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Chaotic noise MOS generator based on logistic map

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ABSTRACT

The Birkhoff's ergodic theorem (BET), bifurcation diagram (BD) and Lyapunov's exponent (LE) are used in order to design a chaotic noise generator that is governed by the logistic map (LM). For this, a MOS analog circuit that operates in the current-mode, which is based on translinear principle (TLP), is used. To iterate the transference function of this circuit and also to maintain the parameter control of the LM, a current amplifier has been used. The specifications of the design are obtained from the analysis of the model. The results demonstrate the correct operation of the circuit, even when a mismatching of 2% is considered between the devices that control the operation region. The statistical distribution of the output signal on the circuit is similar to uniform distribution and it is related with the parameter value that rules the transfer function of the circuit.

1. Introduction

The noise generators on electronic implementation are useful in information protection systems as spread spectrum and watermarking systems. Some works reported which are related with the design of discrete noise generator can be revised in [1,2], whereas the works related with the analog noise generators can be revised in [3]. Electronic design of these devices should be precise [4], so that their behaviour is the awaited one. For that reason, when an electronic circuit is designed, it is necessary to establish a model that describes its dynamics of behaviour, and that allows to find the valid operation regions. Using the Birkhoff's ergodic theorem (BET), it is possible to determine the statistical distribution of the noise signal produced by a dynamic system. On the other hand, with the bifurcation diagram (BD) and the Lyapunov's exponent (LE) the stability islands of the logistic map (LM) can be identified and they must be avoided [5].

In this work, a MOS analog circuit is used, which operates in the current-mode and that uses the translinear principle (TLP) [6,7]. The analysis and design of this electronic circuit are made with techniques of statistical mechanics using BET, BD and LE.

2. Logistic map as noise generator

The basic LM has a chaotic behaviour and it can arise from very simple non-linear dynamical equations [8,9]. This function was popularized in 1976 by the biologist Robert May (see [10]). The LM is defined by the family of parabolic curves shown in Fig. 2 and have in $x=\frac{1}{2}$ their maximum value and it is $\mu/4$. This family curves can be obtained using Eq. (1) with x in the interval (0, 1) and μ in (0, 4)

$$f(x_n) = x_{n+1} = \mu x_n(1-x_n), \tag{1}$$

where μ is the control parameter of the LM.

The dynamics of the LM can be explained by the BD, which is shown in Fig. 1 using $\mu \in (0, 4)$.

For values of $\mu \in (0, 3)$, this function always converges to a fixed point of $f(x_n)$. When the value of μ is close to 3 the system oscillates between two fixed points, which is known as the bifurcation process. As μ is increased beyond 3, the system oscillates between 4 fixed points, then 8, 16 and so on. This behaviour is known as period doubling. Finally, for values of μ greater than $\mu_{\text{chaos}} = 3.5699456\ldots$ [11,12], this period doubling disappears and the system is considered to be in chaotic regime.

Another factor which determines that a system is in chaotic regime is the sensitive dependence to initial conditions. This dependence can be determined by the LE. An iterative dynamic system operates in chaotic regime when the LE is positive [8].

In Fig. 2 the behaviour of the LE is shown. For values of $\mu \geqslant \mu_{\text{chaos}}$ LE is positive. However in this case, there are regions where this exponent becomes negative again. These regions are called stability islands and they show periodicity, which is an undesired condition for the generation of chaotic noise and it must be avoided.

On the other hand, the BET [13] is used to find the statistical distribution of the noise signal produced by the LM. BET implies

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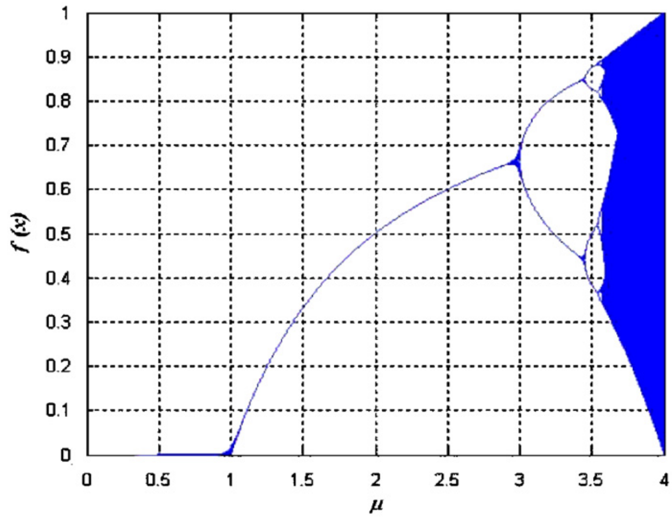

Fig. 1. Bifurcation diagram of the logistic map.

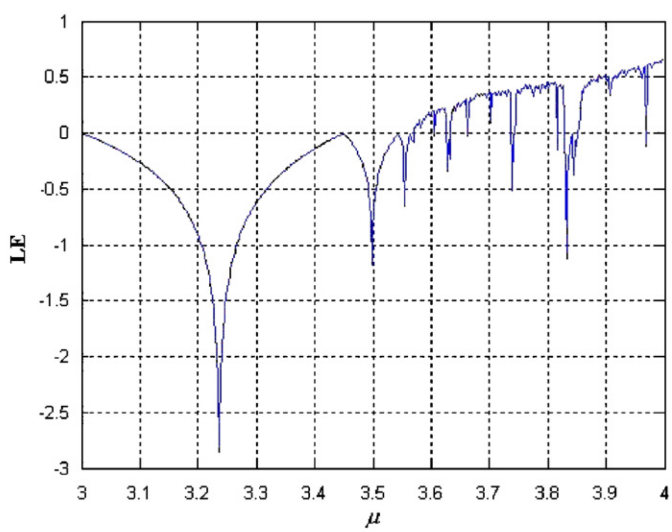

Fig. 2. Lyapunov's exponent of the LM.

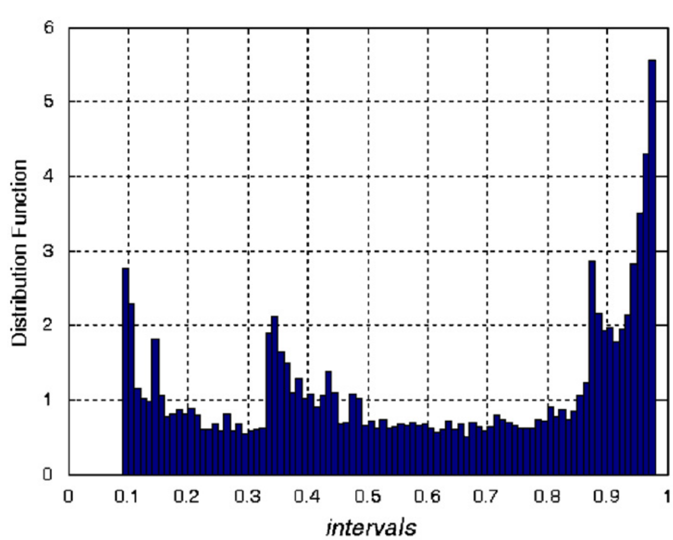

Fig. 3. Statistical behaviour of the LM using $\mu = 3.9$.

that it is equivalent to study the evolution of some initial statistical distribution, so that the LM function should be applied to each point in the initial statistical distribution and when the invariant statistical distribution is obtained, that one distribution corresponds to the statistical distribution of the infinite orbit produced by the LM.

In this way, it is possible to know the statistical behaviour of the chaotic system, though the value that takes some orbit, produced by the mentioned system, could not predict. BET was proved for the first time in 1931 by Birkhoff [14]. The most recent proof was realized by Rudolph in 1990 [15]. Fig. 3 shows the statistical distribution obtained by BET using $\mu = 3.9$ and a MatLab^{TM} program.

3. Circuit design

In the electronic implementation of the logistic noise generator, the logistic function is used as a fundamental component, which is iterated using an amplifier with the gain μ in the feedback loop.

Fig. 4 shows the MOS implementation of the logistic analog noise generator, where the LM is realized by transistors M1–M5,

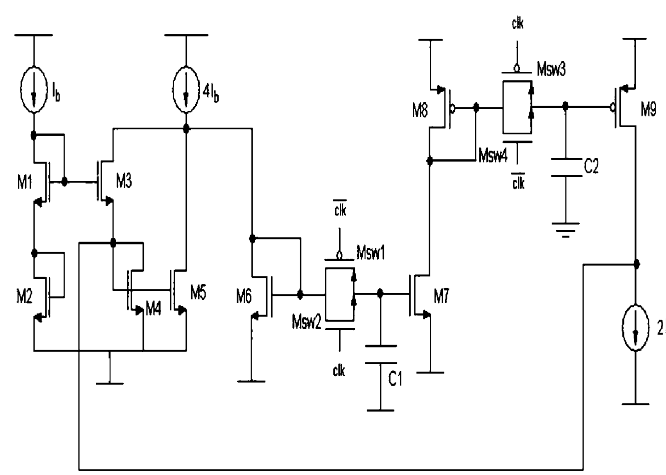

Fig. 4. Chaotic noise generator.

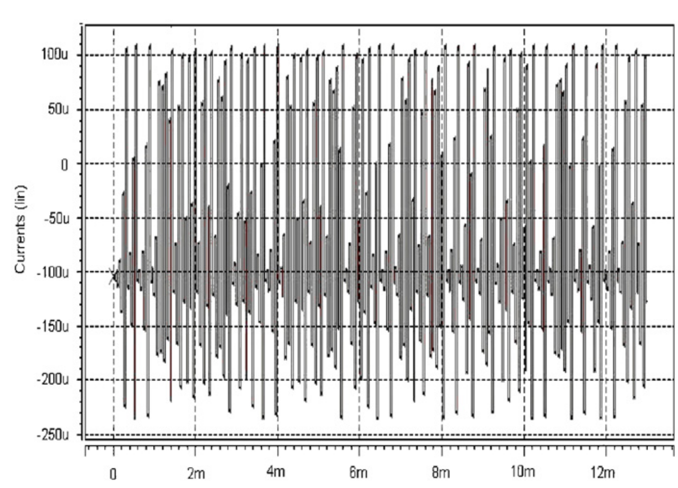

Fig. 5. Transient analysis using $\mu = 3.9$.

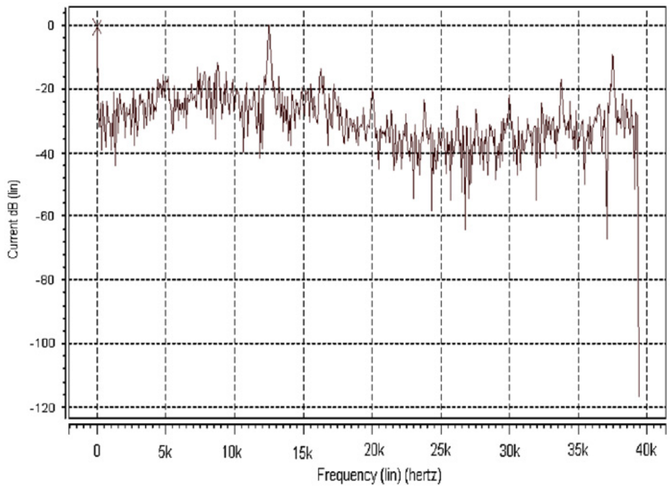

Fig. 6. Density spectrum using $\mu = 3.9$.

and its transfer function is given by

$$I_o = \frac{1}{8I_b} I_{in}(8I_b - I_{in}) \tag{2}$$

The feedback loop is composed of a sample and hold current amplifier with gain μ, which is set by the geometric ratio of M8 and M9. Associating Eq. (1) to the design conditions given by Eq. (2) we obtained

$$x_{n+1} = \mu \frac{x_n}{M}(M - x_n); \quad M = 8I_b \tag{3}$$

4. Results and discussion

The simulations results in Hspice for the circuit of Fig. 4 in 0.35 μm AMS technology and using $I_b = 50\,\mu A$, $V_{dd} = 1.65\,V$, $V_{ss} = -1.65\,V$ and Clk = 25 kHz are shown in Figs. 5 and 6. In Fig. 5 the transient analysis for $\mu = 3.9$ is shown. In this case, the circuit operates in the chaotic regime, according with the BD and LE. Finally, in Fig. 6 the spectral density is shown, according with the results obtained using the BET.

5. Summary

An analog noise generator was designed and simulated as an electronic circuit. The design conditions for the electronic circuit were defined by BD, LE and BET. The circuit exhibited the chaotic behaviour as expected, and the spectral density was similar to that for white noise according with the statistical distribution calculated using BET. The simulations were realized in Hspice for the implementation in 0.35 mm AMS technology.

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