A New Class of Continuous-Time Delay-Compensated Parameter-Varying Low-Pass Elliptic Filters With Improved Dynamic Behavior

Jacek Piskorowski and Miguel Ángel Gutiérrez de Anda, *Member, IEEE*

***Abstract*—In some applications, it is required to have continuous-time low-pass analog filtering systems which simultaneously possess a constant group delay in the passband and a narrow transition band. Usually, these systems are implemented by adding to the output of a filter with steep-amplitude selectivity (an elliptic filter, for instance) an all-pass filter which compensates the unfavorable run of the group delay response of the first filter. However, this compensation process will also increase the duration of the transient response of the resulting filtering system. This paper presents a new class of delay-compensated parameter-varying low-pass elliptic filters with a transient response of short duration. This improvement is achieved by means of a temporary change in the value of the filter parameters when a steplike transition with a minimum amplitude is detected in the input signal. As a consequence of the control strategy used to induce parameter variations in the proposed class of filters, its behavior is nonlinear in nature. Therefore, its stability properties (and particularly their bounded-input bounded-output stability) are also assessed. Simulations verifying the effectiveness of the new class of filters are presented and compared to the performance of delay-compensated and delay-uncompensated elliptic filters which were chosen as study cases.**

***Index Terms*—Elliptic filters, group delay compensation, parameter-varying technique, stability, time-varying systems, transient behavior.**

I. Introduction

THE DESIGN of continuous-time analog filters is based on the approximation of the magnitude or phase specifications in the frequency domain by means of polynomials or rational functions [1]–[3]. In the design of these systems, however, steep-amplitude selectivity and constant group delay in the frequency domain are design specifications which are not orthogonal to each other and, therefore, are difficult to tune simultaneously [4].

Elliptic filters [5]–[11], also known as Cauer filters, achieve the narrowest transition band for the same filter order in comparison to other filter types. On the negative side, they have the most nonlinear phase response over their passband. Ideally, the transfer function of a filter ought to provide a group delay that is constant or nearly constant over the frequency range of interest, typically the passband. If the delay is constant, signal components over a wide frequency band are delayed equally. Therefore, the distortion introduced by the filter on the expected output signal is minimized. This is particularly important, for instance, in filters used for the transmission of baseband digital signals [12] or in pulse-shaping low-pass filters for analog read channels in hard disk drives [13].

A low-pass filter approximation which puts a special emphasis on the phase linearity of its transfer function is the Bessel approximation. Bessel filters give a constant propagation delay across the input frequency spectrum. However, their magnitude response is much less selective as compared to Butterworth, Chebyshev, and elliptic filters. The usual solution considered in the design of continuous-time low-pass analog filters which should simultaneously possess a constant group delay in the passband and a selective magnitude response is to compensate the delay of a given low-pass filter which has been optimized in its magnitude response by means of one or more all-pass filter sections connected in cascade to the output of the low-pass filter. However, this compensation scheme is always done at the expense of an increase in the duration of the transient behavior of the resulting delay-compensated low-pass filter.

In this paper, a new class of continuous-time parameter-varying delay-compensated low-pass elliptic filters with a transient response of short duration will be presented. The improvement attained by this class of filters is based on a temporary change in the value of their parameters when a steplike transition with a given minimum amplitude is detected in the input signal. The strategy proposed for the variation of parameters was used previously in the past with some modifications in a number of applications. For instance, in [15], a parameter-varying low-pass filter was used to eliminate the oscillatory response exhibited by load cells used in weighting applications. Another parameter-varying filter was used in [16] to reduce the time employed in the acquisition of evoked potentials generated through auditive stimuli. The parameter-varying delay-compensated low-pass elliptic filters proposed in this paper may have potential applications as pulse-shaping filters [12]–[14]. It should also be noticed that there are also other classes of continuous-time filters which have control strategies for the variation of their parameters in order to improve their time-domain or frequency-domain response. The interested reader may consult, for instance, [17]–[22].

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J. Piskorowski is with the Department of Electrical Engineering, Szczecin University of Technology, 70-313 Szczecin, Poland (e-mail: jacek.piskorowski@ps.pl).

M. A. Gutiérrez de Anda is with the Electronics Department, National Institute for Astrophysics, Optics and Electronics, 72840 Tonanzintla, Mexico (e-mail: mdeanda@inaoep.mx).

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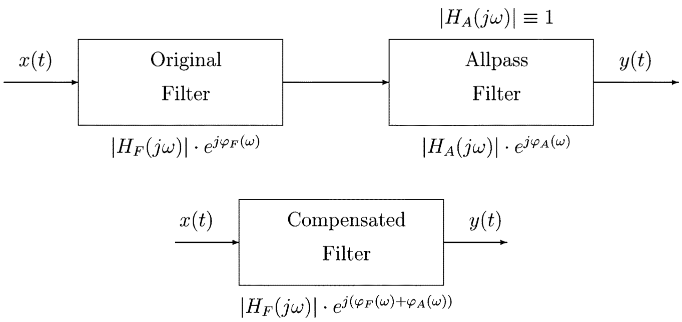

Fig. 1. Block diagram of group delay compensation.

The rest of this paper is organized as follows. In Section II, the problems associated to the group delay compensation are discussed with more detail. The theory behind the technique used to reduce the transient response of a delay-compensated low-pass filter is presented in Section III. In Section IV, a strategy used to detect edge-like transitions in the input signal with a given minimum amplitude is introduced. These transitions lead to the appearance of transient behavior at the filter output. The strategy presented in this section is used to determine in which time instants an increase of the filter parameters is required. Sections III and IV constitute the main contribution of this paper. Finally, some concluding remarks are given in Section V.

II. Group Delay Compensation

A. Delay-Compensation Fundamentals

If a filter is solely designed for a given magnitude response in the frequency domain, the designer has no control over its group delay. Once the magnitude response of the filter has been chosen, its poles and zeros are also determined. Therefore, there remains no degree of freedom to meet any additional filter specifications. In this case, the only option left is to select a filter approximation which gives the "least objectionable" group delay.

If the obtained group delay varies too much for the specifications imposed on the design, the transfer function of the filter may be adjusted as a whole. However, this process has to be made in such a way that the magnitude response of the filter is not affected. For this purpose, an all-pass filter H_A should be connected at the output of the filter H_F with the desired magnitude response as it is shown in Fig. 1.

The magnitudes of the original and the all-pass filters multiply with no contribution from the all-pass module, since $|H_A(j\omega)| \equiv 1$. Moreover, the phases of both filters will add. Since the group delay of an arbitrary filter is obtained from the negative derivative of its phase, the delay $D_F(\omega)$ of the original filter H_F and the delay $D_A(\omega)$ of the all-pass filter H_A will also add

$$D_C(\omega) = -\frac{\mathrm{d}\varphi_F(\omega)}{\mathrm{d}\omega} - \frac{\mathrm{d}\varphi_A(\omega)}{\mathrm{d}\omega} = D_F(\omega) + D_A(\omega). \quad (1)$$

A delay equalizer usually consists of one or more all-pass networks, each of which has the same gain for all frequencies. With a proper design of the all-pass sections, the group delay of the equalized filter approximates a constant group delay response. However, as the equalizer complexity increases, so does the number of circuit components, power consumption, and noise. These tradeoffs must be carefully evaluated to suite the application under consideration. However, these design problems will not be addressed here.

B. Delay-Compensation Example and Conclusions

In this paper, a second-order elliptic filter with passband peak-to-peak ripple $R_p = 0.5$ dB, minimum stopband attenuation $R_s = 60$ dB, and cutoff frequency $\omega_c = 1$ rad/s will be considered as a study case. The transfer function of this filter is of the form

$$H_F(s) = \frac{0.0009967s^2 + 1.4319}{s^2 + 1.4249s + 1.5167}. \quad (2)$$

In order to compensate the group delay of the aforementioned filter, a first-order all-pass section with magnitude equal to one will be used. The transfer function of the first-order all-pass filter considered in this study case is given by

$$H_A(s) = \frac{1 - \mu s}{1 + \mu s} = \frac{-s + \sigma}{s + \sigma}. \quad (3)$$

where μ is a positive constant and $\sigma = \mu^{-1}$.

The group delay functions of the second-order elliptic filter $D_F(\omega)$ and the first-order all-pass filter $D_A(\omega)$ can be expressed as

$$D_F(\omega) = \frac{0.6194\omega^2 + 0.9395}{0.4347\omega^4 - 0.4360\omega^2 + 1} \quad (4)$$

$$D_A(\omega) = \frac{2\mu}{1 + \mu^2\omega^2}. \quad (5)$$

Procedures for group delay compensation are well known from filter theory. However, these methods are based on a trial-and-error approach [2], [3]. In this paper, an analytical method which makes use of the Taylor series expansion around a given frequency point of the group delay functions will be applied. In an analogy to Butterworth filters, whose response is maximally flat when $\omega = 0$, the group delay of the elliptic and all-pass filters will be linearized in this example at $\omega = 0$ in order to obtain a group delay for the compensated filter which is as flat as possible around that frequency. The Taylor expansions in (4) and (5) around $\omega = 0$ may be written as follows:

$$D_F(\omega) \approx 0.9395 + 1.0291\omega^2 + 0.0403\omega^4 + \cdots \quad (6)$$

$$D_A(\omega) \approx 2\mu - 2\mu^3\omega^2 + 2\mu^5\omega^4 + \cdots. \quad (7)$$

The total (compensated) group delay equals the sum of the contributions due to each of the individual sections, i.e., the uncompensated filter and the all-pass module. The same may be said with respect to the Taylor expansions of the group delay functions given in (4) and (5). Therefore, the total group delay is of the form

$$\begin{aligned} D_C(\omega) &= D_F(\omega) + D_A(\omega) \\ &\approx (0.9395 + 2\mu) + (1.0291 - 2\mu^3)\omega^2 \\ &\quad + (0.0403 + 2\mu^5)\omega^4 + \cdots. \end{aligned} \quad (8)$$

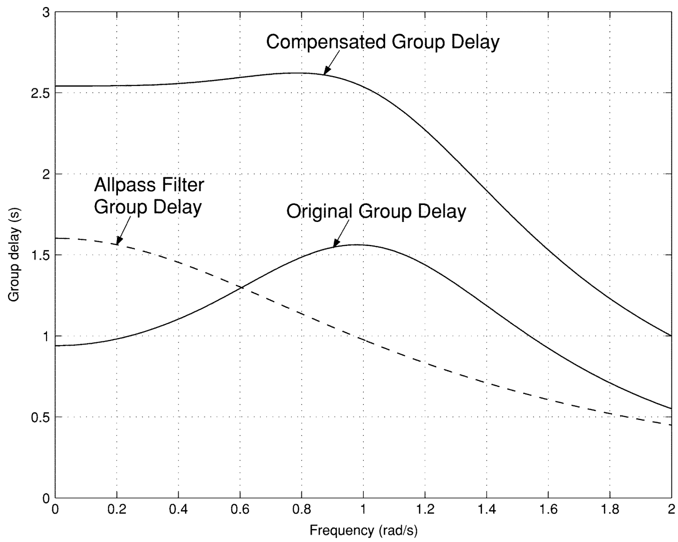

Fig. 2. Results of group delay compensation.

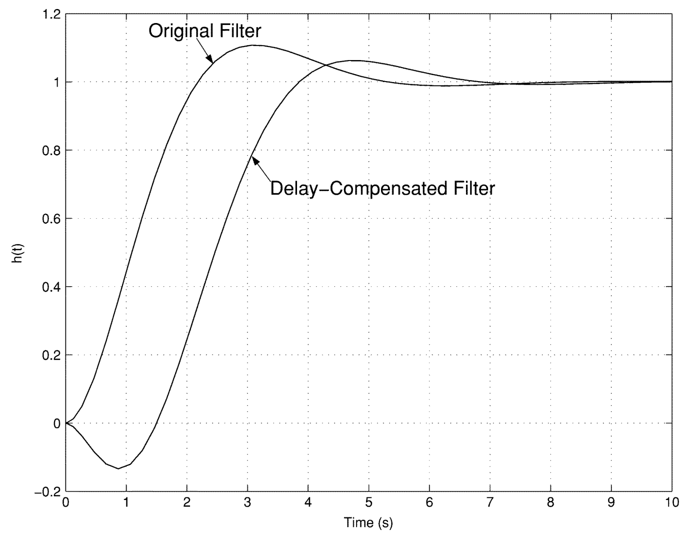

Fig. 3. Step responses of original and compensated filters.

The group delay compensation method used in this paper consists in determining a value of the all-pass filter parameter μ for which the total delay $D_C(\omega)$ is independent of the smallest power of the angular frequency ω. In this case, it is necessary to make equal to zero the constant associated to ω^2 in (8). In other words

$$1.0291 - 2\mu^3 = 0. \quad (9)$$

This equation has a unique real solution, and it is $\mu = 0.8013$.

The results of the group delay compensation of the second-order elliptic filter whose transfer function is given in (2) are shown in Fig. 2. This figure presents the group delay functions of the original filter, the compensated filter, and the all-pass filter section which has been used for the compensation. It is easy to notice that the process of the group delay compensation yielded good results. The group delay response of the compensated filter is considerably flatter in the normalized filter passband than the original one. For the original filter, the minimum and maximum values of the delay in the passband are, respectively, $D_{F\min} = D_F(0) = 0.94$ s and $D_{F\max} = D_F(0.98) = 1.56$ s. Therefore, the variation in the passband is $\Delta D_F = 0.62$ s. On the other hand, the minimum and maximum delay values for the compensated filter are $D_{C\min} = D_C(1) = 2.54$ s and $D_{C\max} = D_C(0.78) = 2.62$ s. Therefore, the delay variation in the passband is $\Delta D_C = 0.08$ s.

From these values, it can be concluded that, after compensation, the delay variation is 7.75 times smaller than the delay variation of the original filter. Of course, the total (compensated) delay is larger than the delay of the original filter. However, the absolute delay is normally of less concern than the delay variation.

In this example, the variations of the delay in the passband were minimized by determining a value of the parameter μ associated to the all-pass filter such that the delay response of the compensated filter is as flat as possible. There are, of course, many other criteria that may be selected to obtain an optimized group delay. For example, it may be desirable to construct the filter such that the delay will have an equal ripple over the passband. In general, computer routines are often needed to solve such requirements, but in some simple cases, closed-form solutions are available.

A group delay compensation process is always made at the expense of an increase in the duration of the transient response of the compensated filter. Fig. 3 shows the step responses of the original and compensated filters which have been considered in our example. It is easy to notice that the transient response of the filter whose group delay has been compensated lasts longer. Additionally, the response of the compensated filter contains an undesirable undershoot. Some ciphers can throw more light into this problem. For the original filter, the 2% settling time is $\bar{t}_s = 4.80$ s, and the overshoot amounts to 10.74%, whereas the compensated filter has, for the same specifications, a settling time of $\bar{t}_{sC} = 6.10$ s and an overshoot of 6.22%. Additionally, the compensated filter has an undershoot which equals to 13.44%. After comparing these ciphers, it may be concluded that, after compensation, the settling time has increased by 27.08%, and the overshoot has been reduced by 42.8%.

In the next sections, the fundamentals of operation of a new class of continuous-time delay-compensated parameter-varying low-pass elliptic filters which have improved transient responses will be presented.

III. Parameter-Varying Technique

A. Introduction

The improvement of the transient behavior of a system for a given set of operating conditions is an old problem which has been considered in many fields of engineering. There is a plethora of techniques used for this aim in adaptive and control systems. In the field of circuit design, there are many situations in which the transient behavior of a given system must be minimized as much as possible. Operational amplifiers used in switched-capacitor circuits are the best example of these systems. There are several techniques proposed in the literature for the design of these circuit blocks which take into account the adjustment of the settling time within a given boundary (see, for instance, [23]–[27]).

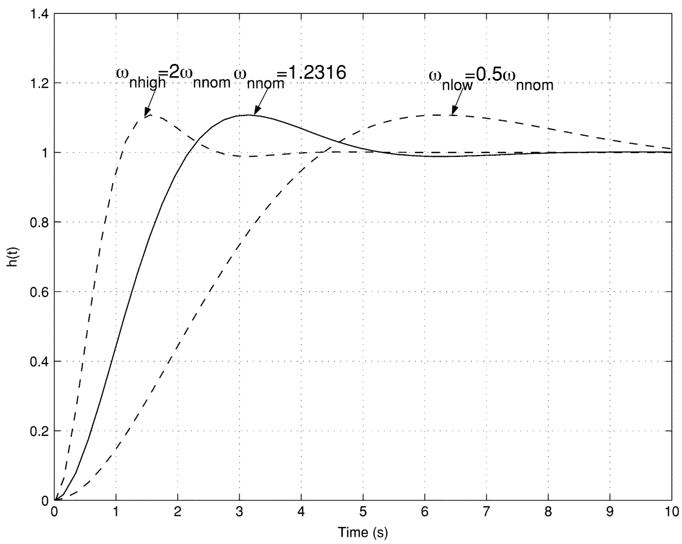

Fig. 4. Step responses for various values of natural frequency ω_n.

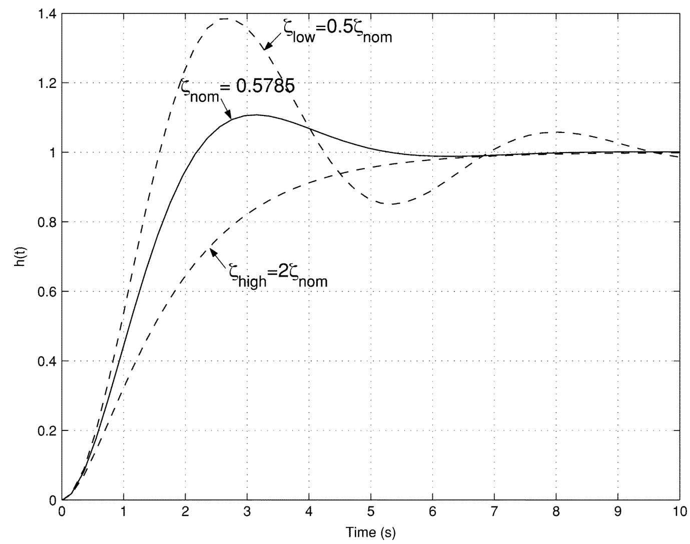

Fig. 5. Step responses for various values of damping ratio ζ.

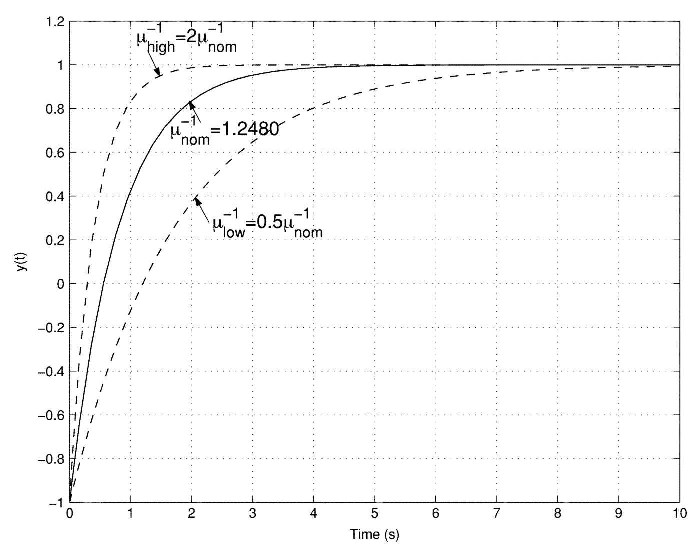

Fig. 6. Step responses for various values of the inverse of all-pass filter parameter μ.

In the particular case of traditional continuous-time analog low-pass filters, there are few options to improve their transient response without disturbing its magnitude or phase response. If timing characteristics of the output signal of the filter for a given stimulus are taken into account during the design process as a design specification, the only option left to the designer is to evaluate the performance in the time domain of different transfer-function approximations which satisfy the required magnitude and phase specifications for his/her design problem and choose the best approximation.

There is another possibility to solve the problem previously described. It is possible to attain a significant reduction in the duration of the transient behavior of a low-pass filter to a given input signal by varying its filter passband as done in [16], [28]–[30]. The variations of the filter passband are achieved by varying the value of the filter coefficients during the time interval where the transient behavior is expected to occur. The technique used to vary the parameters of a prototype linear time-invariant filter as well as the properties of the resulting filtering system will be presented in the next subsections.

B. Filter Dynamics

The dynamic properties of the second-order low-pass elliptic filter are described by the damping ratio ζ and the undamped natural frequency ω_n. The transfer function of such a kind of filter is usually written in the following form:

$$H_F(s) = \frac{h_0\left(\omega_n^{-2}s^2 + k^2\right)}{\omega_n^{-2}s^2 + 2\zeta\omega_n^{-1}s + 1} \tag{10}$$

where h_0 and k are constants. The coefficient h_0 is determined such that $|H_F(j\omega)|_{\max} = 1$. For the second-order elliptic filter which was considered in Section II, its dynamical parameters are as follows: $\omega_n = 1.2316$, $\zeta = 0.5785$, $h_0 = 0.0011$, and $k = 30.7262$. It is not difficult to demonstrate that a larger value of the natural frequency ω_n of the second-order transfer function given in (10) will lead to a shorter duration of its transient behavior to a step input. On the other hand, a larger value of the damping ratio ζ implies a reduction of the expected overshoot of the filter. Figs. 4 and 5 show simulation results of the step responses of the transfer function given in (10) when ω_n and ζ are varied separately.

In a similar fashion, the inverse of the all-pass filter parameter μ has an effect in the control of the transient behavior of the all-pass filter which may be comparable to the role of the natural frequency ω_n in the second-order low-pass filter. The relation between the duration of the transient behavior in the all-pass filter and the value of μ^{-1} is shown in Fig. 6. As shown, the transient behavior strongly depends on the dynamical parameters of the filter. By changing these parameters in time, it is possible to improve the dynamics of the filter and obtain a significant reduction of the transient duration.

C. Time-Varying Filter Model for Reducing the Duration of the Transient Behavior in Delay-Compensated Elliptic Filters

The proposed technique for the variation of parameters of the delay-compensated elliptic filter is the result of modeling the scalar single-input–single-output ordinary differential equations associated to the transfer functions given in (3) and (10) with time-varying coefficients. In order to improve the time-domain response of the compensated filter, it was assumed that its dynamic parameters are varied in time. The delay-compensated elliptic filter with time-varying parameters may be mathematically represented as follows:

$$\omega_n^{-2}(t)y_1''(t) + 2\zeta(t)\omega_n^{-1}(t)y_1'(t) + y_1(t) = h_0\left[\omega_n^{-2}(t)x''(t) + k^2x(t)\right] \tag{11a}$$

$$y'(t) + \sigma(t)y(t) = -y_1'(t) + \sigma(t)y_1(t) \tag{11b}$$

where $x(t)$ and $y(t)$ are the input and output of the filter, respectively. Functions $\omega_n(t)$, $\zeta(t)$, and $\sigma(t)$ define the parameter variations of the natural frequency, damping ratio, and the inverse of the all-pass filter parameter μ, respectively. It should be noticed that, in (11a), parameters h_0 and k are not time-varying, since they do not influence the speed of the filter response. For the same reason, the magnitude of the all-pass filter has not been included as an additional parameter which may be varied to improve the speed of the filter response.

Equation (11a) describes the dynamics of the low-pass elliptic filter when its parameters are being varied, and (11b) represents the all-pass filter section. In the proposed model, it has been assumed that individual filter sections do not load each other. It is clearly seen from (11) that the output of the elliptic filter denoted by $y_1(t)$ is, at the same time, the input signal for the all-pass filter section.

D. Parametric Functions of the Filter

In Section II-B, it was shown how the natural frequency, the damping ratio, and the all-pass filter parameter influence the transient behavior of the filter. The analysis pointed out that, for larger values of the parameters ω_n and μ^{-1}, the duration of the transient behavior is diminished. On the other hand, the overshoot of the filter is reduced for increasing values of ζ. If these rules are taken as a departure point, it may be concluded that, in order to improve the dynamic behavior of the compensated filter, a temporary increase of the filter parameters has to take place when the filter is expected to display transient behavior at its output. Therefore, the functions responsible for the variation of the filter parameters $\omega_n(t)$, $\zeta(t)$, and $\sigma(t)$ have been formulated as follows:

$$\omega_n(t) = \bar{\omega}_n \cdot [1 + (d_\omega - 1) \cdot h_S(t)] \tag{12}$$

$$\zeta(t) = \bar{\zeta} \cdot [1 + (d_\zeta - 1) \cdot h_S(t)] \tag{13}$$

$$\sigma(t) = \bar{\mu}^{-1} \cdot [1 + (d_\sigma - 1) \cdot h_S(t)] \tag{14}$$

where $\bar{\omega}_n$ and $\bar{\zeta}$ are the natural frequency and the damping ratio which come from the low-pass elliptic-filter approximation, and $\bar{\mu}^{-1}$ is the all-pass filter parameter which comes from the delay-compensation calculation. The coefficients d_ω, d_ζ, and d_σ define variation ranges of the functions $\omega_n(t)$, $\zeta(t)$, and $\sigma(t)$, respectively. These parameters are given by

$$d_\omega = \frac{\omega_n(0)}{\bar{\omega}_n}, \quad d_\zeta = \frac{\zeta(0)}{\bar{\zeta}}, \quad d_\sigma = \frac{\sigma(0)}{\bar{\mu}^{-1}}. \tag{15}$$

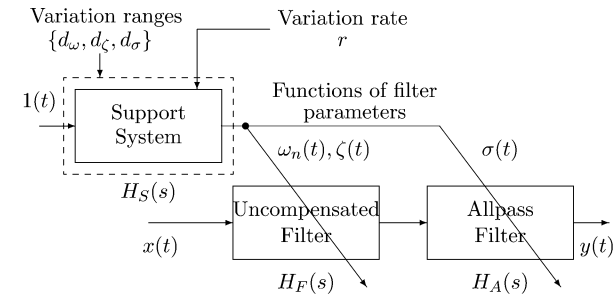

Fig. 7. Block diagram of the delay-compensated parameter-varying low-pass elliptic filter.

For $\{d_\omega, d_\zeta, d_\sigma\} > 1$, functions (12)–(14) decrease in the variation interval. The choice of functions (12)–(14) is related to the easiness of their generation by means of analog circuitry. The function $h_S(t)$ in (12)–(14) is the step response of the first-order support system $H_S(s)$ which is assumed to adopt the following form:

$$H_S(s) = \frac{r \cdot s}{r \cdot s + 1}. \tag{16}$$

The step response $h_S(t)$ of $H_S(s)$ can be written as follows:

$$h_S(t) = \mathcal{L}^{-1}\left[\frac{1}{s} \cdot H_S(s)\right] = \mathcal{L}^{-1}\left[\frac{r}{r \cdot s + 1}\right] \tag{17}$$

where $\mathcal{L}^{-1}$ is the inverse Laplace transform and r is the time constant of the first-order support system. Constant r can be denoted as the exponential variation rate of functions $\omega_n(t)$, $\zeta(t)$, and $\sigma(t)$. In the time domain, (17) can be written in the following form:

$$h_S(t) = \mathrm{e}^{-\frac{t}{r}} \cdot 1(t) \tag{18}$$

where $1(t)$ stands for the unit step applied at $t = 0$.

Fig. 7 shows a block diagram of the proposed parameter-varying delay-compensated low-pass elliptic filter. This diagram presents in a general way how the support system influences the dynamics of the elliptic and all-pass filters by means of the functions given in (12)–(14).

E. Functions Restrictions

In order to not alter the transfer characteristics of the delay-compensated filter when its transient behavior has to be reduced in duration, (12)–(14) must settle, respectively, to the values $\bar{\omega}_n$, $\bar{\zeta}$, and $\bar{\mu}^{-1}$ during the time interval $[0, \bar{t}_s]$. Parameter $\bar{t}_s$ stands for the settling time of the original time-invariant filter for an assumed accuracy factor α. In other words, it is required that the adjustment of the filter parameters is done only during the existence of transient behavior in the filter. At the same time, it must be guaranteed that the behavior of the filter itself will remain stable.

The stability properties of the set of equations (11) will be discussed first. In [31], the stability properties of the differential equation

$$y''(t) + 2\zeta(t)\omega_n(t)y'(t) + \omega_n^2(t)y(t) = 0 \tag{19}$$

are studied, and a set of conditions are given for functions $\omega_n(t)$ and $\zeta(t)$ such that the solutions of (19) are asymptotically stable. These conditions were determined by means of the second Lyapunov method and are given as follows:

$$\omega_n(t) > 0 \tag{20a}$$
$$\zeta(t) > 0 \tag{20b}$$
$$|\omega_n'(t)| < \left|\omega_n^2(t)\zeta(t)\right|. \tag{20c}$$

Functions $\omega_n(t)$ and $\zeta(t)$ defined in (12) and (13) satisfy the constraints given in (20a) and (20b). If the definitions for $\omega_n(t)$ and $\zeta(t)$ are substituted in condition (20c), the following inequality may be obtained for $t > 0$:

$$\begin{aligned}\left|a\mathrm{e}^{-\frac{t}{r}}\right| &< \left|\bar{\omega}_n^2\bar{\zeta}\left(1 + b_1\mathrm{e}^{-\frac{t}{r}} + b_2\mathrm{e}^{-\frac{2t}{r}} + b_3\mathrm{e}^{-\frac{3t}{r}}\right)\right| \\ &\leq \bar{\omega}_n^2\bar{\zeta}\left(1 + \left|b_1\mathrm{e}^{-\frac{t}{r}}\right| + \left|b_2\mathrm{e}^{-\frac{2t}{r}}\right| + \left|b_3\mathrm{e}^{-\frac{3t}{r}}\right|\right)\end{aligned} \tag{21}$$

where the coefficients a, b_1, b_2, and b_3 are given as follows:

$$a = \frac{(d_\omega - 1)\bar{\omega}_n}{r} \tag{22a}$$
$$b_1 = 2d_\omega + d_\zeta - 3 \tag{22b}$$
$$b_2 = d_\omega(d_\omega + d_\zeta - 4) - 2d_\zeta - 3 \tag{22c}$$
$$b_3 = d_\omega(2 - d_\omega - d_\zeta + d_\omega d_\zeta) + d_\zeta - 1. \tag{22d}$$

In order to guarantee that the inequality (20c) holds, it suffices to guarantee that $a < \bar{\omega}_n^2\bar{\zeta}b_1$. This implies that coefficient r must satisfy the following inequality:

$$r > \frac{d_\omega - 1}{\bar{\omega}_n\bar{\zeta}(2d_\omega + d_\zeta - 3)} > 0. \tag{23}$$

This relation is valid provided that $d_\omega > 1$ and $d_\zeta > 1$. As it will be seen soon, the stability criterion given in (23) is very useful for the determination of the parameters r, d_ω, d_ζ, and d_σ in functions (12)–(14).

Asymptotic stability as such does not guarantee bounded-input bounded-output (BIBO) stability. However, if the homogeneous solutions of (19) are exponentially asymptotically stable and the coefficients of (19) are bounded for all t, (11a) has BIBO stability [32], [33]. Unfortunately, it is not possible to analytically assess whether (19) has solutions which are exponentially asymptotically stable or not. Later on, in Section III-F, a method will be presented to determine numerically whether the solutions of (19) are exponentially bounded or not.

In the particular case of the system described by (11b), its general homogeneous solution may be written as

$$y(t) = C\exp\left(-\int\sigma(t)\mathrm{d}t\right) \tag{24}$$

where C is an arbitrary constant. In order to guarantee the asymptotic stability of the previous function, it suffices that function $\sigma(t)$ is always positive. Once more, the parameter-varying all-pass filter described by (11b) will generate a bounded output if its homogeneous solution is stable and its input is bounded. If the output of the parameter-varying low-pass elliptic filter generates a bounded output, the output of the parameter-varying all-pass filter will also be bounded.

The dynamics of the elliptic filter may be improved when the settling time t_{sf} of the functions given in (12)–(14) will equal the settling time of the original time-invariant filter $\bar{t}_s$, i.e.,

$$t_{\mathrm{sf}} = \bar{t}_s \approx \frac{-\ln\alpha}{\bar{\omega}_n\bar{\zeta}}. \tag{25}$$

where α is a constant which defines the maximum permissible error for the settling-time measurement. For an error of 2%, $\alpha = 0.02$. The settling time of $\omega_n(t)$ can be derived from the following relation:

$$\omega_n(t_{\mathrm{sf}}) = (1 + \alpha)\cdot\bar{\omega}_n. \tag{26}$$

Substituting (12) and (18) into the left side of this relation, we have

$$\bar{\omega}_n\cdot\left[(d_\omega - 1)\cdot\exp\left(-\frac{t_{\mathrm{sf}}}{r}\right) + 1\right] = (1 + \alpha)\cdot\bar{\omega}_n. \tag{27}$$

The settling time for function $\omega_n(t)$ can be determined from this equation and is given by

$$t_{\mathrm{sf}} = -r\cdot\ln\frac{\alpha}{d_\omega - 1}. \tag{28}$$

From this last relation, it is possible to calculate the exponential variation rate r of the functions $\omega_n(t)$, $\zeta(t)$, and $\sigma(t)$. Using (25) and (28), the exponential variation rate r can be written as

$$r = \frac{\ln\alpha}{\bar{\omega}_n\bar{\zeta}\left(\ln\alpha - \ln(d_\omega - 1)\right)}. \tag{29}$$

If d_ω and α are known, r may be readily determined. In order to preserve the stability of the system, the value of d_ζ must be selected such that the condition given in (23) holds. Finally, parameter d_σ must be equal to d_ω to guarantee that function (14) will have the same settling time.

F. Parameter Selection

In the previous section, a lower bound for the exponential variation rate r of functions $\omega_n(t)$, $\zeta(t)$, and $\sigma(t)$ was determined. Moreover, it was also concluded that the value of d_ω determines automatically the value of d_σ and establishes a maximum limit for the value that d_ζ may have if the stability condition given in (23) is taken into account. Here, it will be explained how to determine an optimal value of d_ω and d_ζ.

Suitable values for d_ω and d_ζ must be determined via computer simulations because a complete analytical solution of the differential equation given in (11a) is unfortunately not available. The search strategy for these values is described as follows. First, an interval $(1, d_{\omega_{\max}}]$ must be proposed in which the optimal value of d_ω will be searched. Using $d_\omega = d_{\omega_{\max}}$, the exponential variation rate r associated to this value should be calculated. In this step, it is assumed that the parameter α is already known and given as a design specification. Using condition (23), a second interval must be estimated for d_ζ from r and the maximum proposed value of d_ω in order to determine a search space. Once a space search has been defined, the optimal values of d_ω and d_ζ are selected by means of an optimization strategy in which the total efficiency factor η has to be minimized. The efficiency factor η is expressed as the product of the time efficiency factor η_T and the frequency efficiency factor η_Ω

$$\eta = \eta_\Omega \cdot \eta_T. \tag{30}$$

The time efficiency factor η_T is defined as the ratio of the settling time $\tilde{t}_{sC}$ of the delay-compensated filter with varying parameters and the settling time $\bar{t}_s$ of the original time-invariant filter with no phase compensation. On the other hand, the frequency efficiency factor η_ω stands for the ratio of the cutoff frequency $\tilde{\omega}_{C0}$ of the time-invariant filter which corresponds to the parameter-varying filter for $t = 0$ and the cutoff frequency $\bar{\omega}_c$ of the original uncompensated time-invariant filter

$$\eta_T = \frac{\tilde{t}_{sC}}{\bar{t}_s} \quad \eta_\Omega = \frac{\tilde{\omega}_{C0}}{\bar{\omega}_C}. \tag{31}$$

Therefore, optimal values for the variation ranges d_ω and d are found for $\eta = \min$. A smaller value of the efficiency factor η implies that there has been an improvement in the transient behavior of the designed parameter-varying filter. If the value of the total efficiency factor satisfies the condition $\eta < 1$, then the designed filter has better properties than the original time-invariant filter.

Finally, once the parameters r, d_ω, d_ζ, and d_σ have been chosen for the functions (12)–(14), it is necessary to assess numerically whether the resulting filtering system has BIBO stability or not. For this purpose, it suffices to obtain the magnitude of the modes associated to the homogeneous response of (11a). At this point, it is pertinent to introduce the concept of a mode. In systems theory, the equation

$$\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t) \tag{32}$$

where $\mathbf{x}(t)$ is a vector of state variables and $\mathbf{A}$ is the system matrix, has one and only one mode excited if its solution for a given initial condition $\mathbf{x}(t_0) = \mathbf{x}_0$ adopts the form

$$\mathbf{x}(t) = \mathbf{c}e^{\lambda t}. \tag{33}$$

In this expression, $\mathbf{c}$ is a constant vector, and λ is an eigenvalue associated to the system matrix $\mathbf{A}$ [34]. Assuming that the eigenvalues of $\mathbf{A}$ are different, it is possible to represent the response of system (32) as a sum of modes of the form (33) which will be linearly independent from each other. An important property of the mode given in (33) is that it defines not only a possible direction of growth of the general response of (32) but also its exponential rate of growth through the eigenvalue λ.

The previous concept can be extended for linear time-varying systems of the form

$$\mathbf{x}'(t) = \mathbf{A}(t)\mathbf{x}(t) \tag{34}$$

as it was proposed in [35]. According to [35], the magnitude of each of the modes present in (34) may be obtained from the magnitude of the orthogonalized columns of the transition matrix of (34). The magnitude of each of the orthogonalized vectors will contain a measure of their growth, which does not necessarily has to decrease or increase exponentially. If the magnitudes of each of the modes of system (34) are functions which tend to zero and are exponentially bounded for a given $t > t_0$, then system (34) is exponentially asymptotically stable [35].

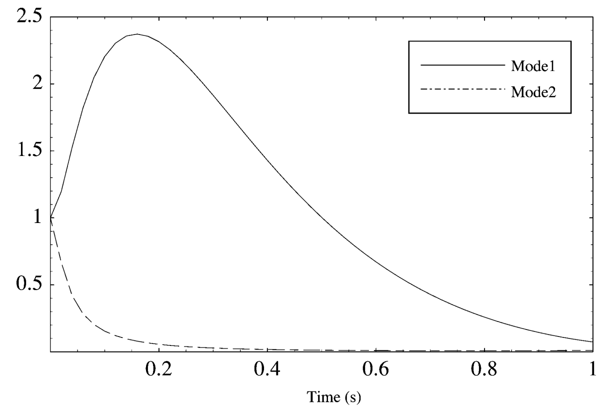

Fig. 8. Magnitudes of the modes of the filter represented by (11a).

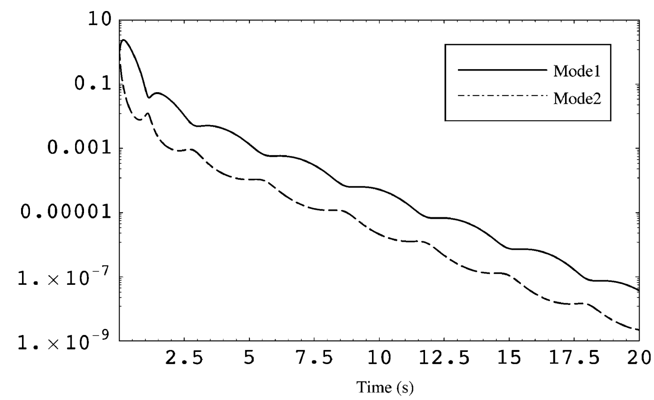

Fig. 9. Magnitudes of the modes of the filter represented by (11a) for a sufficiently large time interval.

In order to determine whether (11a) has exponential asymptotic stability or not, this equation may be cast into state form using phase variables. After this step, the magnitudes of its modes may be computed with the aforementioned method. If the resulting state equation is not exponentially asymptotically stable, then it is not possible to guarantee the BIBO stability of the filtering system. In that case, new constants r, d_ω, d_ζ, and d_σ have to be selected such that the resulting filtering system has exponential asymptotic stability. However, from the numerical experiments conducted so far, it has not yet detected a filtering system which does not have BIBO stability, so it seems unlikely that the previously mentioned parameters have to be recalculated.

G. Results

Applying the method presented in the previous subsection, the following parameter values for the filtering system were determined for $\alpha = 0.02$: $r = 1.05$, $d_\omega = d_\sigma = 5$, and $d_\zeta = 1.6$. The magnitudes of the modes associated to (11a) are shown in Fig. 8. As shown, the magnitude of each of the modes tends asymptotically to zero. If the magnitudes of the modes are plotted in a time interval sufficiently large and a logarithmic scale is used for the ordinate as in Fig. 9, it can be further verified that these functions are exponentially bounded for $t > 2.5$ s. Therefore, the resulting filtering system is BIBO stable.

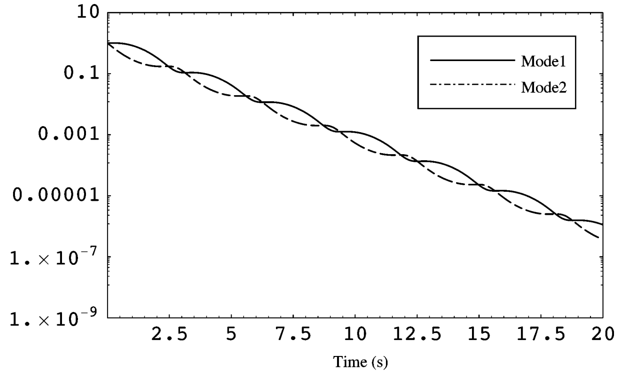

Fig. 10. Magnitudes of the modes of the uncompensated linear time-invariant elliptic filter for a sufficiently large time interval.

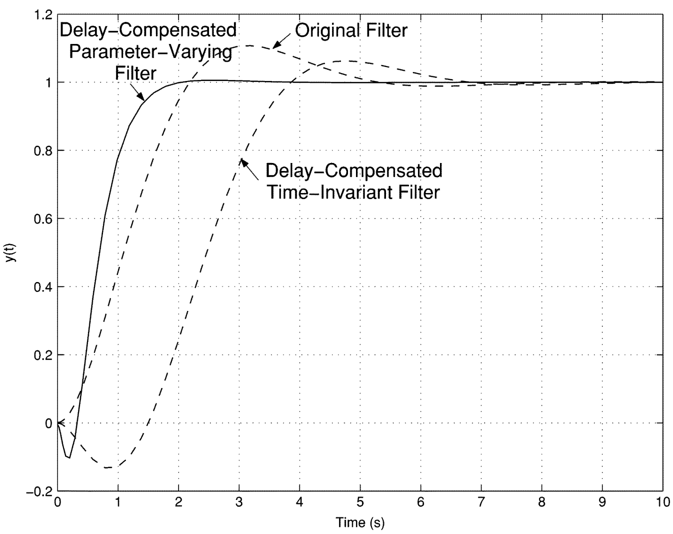

Fig. 11. Step responses of original, delay-compensated time-invariant, and parameter-varying delay-compensated low-pass elliptic filters ($d_\omega = d_\sigma = 5, d_\zeta = 1.6, \alpha = 0.02, r = 1.05$).

The magnitudes of the modes of the original linear time-invariant uncompensated elliptic filter described in Section II are shown in Fig. 10. If the magnitudes of the modes of the linear time-invariant elliptic filter are compared to the magnitudes of the modes of the parameter-varying filter, it may be seen that both pairs have the same decrease rate when $t > 2.5$ s. This is to be expected since the time-varying coefficients of (11a) must tend for large t to the coefficients of the original elliptic filter. However, the magnitudes of the modes associated to (11a) are significantly smaller in the same time interval. Moreover, the difference in magnitude between the modes of (11a) is greater than in the linear time-invariant case. These phenomena are a consequence of the temporary increase of the parameters in (11a) and need to be further studied.

Fig. 11 shows the simulation results of the original, delay-compensated time-invariant, and parameter-varying delay-compensated elliptic filters when a step signal is used as an input. It is easy to notice that the parameter-varying technique achieved a considerable reduction of the duration of the transient behavior. Moreover, the simulated parameter-varying filter is even faster than the original one, and undesirable overshoot has been eliminated from its response.

The settling time of the parameter-varying delay-compensated filter when $\alpha = 0.02$ is equal to 1.70 s. This time has to be compared to the settling times of the original and the delay-compensated filters for the same specification parameter α. The settling times of the original and the delay-compensated elliptic filters are, respectively, equal to 4.80 and 6.10 s. Therefore, the settling time of the parameter-varying delay-compensated filter is over 3.5 times shorter than the settling time of the compensated time-invariant filter and over 2.8 times shorter than the settling time of the original filter. On the other hand, the undershoot of the parameter-varying filter is of 10.28%. In other words, it underwent a reduction of 23.5% as compared to the undershoot of the delay-compensated time-invariant filter.

Fig. 12 shows a detailed model of the second-order parameter-varying delay-compensated low-pass elliptic filter which has been discussed in this paper. In this model, the uncompensated low-pass elliptic filter and the all-pass filter module are specified. The remaining part of the model describes the system which generates the functions of the filter parameters. A classical implementation of the parameter-varying filter as shown in Fig. 12 requires the use of multipliers, adders, and one additional integrator. As it can be noticed, the overall complexity of the system underwent a significant increase. However, in situations in which the transient behavior generated by the filter should be as short as possible, this complexity increase may be profitable. It seems likely that this filter could be implemented in CMOS technology using transconductors and capacitors to build the required integrators and multipliers. A linear time-varying filter presented in [36] which emulated the behavior of a linear time-invariant current-mode filter was built using such elements.

IV. Control Strategy for the Parameter Increase in the Parameter-Varying Filter

Until now, a strategy has been developed to reduce the duration of the transient behavior of the second-order delay-compensated low-pass elliptic filter when a step function is given as a stimulus at $t = 0$. Under normal conditions, an arbitrary input signal may be expected to have fast variations in its amplitude which are not related to the presence of noise such as the signal shown in Fig. 13. Such variations may lead to the appearance of transient behavior at the output of the filter. In a first approximation, these fast transitions may be treated as step signals. In this section, a method to detect steplike transitions in the input signal with a minimum amplitude will be proposed. This method will be used to determine in which time instants a temporary increase of the value of the filter parameters is required to reduce the duration of the transient behavior of the filter.

In order to make useful the technique proposed in the previous section for the filtering of signals containing steplike transitions in the time domain which are not due to the presence of noise, it is necessary to have another system which will be able to detect these transitions in the input signal [28]. This is particularly possible when the signal is distorted by a small amount of additive noise, as in Fig. 13. For this purpose, a system which delays the input signal $x(t)$ by a given time interval τ may be used.

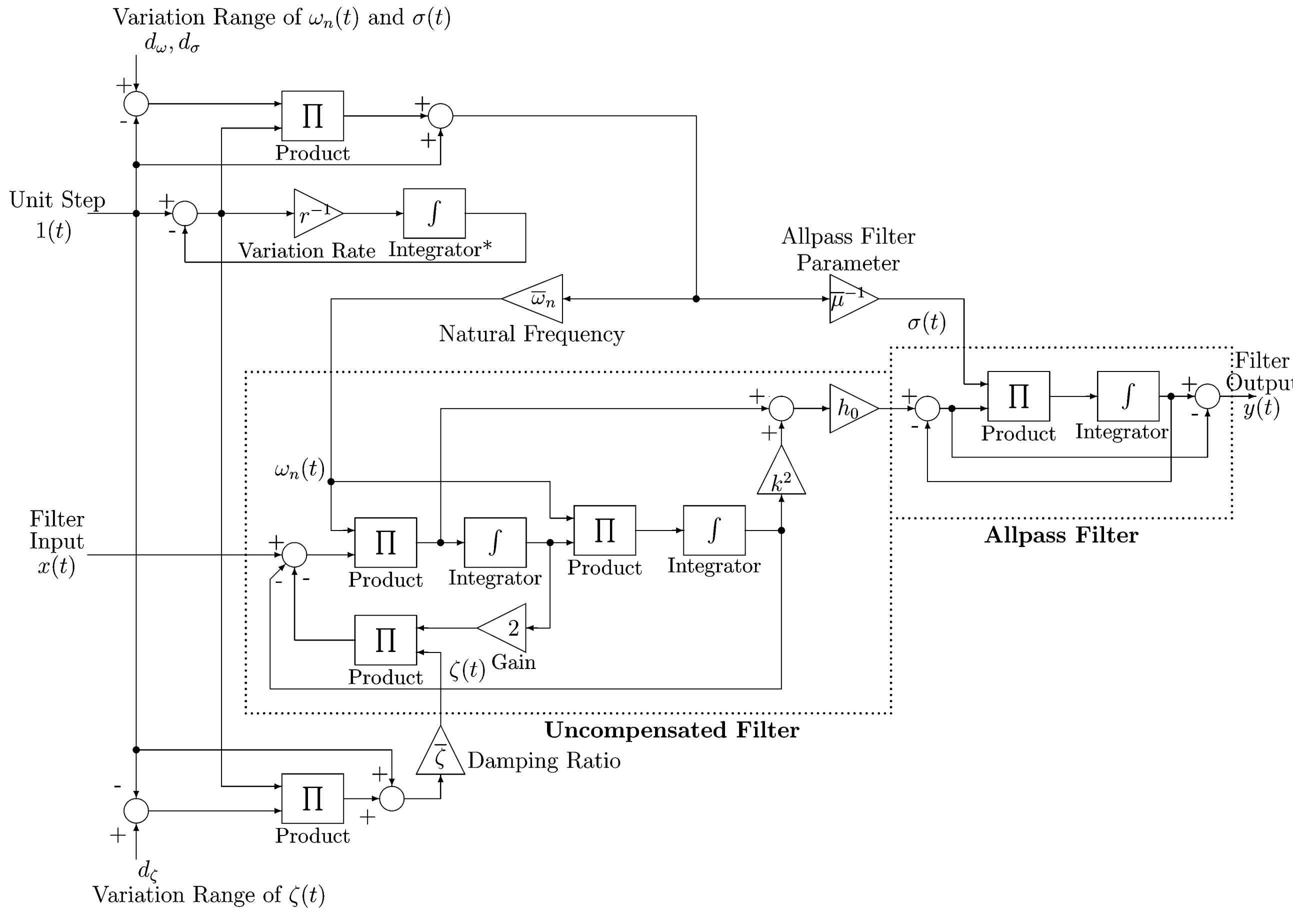

Fig. 12. Detailed model of the second-order parameter-varying delay-compensated low-pass elliptic filter.

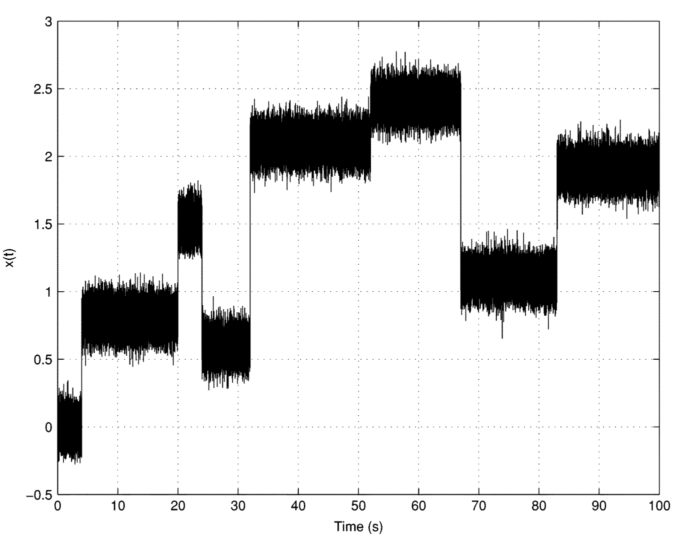

Fig. 13. Input signal with additive noise.

This time should be chosen in such a way that the most significant changes in the amplitude of the input signal are detected. A block diagram of the system required for this aim is shown in Fig. 14. The absolute value of the difference between the input signal and the delayed input signal $\gamma(t) = |x(t) - x(t-\tau)|$ is fed to the comparator input. The comparator compares the actual value of the signal $\gamma(t)$ with the activation threshold c. The

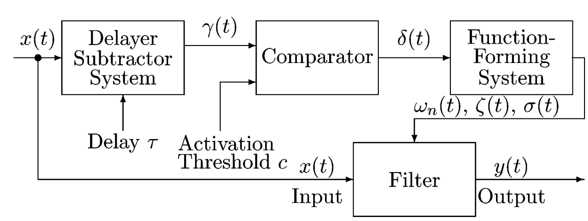

Fig. 14. Triggering system required for the control of the parameter variation in the proposed filter.

value of this threshold is chosen according to the variance of the input noise. A larger value of the activation threshold will be used when the level of input noise is higher. In many applications (e.g., in measurement systems), the noise level is small, and thus, it is possible to set the activation threshold of the designed filter to a given value.

If the condition $\gamma(t) \geq c$ is met, the comparator generates the trigger signal $\delta(t) = 1$, which means that a steplike transition which is not due to additive noise was detected in the input signal; otherwise, $\delta(t) = 0$. Then, the signal $\delta(t)$ is fed to the integrator and to its reset input. This integrator forms the support filter structure and is marked with a star in the diagram shown in Fig. 12. The trigger signal $\delta(t)$ fed to the reset input of the integrator is responsible for the generation of all functions of the filter parameters. Any rising edge of the signal $\delta(t)$ will restart

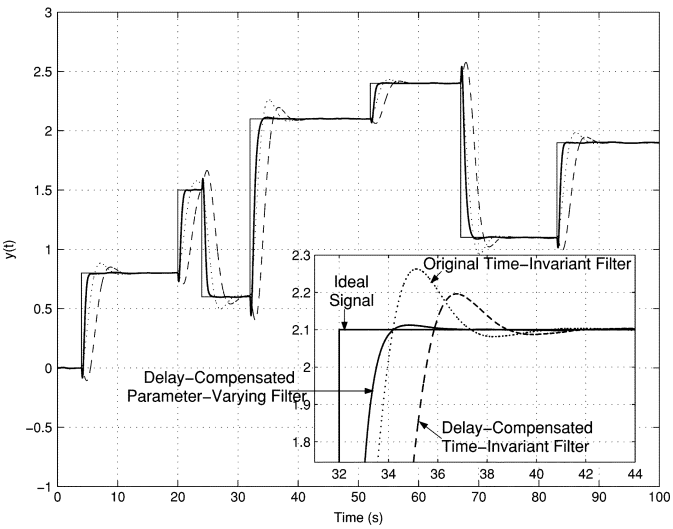

Fig. 15. Responses of the original filter, delay-compensated filter, and parameter-varying delay-compensated filter with triggering system to the signal shown in Fig. 13.

the support-system integrator, which in turn causes the generation of the functions $\omega(t)$, $\zeta(t)$, and $\sigma(t)$. Given that the trigger signal is generated by means of a static nonlinearity, the system shown in Fig. 12 will be BIBO stable, since its dynamics will still be largely dependent on the dynamics of the set of equations given in (11). It has to be noticed that the resulting filtering system after the addition of the triggering system will be nonlinear in nature.

Fig. 15 shows the simulation results of the traditional time-invariant elliptic filter, delay-compensated time-invariant filter, and the proposed parameter-varying delay-compensated filter with triggering system when the signal shown in Fig. 13 is applied as a stimulus. It is easy to notice that the application of the parameter-varying filter in the processing of signals with steplike transitions which are distorted by additive noise yields much better results as compared to the performance of time-invariant filters. If a steplike transition is detected in the input signal, the parameter-varying filter is considerably faster than the traditional time-invariant one. It is necessary to add that the group delay response is also equalized. This occurs when the filter parameters have already settled to a constant value. If the amplitude of a steplike transition present in the input signal is smaller than the threshold value used in the triggering system, a step will not be detected. Therefore, the filter will work in the delay-compensated time-invariant mode.

The proposed filter may be applied in many signal-processing and data-acquisition systems. There are, of course, some limitations. The noise level must be small as compared to the steplike transitions present in the input signal, and the changes of the input signal level should not be faster than the settling time of the filter. There are also data-acquisition processes in which it is known when the level changes in the input signal occur, and in this case, the triggering system is not required. Such a situation is observed, for instance, in the processing of analog time-multiplexed signals such as those present in the read channels in hard disk drives [14]. In this particular case, the increase of the filter parameters may be needed when a different input of the multiplexer is selected [30].

V. Conclusion

In this paper, the parameter-varying technique has been used in order to generate a new class of continuous-time delay-compensated parameter-varying low-pass elliptic filters with improved dynamic response. A block diagram of this class of filters has also been presented. Parameter-varying delay-compensated low-pass elliptic filters possess selective magnitude response and constant group delay over the passband when the filter parameters have settled to a constant value. It was demonstrated via simulations that the new class of filters achieved a considerable reduction of the duration of the transient response as compared to the traditional and the delay-compensated low-pass elliptic filters which were used as prototypes. Moreover, the simulated parameter-varying delay-compensated filter is even faster than the uncompensated filter and has a small overshoot in its response. The parameter-varying filter presented in this paper also satisfies a number of conditions for asymptotic stability and displays BIBO stability. Finally, although the proposed continuous-time parameter-varying delay-compensated low-pass analog filter is a system whose implementation may not be easy, preliminary analyses have confirmed that the proposed technique may be applied in practice successfully.

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Jacek Piskorowski was born in Pila, Poland, in 1977. He received the M.Sc. degree in electronic engineering and the Ph.D. degree from Szczecin University of Technology, Szczecin, Poland, in 2002 and 2006, respectively.

Since 2002, he has been with the Institute of Control Engineering, Department of Electrical Engineering, Szczecin University of Technology, where he is currently an Assistant Professor. His research activity is mainly focused on analysis and synthesis of systems and circuits with time-varying parameters.

Miguel Ángel Gutiérrez de Anda (M'07) was born in Puebla, Mexico, in 1973. He received the M.Sc. degree in electronics from National Institute for Astrophysics, Optics and Electronics (INAOE), Tonanzintla, Mexico, in 1998 and the Ph.D. degree in electrotechnical engineering from Delft University of Technology, Delft, The Netherlands, in 2003.

In 2003, he was a Summer Intern with Nokia Telecommunications, Haukipudas, Finland. Between 2003 and 2004, he was a Postdoctoral Scholar in the frame of the FP5 European Project System for European Water Monitoring with Warsaw University of Technology, Warsaw, Poland. From 2005 to 2006, he was a Visiting Professor of analog electronics at the Autonomous Metropolitan University, Iztapalapa, Mexico. Since 2007, he has been an Associate Researcher with the Electronics Department, INAOE. He was a Guest Scholar at Szczecin University of Technology, Szczecin, Poland, for two short periods in 2007. His current areas of interest are in linear-time varying circuits and systems, applied mathematical methods for the analysis and design of electronic circuits, and design of analog circuits.