$.

In summary, we have discussed a PH for encoding an arbitrary nondiffracting Bessel beam, whose transmittance coincides with the phase modulation of the beam itself. We established that the peak amplitude of the encoded beam is higher than the hologram amplitude, in the plane of this device. This remarkable feature of the PH enables the generation of the encoded beam with high efficiency. The numerical simulations and experimental results provide a satisfactory validation of the proposed holographic method.

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