A General Method to Design GCF Compensation Filter

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***Abstract*—This brief addresses the design of the compensation filter of a generalized comb filter (GCF). The proposed method is general and can be applied to different design constraints, i.e., maximally flat, least square, and minimax. The coefficients of the proposed filter for all three cases can be obtained by solving two simple linear equations. The filter operates at a low rate and considerably reduces the passband droop of the GCF.**

***Index Terms*—Finite-impulse-response (FIR) filters, generalized comb filters (GCFs), least square, maximally flat, minimax.**

I. Introduction

THE SIMPLEST decimation filter, which was proposed by Hogenauer [1], is the cascaded-integrator-comb (CIC) filter. However, this filter has a high passband droop and low stopband attenuation. Different methods have been proposed to improve the passband and stopband characteristics of the CIC filter [2]–[6]. Recently, a generalized CIC decimation filter [generalized comb filter (GCF)] has been proposed in [6], resulting in increased stopband attenuation and extended bands around the zeros of the magnitude characteristic of the CIC filter (folding bands), i.e., at frequency points $2\,\pi k/D$, where D is the decimation factor and $k = 1, \ldots, D-1$ [6], [7]. The modified CIC filters proposed in [8] and [9] are special cases of the GCFs. A polyphase structure of the GCF is analyzed in [7].

The transfer function of the GCF is expressed as [6]

$$H_{\mathrm{GCF}_N}(z) = \prod_{n=1}^{N} \frac{\sin(\alpha_n/2)}{\sin(\alpha_n D/2)} \prod_{n=1}^{N} \frac{1 - z^{-D} e^{-j\alpha_n D}}{1 - z^{-1} e^{-j\alpha_n}} \qquad (1)$$

where D is the decimation factor, and α_n, $n = 1, \ldots, N$, are rotation parameters that were optimized such that the minimum attenuation within folding bands is maximized [6].

The discrete-time Fourier transform (DTFT) of $H_{\mathrm{GCF}_N}(z)$ is

$$H_{\mathrm{GCF}_n}(e^{j\omega}) = H(\omega) \exp\left(-j\frac{(D-1)}{2}\left(\omega N + \sum_{n=1}^{N} \alpha_n\right)\right) \qquad (2)$$

where

$$H(\omega) = \prod_{n=1}^{N} \frac{\sin(\alpha_n/2)}{\sin(\alpha_n D/2)} \prod_{n=1}^{N} \frac{\sin\left((\omega + \alpha_n)D/2\right)}{\sin\left((\omega + \alpha_n)/2\right)}. \qquad (3)$$

In the general case, $H_{\mathrm{GCF}_N}(z)$ has linear-phase characteristics and complex-valued coefficients [see (2)]. The real-valued filter coefficients of $H_{\mathrm{GCF}_N}(z)$ are obtained by satisfying $\alpha_n = -\alpha_{N-n}$ [6]. A useful choice for α_n is $\alpha_n = q_n\pi/\nu D$, where ν is a positive integer and q_n is a real value in the range $[-1, 1]$ [6]. The traditional CIC filter is obtained by setting $\alpha_n = 0$, $n = 1, \ldots, N$.

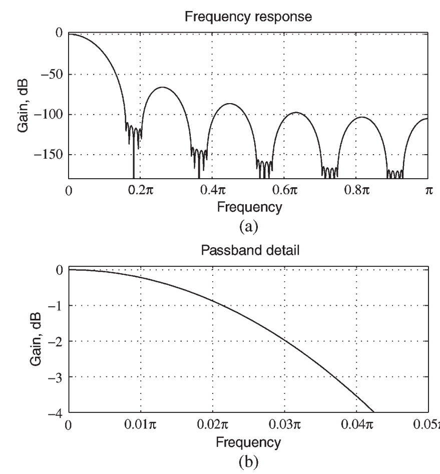

Fig. 1. Magnitude response of the GCF for $N = 5$, $D = 11$, and $\nu = 4$.

As an example, consider the design of a GCF using the following parameters: $N = 5$, $D = 11$, $\nu = 4$, and $q_n = [-0.55, -0.93, 0, 0.93, 0.55]$ [6]. Fig. 1(a) shows the magnitude response of the resulting GCF, whereas the passband detail is shown in Fig. 1(b). Notice the increased width and attenuations around zeros. However, the GCF exhibits a high passband droop [see Fig. 1(b)].

The review of existing compensation methods shows that compensation filters are mainly presented for traditional CIC filters and not for GCFs (see, e.g., [2]–[4]). To this end, in this brief, we introduce a general design method for the GCF passband compensation, which also includes the CIC passband compensation as a special case. The maximally flat, least-square, and minimax designs are included in this design.

The rest of this brief is organized as follows: Section II introduces the proposed second-order compensation filter along with maximally flat, least-square, and minimax designs. Discussions of the results are presented in Section III.

II. GCF Compensation Filter

The transfer function of the proposed GCF compensation filter is

$$P(z^D) = a + bz^{-D} + az^{-2D} \qquad (4)$$

where a and b are real-valued constants.

The compensation filter is cascaded with the GCF, as shown in Fig. 2(a). Using the multirate identity [10], filter $P(z^D)$ can

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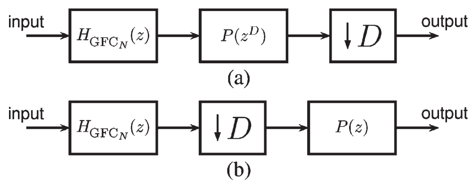

Fig. 2. Decimation block diagrams. (a) Generalized CIC filter $H_{\mathrm{GCF}_N}(z)$ and compensation filter. (b) Efficient structure for decimation.

be moved to a lower rate, resulting in a more efficient structure shown in Fig. 2(b).

The cascade of the compensation filter $P(z^D)$ and the GCF yields the following overall transfer function:

$$G(z) = H_{\mathrm{GCF}}(z)P(z^D). \tag{5}$$

By performing DTFT, (5) becomes

$$G(e^{j\omega}) = e^{-j\omega((D-1)N+2D)/2}H(\omega)P_R(D\omega) \tag{6}$$

where $P_R(D\omega)$ is the amplitude response of $P(e^{j\omega D})$, which is given by

$$P_R(D\omega) = b + 2a\cos(D\omega). \tag{7}$$

The next issue is to find coefficients a and b. We define the error function in the frequency range $[0, \omega_p]$, where ω_p is the upper edge frequency of the signal band, which is necessary to preserve after decimation, i.e.,

$$E(\omega) = H(\omega)P_R(D\omega) - 1. \tag{8}$$

In order to find coefficients a and b, we impose the condition in which the error function should be zero at frequencies $\omega = 0$ and $\omega = \omega_0$, where ω_0 is less than or equal to ω_p.

For $\omega = 0$, from (3), (6)–(8), it follows that

$$2a + b = 1. \tag{9}$$

Similarly, for $\omega = \omega_0$ [see (6)–(8)], the imposed condition results in

$$H(\omega_0)\left(2a\cos(D\omega_0) + b\right) = 1. \tag{10}$$

Solving (9) and (10), the values of a and b are given by

$$a = \frac{1}{2}\frac{1 - 1/H(\omega_0)}{1 - \cos(D\omega_0)} \tag{11}$$

$$b = 1 - 2a \tag{12}$$

respectively.

Equations (11) and (12) are general equations, where the value of ω_0 depends on the method used for the minimization of error, as described in the succeeding sections.

A. Maximally Flat Case

Here, we consider error function $E(\omega)$ to be maximally flat at $\omega = 0$, i.e., the error function has as many derivatives that are vanishing at $\omega = 0$ as possible [11].

Since the error function is an even function of ω, its odd-indexed derivatives evaluated at $\omega = 0$ are automatically zero. Therefore, it follows that

$$\left.\frac{d^2E(\omega)}{d\omega^2}\right|_{\omega\to 0} = 0. \tag{13}$$

Thus, solving (13) and using (12), the value of a is given by (see details in the Appendix)

$$a = \frac{1}{8D^2}\sum_{n=1}^{N}\left(\frac{1}{\sin^2(\alpha_n/2)} - \frac{D^2}{\sin^2(\alpha_n D/2)}\right). \tag{14}$$

For the traditional CIC compensation filter ($\alpha_n = 0$ for $n = 1, \ldots, N$), the value of a reduces to

$$a = \frac{N(1 - D^2)}{24D^2}. \tag{15}$$

It can easily be shown that the coefficient a for the maximally flat case defined in (14) can also be obtained from general equation (11) by replacing ω_0 with zero and applying the L'Hôpital rule.

B. Least-Square Case

For the least-square compensation filter, the error is minimized over band $[0, \omega_p]$, i.e.,

$$\min_a \epsilon \tag{16}$$

where

$$\epsilon = \int_0^{\omega_p} E^2(\omega)d\omega. \tag{17}$$

Condition (16) means that the first derivative of ϵ with respect to a is equal to zero and results in

$$a = \frac{1}{2}\frac{\int_0^{\omega_p} H(\omega)\left(H(\omega) - 1\right)\left(1 - \cos(D\omega)\right)d\omega}{\int_0^{\omega_p} H^2(\omega)\left(1 - \cos(D\omega)\right)^2 d\omega}. \tag{18}$$

Coefficient b is obtained from (12) and (18).

Combining general equation (11) and that obtained in (18), we have

$$\frac{1-1/H(\omega_0)}{1-\cos(D\omega_0)} = \frac{\int_0^{\omega_p} H(\omega)\left(H(\omega)-1\right)\left(1-\cos(D\omega)\right)d\omega}{\int_0^{\omega_p} H^2(\omega)\left(1-\cos(D\omega)\right)^2 d\omega}. \tag{19}$$

To relate frequency ω_p and frequency ω_0, we use $N = 3, 4, 5, 6$; $D = 3, 5, 7, 11$; and $\nu = 4$. We then solve (19). (The values of N and ν are chosen from [6].) Fig. 3(a) shows frequency ω_p as a function of ω_0, whereas Fig. 3(b) shows an extended view of the upper branch of Fig. 3(a). By observing the plot in Fig. 3, we can notice that, in band $[0, \pi/2D]$, frequency ω_p practically does not depend on N and D, and is linearly related to frequency ω_0, i.e.,

$$\omega_p \approx \sqrt{\frac{7}{5}}\omega_0, \qquad 0 < \omega_0 < \pi/2D. \tag{20}$$

Relation (20) is also confirmed by solving (19). (Details are shown in the Appendix.) This result demonstrates that the optimum compensation filter in the least-square sense can be obtained from general equation (11) by replacing ω_0 with $\omega_0 = \sqrt{5/7}\omega_p$.

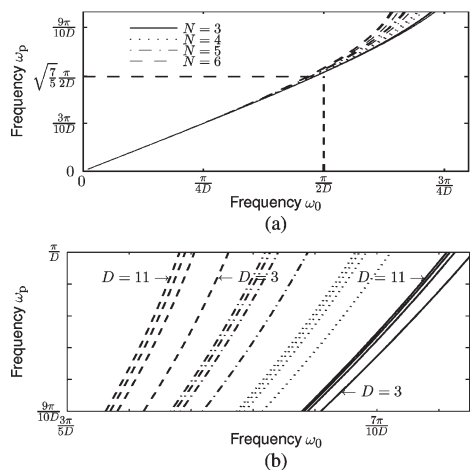

Fig. 3. Frequencies ω_p for the least-square compensation filters as a function of frequency ω_0 for $N = 3, 4, 5, 6$; $D = 3, 5, 7, 11$; and $\nu = 4$.

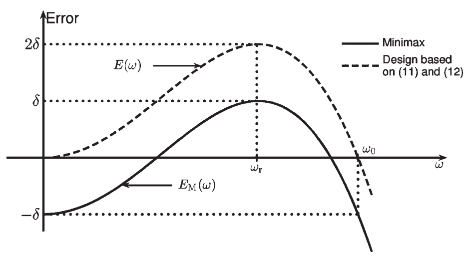

Fig. 4. Error function for the minimax design and the proposed design.

C. Minimax Case

We now turn our attention to the design of a compensation filter using the minimax criterion defined as

$$\delta = \min_{a_M, b_M} \max_{[0,\omega_0]} |E_M(\omega)| \tag{21}$$

where, for the minimax design, $E_M(\omega)$ is the error function, and a_M and b_M are the corresponding filter coefficients.

Error function $E_M(\omega)$ alternates between $\pm\delta$, where δ is the maximum magnitude of $E_M(\omega)$, at consecutive extremal frequencies for an optimal $P_R(\omega)$ [12]. The extremal frequencies, in this case, are $\omega_1 = 0, \omega_2 = \omega_r$, and $\omega_3 = \omega_0$. This means that $E_M(0) = -\delta$, $E_M(\omega_r) = \delta$, and $E_M(\omega_0) = -\delta$ [see Fig. 4 (solid line)].

From (7) and (8), in the range $[0, \omega_0]$, we have

$$P_{\rm RM}(D\omega) = b_M + 2a_M \cos(D\omega) \tag{22}$$

$$E_M(\omega) = H(\omega)P_{\rm RM}(D\omega) - 1. \tag{23}$$

Using conditions $E_M(0) = -\delta$ and $E_M(\omega_r) = \delta$, from (22) and (23)

$$2a_M + b_M = 1 - \delta \tag{24}$$

$$H(\omega_r)\left(2a_M \cos(D\omega_r) + b_M\right) = 1 + \delta. \tag{25}$$

Thus, coefficients a_M and b_M are given by

$$a_M = \frac{1}{2}\frac{1 - 1/H(\omega_r)}{1 - \cos(D\omega_r)} - \frac{\delta}{2}\frac{1 + 1/H(\omega_r)}{1 - \cos(D\omega_r)} \tag{26}$$

$$b_M = 1 - \delta - 2a_M. \tag{27}$$

In the following, we show how to achieve the minimax design using the proposed approach based on (11) and (12).

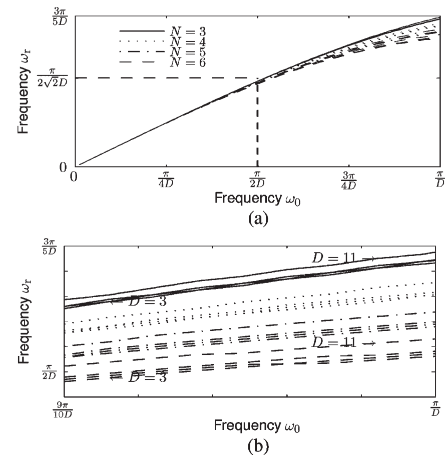

Fig. 5. Extremal frequency ω_r as a function of frequency ω_0 for $N = 3, 4, 5, 6$; $D = 3, 5, 7, 11$; and $\nu = 4$.

Recall that the error function $E(\omega)$ based on (11) and (12) is equal to zero at $\omega = 0$ and $\omega = \omega_0$, as shown in Fig. 4 (dashed line). Therefore, error functions $E(\omega)$ and $E_M(\omega)$ satisfy

$$E(\omega) = E_M(\omega) + \delta \tag{28}$$

$$E(\omega_r) = 2\delta. \tag{29}$$

In this way, frequency ω_0 can be related with frequency ω_r using

$$\frac{dE(\omega)}{d\omega} = 0. \tag{30}$$

Fig. 5(a) shows extremal frequency ω_r as a function of ω_0, with $N = 3, 4, 5, 6$; $D = 3, 5, 7, 11$; and $\nu = 4$. Fig. 5(b) shows an extended view of the upper branch of Fig. 5(a). It is worth noting that the relationship between ω_r and ω_0 is approximately linear in band $[0, \pi/2D]$ and that quantity ω_r is practically independent of N, D, and ν in this band, resulting in (see details in the Appendix)

$$\omega_r \approx \frac{\omega_0}{\sqrt{2}}, \qquad 0 < \omega_0 < \pi/2D. \tag{31}$$

Now, we combine the coefficients of the minimax compensation filter given in (26) and (27) with the general results (11) and (12).

From (8) and (29), the value of δ is expressed as

$$\delta = \frac{H(\omega_r)P_R(D\omega_r)1}{2}. \tag{32}$$

Substituting (32) into (26) and (27), and using (7), filter coefficients a_M and b_M become

$$a_M = -\frac{1}{2}\frac{1 + H(\omega_r)}{1 - \cos(D\omega_r)}\left(a\cos(D\omega_r) + \frac{b}{2}\right) - \frac{1}{4H(\omega_r)}\frac{1 - 3H(\omega_r)}{1 - \cos(D\omega_r)} \tag{33}$$

$$b_M = \frac{1 + H(\omega_r)\cos(D\omega_r)}{1 - \cos(D\omega_r)}\left(a\cos(D\omega_r) + \frac{b}{2}\right) + \frac{1}{2H(\omega_r)}\frac{1 - 3H(\omega_r)\cos(D\omega_r)}{1 - \cos(D\omega_r)} \tag{34}$$

where ω_r, a, and b are given by (31), (11) and (12), respectively.

Note that, in this design, frequency ω_0 is equal to ω_p.

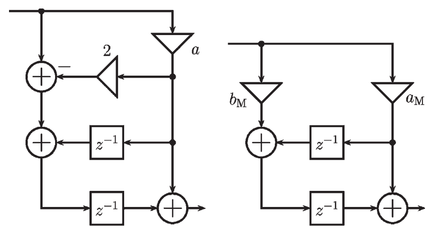

Fig. 6. Compensation filter structure $P(z)$. (a) Maximally flat and least square. (b) Minimax.

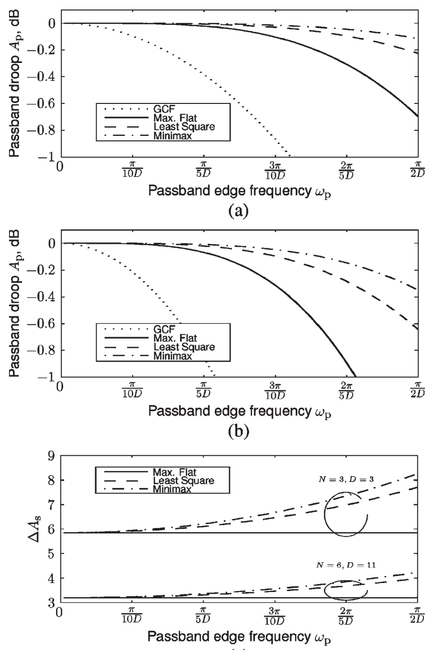

Fig. 7. (a) Passband droops for the case $(N, D) = (3, 3)$. (b) Passband droops for the case $(N, D) = (6, 11)$. (c) Stopband attenuation decrease as a function of frequency ω_p.

III. Discussion of Results

Fig. 6(a) shows the resulting filter structure for the maximally flat and least-square cases, whereas the minimax structure is shown in Fig. 6(b). Observe that the structure shown in Fig. 6(a) requires one multiplier and three adders. Similarly, two multipliers and two adders are needed in the structure shown in Fig. 6(b) for the minimax design.

In the following, we analyze passband droop A_p after compensation and stopband attenuation decrease $\Delta A_s = A_{s1} - A_{s0}$, where A_{s1} is the stopband attenuation after compensation and A_{s0} is the stopband attenuation without compensation.

Fig. 7(a) and (b) shows the passband droops as a function of frequency ω_p for $(N, D) = (3, 3)$, $(N, D) = (6, 11)$, and $\nu = 4$. Notice that the best passband droop compensation in the wideband is obtained by the minimax method. Additionally, observe that the passband droop is less than 1 dB.

In a similar way, Fig. 7(c) compares the decrease in the stopband attenuation for the same values of N and D [see Fig. 7(a) and (b)]. In general, the maximally flat design has the smallest attenuation decrease in ΔA_s.

From Fig. 7, we note that the least-square design has almost similar characteristics as the minimax design.

Next, we relate our work with recent compensation filters for the traditional CIC filter $(\alpha_n = 0, n = 1, \ldots, N)$ [3], [4].

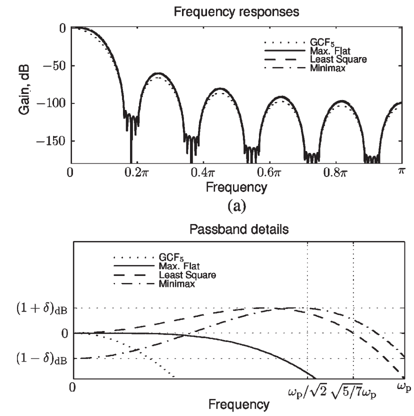

Fig. 8. Overall magnitude responses of the GCF and the compensation filters in the design example.

TABLE I
Parameters in the Design Example

	GCF_5	Max. Flat	Least Square	Minimax
A_p	-3.71	-1.08	-0.35	-0.18
A_s	-66.15	-60.99	-59.77	-59.38
a		-0.209665	-0.279849	-0.309826
b		1.419331	1.559699	1.597619
ω_0		0	$\sqrt{5/7}\omega_p$	ω_p
ω_r				$\omega_p/\sqrt{2}$
δ				0.022033

Kim *et al.* [3] considered a second-order compensation filter based on least-square minimization. It is worth highlighting that we obtain the same result as [3] if we set $\omega_0 = 0.4\,\pi/D$ and $\omega_0 = 0.6\,\pi/D$ (see [3, Table 2]).

The compensation filter in [4] is a multiplierless filter with only three adders. This filter exhibits better compensation in narrowband and has less computational load than [3], whereas the method in [3] has better results in wideband compensation (for more details, see [4]).

We illustrate the proposed design with one example.

Example: Consider the design of a GCF compensation filter with the following parameters: $D = 11$, $N = 5$, $\omega_p = 0.45\,\pi/D$, and $\alpha_n = q_n\pi/4D$ for $n = 1, 2, 3, 4, 5$, where $q_n = [-0.55, -0.93, 0, 0.93, 0.55]$ [6]. The corresponding passband droop and stopband attenuation of the GCF are $A_p = -3.71$ dB and $A_s = -66.15$ dB, respectively, as shown in Fig. 8.

First, we consider the *maximally flat* case. Using (14), it follows that $a = -0.209665$ and $b = 1.419331$. The passband droop after compensation is -1.08 dB. Similarly, the stopband attenuation becomes -60.99 dB.

Next, we apply the *least-square* design. Frequency ω_0 is computed using (20), i.e., $\sqrt{5/7}\omega_p$. The corresponding values of a and b are -0.279849 and 1.559699, respectively [see (11) and (12)]. The resulting passband droop and stopband attenuation are $A_p = -0.35$ dB and $A_s = -59.77$ dB.

Finally, to obtain the *minimax* design, we need to compute the δ value. From (31) and (32), with $\omega_0 = \omega_p$, we have $\omega_r = \omega_p/\sqrt{2}$ and $\delta = 0.022033$. Therefore, filter coefficients a_M and b_M are -0.309826 and 1.597619, respectively [see (33) and (34)]. The corresponding passband droop and stopband attenuation become $A_p = -0.18$ dB and $A_s = -59.38$ dB.

The results are summarized in Table I.

The passband details for the GCF and the compensation filters are shown in Fig. 8(b). As expected, the maximally flat design has better magnitude characteristics around the origin than the least-square and minimax methods. Additionally, the minimax design has the best magnitude characteristic in the band $[0, \omega_p]$. However, the best passband compensation is achieved by the least-square method in the band $[0, \sqrt{5/7}\omega_p]$.

From Fig. 8(a), it is worth highlighting that the reduction in the attenuation impacts regions $[2\pi k/D + \pi/\nu D, 2\pi(k+1)/D + \pi/\nu D]$ for $k = 1, \ldots, D-2$ (don't care regions [6]) and not the folding bands.

IV. Concluding Remarks

A novel approach for a GCF compensation filter design has been presented. The technique is based on a $2D$-order compensation filter, which becomes a second-order filter after moving to a low rate [10]. The proposed method includes the maximally flat, least-square, and minimax designs. Considering that the maximally flat and least-square designs need only one multiplier, both designs could be a good choice for narrow-passband compensation. However, for wide-passband compensation, the best choice is the minimax design, which requires two multipliers.

Appendix
Proofs of (14), (20), and (31)

This appendix gives a more detailed explanation of (14), (20) and (31).

Proof of (14)

After small computations from (9), (10) and (13), we get

$$a = \left. \frac{H''(\omega)}{2D^2} \right|_{\omega=0} \quad (35)$$

where $H''(\omega)$ is the second derivative of $H(\omega)$ with respect to ω.

Now, we obtain a closed-form equation for $H''(\omega)$. First, consider the following derivative:

$$\frac{d}{d\omega}\left\{\frac{\sin((\omega+\alpha_n)D/2)}{\sin((\omega+\alpha_n)/2)}\right\} = \frac{\sin((\omega+\alpha_n)D/2)}{\sin((\omega+\alpha_n)/2)} \times \left(\frac{D}{2}\cot((\omega+\alpha_n)D/2) - \frac{1}{2}\cot((\omega+\alpha_n)/2)\right). \quad (36)$$

Using (3), (36), and the product rule for derivatives, the first derivative of $H(\omega)$, i.e., $H'(\omega)$, is

$$H'(\omega) = \frac{H(\omega)}{2} \times \sum_{n=1}^{N}\left(D\cot\left(D\frac{\omega+\alpha_n}{2}\right) - \cot\left(\frac{\omega+\alpha_n}{2}\right)\right). \quad (37)$$

Notice that the evaluation of (37) at $\omega = 0$ is equal to zero since $\alpha_n = -\alpha_{N-n}$ and $\cot(\cdot)$ is an odd function.

From (37), function $H''(\omega)$ is expressed as

$$H''(\omega) = \frac{H(\omega)}{4}\left[\sum_{n=1}^{N}\left(\csc^2\left(\frac{\omega+\alpha_n}{2}\right) - D^2\csc^2\left(D\frac{\omega+\alpha_n}{2}\right)\right) + \left(\sum_{n=1}^{N}\left(D\cot\left(D\frac{\omega+\alpha_n}{2}\right) - \cot\left(\frac{\omega+\alpha_n}{2}\right)\right)\right)^2\right]. \quad (38)$$

Substituting $\omega = 0$ into (38) and (35) proves (14).

Proof of (20)

At first, consider the two-term Taylor polynomials of $H(\omega)$ and $\cos(D\omega)$ around $\omega = 0$, i.e.,

$$H(\omega) \approx 1 - H(0)\frac{\omega^2}{2} \quad (39)$$

$$\cos(D\omega) \approx 1 - \frac{D^2\omega^2}{2}. \quad (40)$$

Substituting (39) and (40) into (19), we arrive at

$$\frac{1}{H''(0)\omega_0^2 - 2} \approx \frac{9\left(5H''(0)\omega_p^2 - 14\right)}{35H''^2(0)\omega_p^4 - 180H''(0)\omega_p^2 + 252}. \quad (41)$$

Solving (41) for ω_0 and neglecting higher order terms of ω_p, (20) holds.

Proof of (31)

The first derivative of $E(\omega)$ with respect ω is

$$\frac{dE(\omega)}{d\omega} = \frac{d\left\{H(\omega)\left(b + 2a\cos(D\omega)\right)\right\}}{d\omega}. \quad (42)$$

Substituting (11) and (12), (39) and (40) into (42), we have

$$\frac{dE(\omega)}{d\omega} \approx \frac{\omega H''^2(0)}{H''(0)\omega_0^2 - 2}\left(2\omega^2 - \omega_0^2\right). \quad (43)$$

Combining (30) and (43) proves (31).

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