On MATLAB Demonstrations of Narrowband Gaussian Noise

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ABSTRACT: A demo program for teaching characteristics of narrowband Gaussian noise (NBGN) by using MATLAB environment is presented in this article. Programs are developed in MATLAB tool makeshow, and users are led step by step through the statistical characteristics of the NBGN including probability density and distribution functions, and the corresponding probability. The evaluation of the software by users is also included. © 2009 Wiley Periodicals, Inc. Comput Appl Eng Educ 19: 598–603, 2011; View this article online at wileyonlinelibrary.com/journal/cae; DOI 10.1002/cae.20340

Keywords: narrowband Gaussian process; MATLAB GUI; envelope; phase; probability density and distribution functions

INTRODUCTION

An attractive and instructive approach for teaching the concept of narrowband Gaussian noise (NBGN) by using MATLAB environment is presented in this article. Narrowband Gaussian process is important in communication systems [1–8], control, electronic and computer engineering [9], random vibrations [10], biomedical engineering [11], radar-sonar processing [12], underwater oceanic engineering [13], astronautics and aeronautics [14], among others.

Narrowband Gaussian noise is defined as the output of a bandpass filter whose bandwidth B is very small compared to the center frequency f_c, the input being Gaussian white noise [1]. For small but finite B/f_c, narrowband process looks like a sinusoidal wave of frequency f_c but with slow and random amplitude and phase variations. Spectral power density $S(f)$ of the NBGN can be expressed as

$$S(f) = \frac{N_0}{2}|H(f)|^2 \tag{1}$$

where N_0 is the spectral power density of the Gaussian white process, and $H(f)$ is the spectral characteristic of the bandpass system.

For theoretical Gaussian narrowband process whose power spectral density is shown in Fig. 1 the total power of the process is $P = N_0B$, with no dc component, yielding in

$$\bar{n} = 0, \qquad \bar{n}^2 = \sigma_n^2 = P = N_0B \tag{2}$$

Quadrature-carrier representation of GNB noise is given by the following equation

$$n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \tag{3}$$

where f_c is the carrier frequency and n_c and n_s are the in-phase and quadri-phase components, respectively. Both quadrature components are Gaussian processes having the same mean value and variance,

$$\bar{n}_c = \bar{n}_s = 0, \qquad \bar{n}_c^2 = \bar{n}_s^2 = \sigma_n^2 = \sigma^2 \tag{4}$$

Phasor interpretation of Equation (3) is given in Figure 2 [1].

Equivalent useful representation of Equation (3), known as an *envelope-and-phase representation*, is given in Equation (5)

$$n(t) = u_n(t)\cos[2\pi f_c t + \varphi(t)] \tag{5}$$

where $u_n(t)$ is the random envelope (defined to be nonnegative) and $\varphi(t)$ is the random phase. From phasor interpretation given in Figure 3, it follows

$$\begin{aligned} u_n^2 &= n_c^2 + n_s^2 \\ \varphi_n &= \arctan\frac{n_s}{n_c} \end{aligned} \tag{6}$$

It can be shown that the envelope and phase are independent. The envelope has the Rayleigh density function [15–17],

$$f_{u_n}(u_n) = \frac{u_n}{\sigma_n^2}\,e^{-u_n^2/2\sigma_n^2}, \quad u_n \geq 0 \tag{7}$$

while the phase has the uniform density in the range $[-\pi, \pi]$

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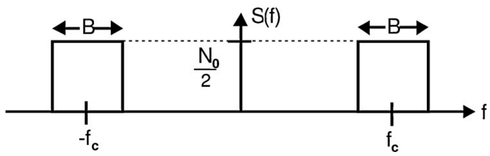

Figure 1 Spectral density of a theoretical Gaussian Narrowband noise.

[15–17],

$$f_{\varphi_n}(\varphi_n) = \frac{1}{2\pi}, \quad |\varphi_n| \leq \pi \tag{8}$$

The sinusoidal signal of the amplitude A is added to the NBG noise (3)

$$nn(t) = n(t) + A\cos 2\pi f_c t = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t + A\cos 2\pi f_c t \tag{9}$$

Envelope-and-phase representation, of Equation (9) is given by [17]

$$nn(t) = u_{nn}(t)\cos[2\pi f_c t + \varphi_{nn}(t)] \tag{10}$$

where the envelope u_{nn}, and phase φ_{nn} are no more independent, and the envelope is the *Rice* random variable.

MOTIVATION

Learning sciences research indicates that students learn much better "by doing" rather than "by listening," and engineering-related educators are recognizing the need for more active learning pedagogy. Thus, passive learning, that is, the traditional lecture is beginning to share time in classrooms with more active learning that emphasizes student problem solving, discussion, presentation, and other learning-by doing activities [18–20].

We made the demo program for Narrowband Gaussian noise described in this article. Narrowband Gaussian noise has traditionally been stated in terms of an abstract mathematical description. However students are very often confused with the terms amplitude and envelope of the noise and the probability that envelope exceed given value. Therefore the principal motivation to make this demo program was to give students the visual and more intuitive representation of the Narrowband Gaussian noise.

DESCRIPTION OF THE PROGRAM

We choose MATLAB because MATLAB along with the accompanying toolboxes is the tool of choice for most educational and research purposes. The MATLAB *slideshow* file for use with *playshow*, and *makeshow* files provides friendly and visual approach to the programs.

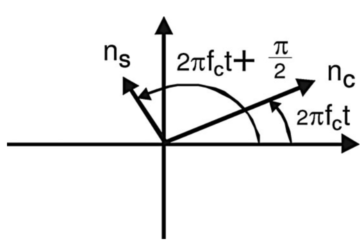

Figure 2 Phasor interpretation of Equation (3).

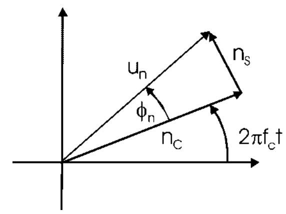

Figure 3 Phasor interpretation of Equation (5).

The content of the demo program is given in the Menu shown in Figure 4.

The programs are executed by pressing the corresponding button in the Menu. There are two windows in each slide as demonstrated in Figure 5.

We used the upper windows for the title of the presentation or for graphics. The bottom window was used for general explanations, or for the explanations of the upper graphics. The order of the slides and the command buttons are on the right side of each slide.

The student can change the slides automatically using the option *AutoPlay*. In this case the student may use the button *Stop* to stop the presentation and later hit *Continue* to continue it. The student can also choose the option to change slides manually with the mouse, by clicking *Next* to go forward or *Prev*, to go backward. The buttons *Reset* and *Close* are used to reset the demos and exit the program, respectively.

DESCRIPTION OF DEMO PROGRAMS

The demo program `QUADRATURE PRESENTATION` demonstrates that the amplitudes of the NB Gaussian noise and its quadrature components are Gaussian random variables. First, quadrature components are generated and the probability density functions and distribution functions are estimated. Finally, the NBG noise is generated from its quadrature components. The probability density and distribution functions are estimated to demonstrate that the amplitude of NBG noise is also Gaussian random variable with the mean value zero and the variance equal to that of its quadrature components as illustrated in Figure 6. The presentation `ENVELOPE` is devoted to the demonstration of the statistical characteristic of the envelope. The probability density function and the distribution function of the envelope are

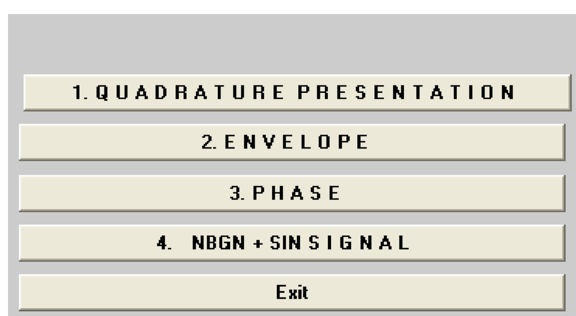

Figure 4 Menu.

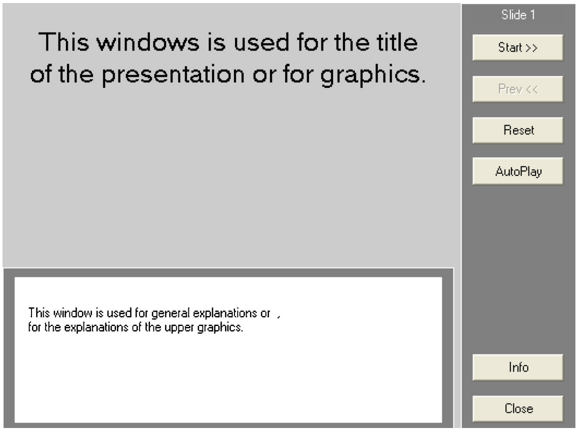

Figure 5 The form of the slides.

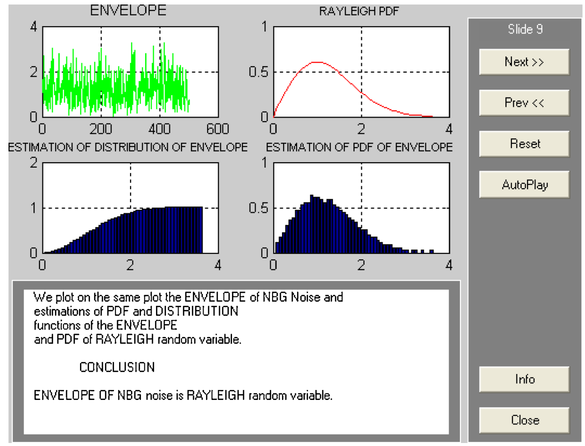

Figure 7 Demo ENVELOPE: PDF and Distribution.

estimated and compared with that of the Rayleigh random variable. The slide in Figure 7 illustrates this demonstration.

It is also illustrated the meaning of the probability that envelope is less than given value *A*, as shown in Figure 8. The estimation of probability density function of random phase of the NBG noise which is demonstrated in demo `PHASE` shows that the random phase is the uniform random variable. Figure 9 illustrates the demo program `PHASE`.

The demo `NBG NOISE + SIN SIGNAL` describes random envelope of the NBG noise with added sinusoidal signal. It is demonstrated that the probability density of the envelope is the Rice random variable. Figure 10 illustrates the demo.

Three cases are demonstrated depending on the relation of amplitude of sinusoidal signal and standard deviation (SD) of the NBG noise.

As en example, Figure 11 illustrates the case where the noise is dominant compared to the additional sinusoidal signal.

EVALUATION

This program has been used as a complementary tool in teaching graduate students enrolled in: Introduction to Communications in Electronics (group E) and Random Vibration in Mechanical Engineering (group M). We consider that is very important to gather information from students about the usefulness of the software in the teaching-learning process. To this end we developed a suitable tool to evaluate quality of the software in the teaching-learning process.

Reviewing literature [18–25] and our previous experience [26–28] helped us to define the general topics to be evaluated. Finally we adopted a classification consisting of two categories which cover the more important concepts:

(1) Teaching contents and methodology.
(2) Software and design features.

All questions in the evaluation form are rated with marks varying from 1 to 4; with the latter being the highest mark.

Teaching Contents and Methodology

This set of questions attempted to test the usefulness of the software.

The following questions were asked:

(1) Justification for the computer use in teaching NBGN (1 = unjustified; 4 = absolutely justified).

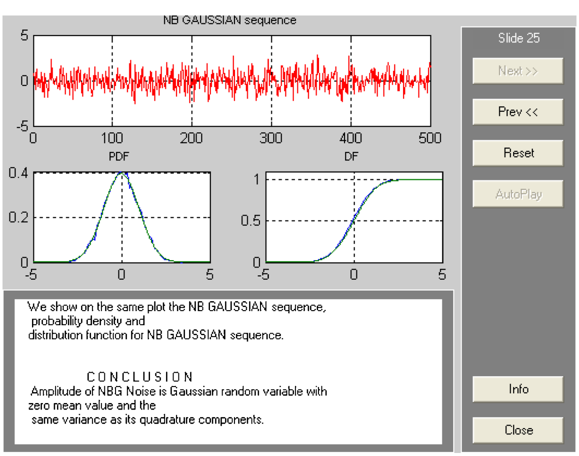

Figure 6 Demo QUADRATURE PRESENTATION.

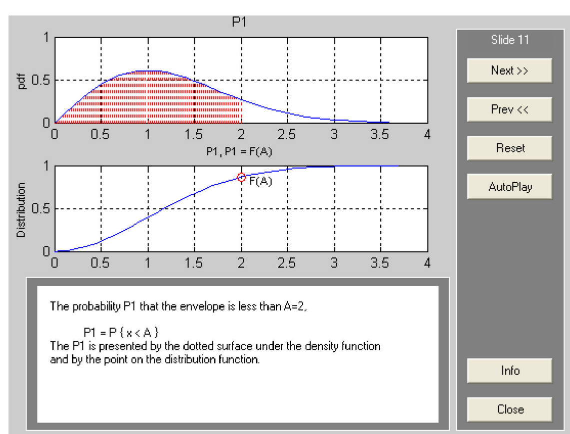

Figure 8 Demo ENVELOPE: Probability.

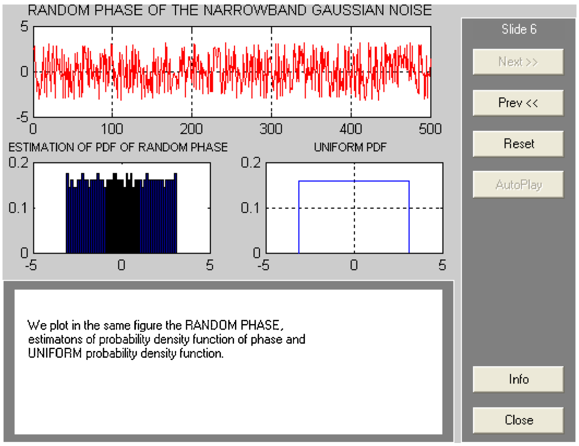

Figure 9 Demo PHASE.

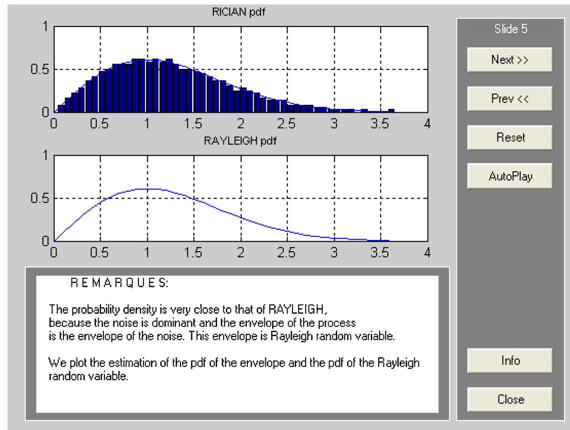

Figure 11 NBG NOISE + SIN SIGNAL: Special case.

(2) Contribution to study of NBGN by demo program use (1 = irrelevant; 4 = very effective).
(3) Clarity of explanations and features of demo (1 = confusing; 4 = absolutely clear).
(4) Did this demo help you to understand better the quadrature representation of NBGN? (1 = NO; 4 = Absolutely YES).
(5) Did this demo help you understand better the envelope and phase of NBGN? (1 = NO; 4 = Absolutely YES).
(6) Did this demo help you understand better the addition of sinusoidal signal to NBGN? (1 = NO; 4 = Absolutely YES).
(7) Do you think that the demo software can completely replace traditional classes on NBGN? (1 = NO; 2 = I am not sure; 3 = Only as a complementary tool; 4 = YES).

Results are given in Table 1 in terms of percentage of students in group E or group M, giving the mark indicated in the first column of the corresponding row. The average marks for both groups are given in plots in Figure 12.

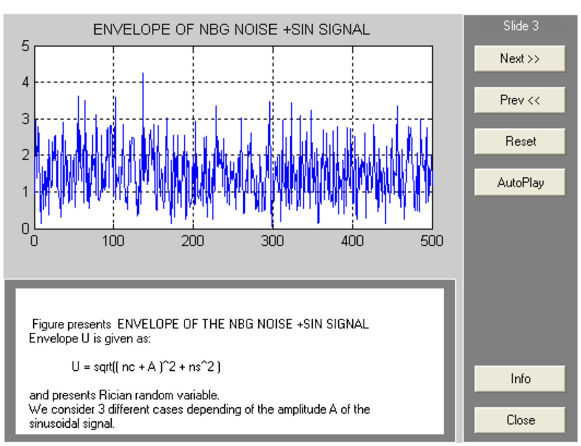

Figure 10 Demo NBG NOISE + SIN SIGNAL: Envelope.

Software and Design Features

Answers in this group of questions point directly to the program design aspects:

(1) Special knowledge or programming skills required (1 = excessive; 4 = none).
(2) Ease of operation (1 = complex; 4 = very easy).
(3) Flexibility and Repeatability (you can come back to previous slide/slides and repeat it/them many times) (1 = unnecessary; 4 = very useful).
(4) General quality of presentation (figures, resolution, visibility, etc) (1 = poor; 4 = excellent).

Table 2 provides the corresponding answers and Figure 13 shows the average marks for both groups.

DISCUSSION OF RESULTS

Results of the evaluation confirm that students from electrical engineering, as well as students from mechanical engineering consider that the demo program is very useful in teaching the characteristics of NBGN. The demo helped them better understand the quadrature representation of NBGN, envelope and phase of NBGN, and the effect of addition of sinusoidal signal to NBGN. However majority of users prefer demo programs as a complementary teaching tool, rather than, as a replacement of the traditional teaching. As far as Software and design features are concerned both group of students rated with the highest mark the following software features: No special knowledge or programming skills required, Ease of operation and Flexibility and Repeatability.

CONCLUSIONS

Computer-aided learning has become an important educational research activity in various engineering disciplines and there has been a growing interest in the development of educational software in all areas of study. Our experience at the National

Table 1 Questioner Result: Teaching Contents and Methodology

	Questions						
Marks	Q1	Q2	Q3	Q4	Q5	Q6	Q7
1	E: 0%, M: 0%	E: 0%, M: 0%	E: 0%, M: 0%	E: 0%, M: 0%	E: 0%, M: 0%	E: 0%, M: 0%	E: 20%, M: 23%
2	E: 2%, M: 0%	E: 0%, M: 0%	E: 8%, M: 10%	E: 0%, M: 5%	E: 0%, M: 5%	E: 1%, M: 6%	E: 5%, M: 9%
3	E: 8%, M: 0%	E: 10%, M: 5%	E: 20%, M: 20%	E: 5%, M: 5%	E: 4%, M: 5%	E: 5%, M: 7%	E: 75%, M: 68%
4	E: 90%, M: 100%	E: 90%, M: 95%	E: 72%, M: 70%	E: 95%, M: 90%	E: 96%, M: 91%	E: 94%, M: 87%	E: 0%, M: 0%

"E" means group E; "M" means group M.

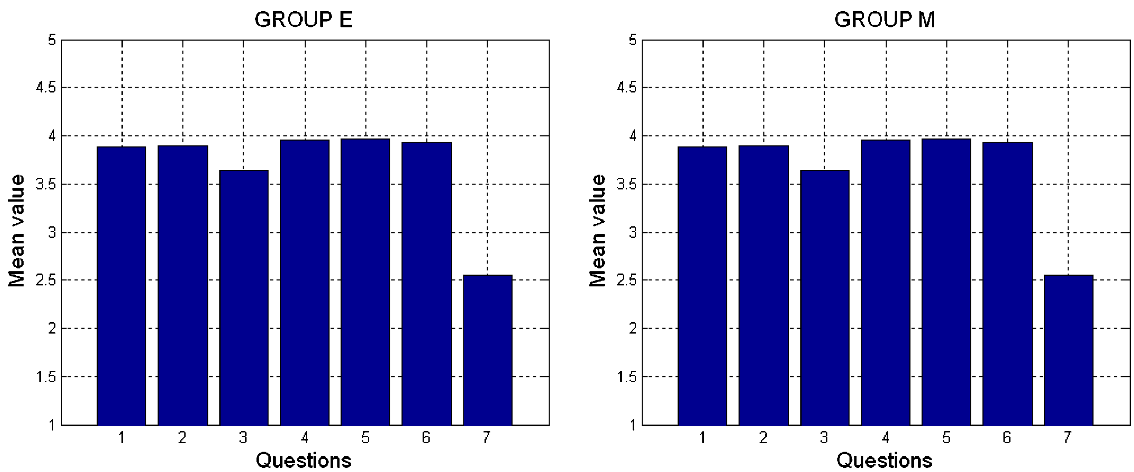

Figure 12 Rating scheme: Contents and Methodology Questions.

Table 2 Questioner Result: Software and Design Features

	Questions			
Marks	Q1	Q2	Q3	Q4
1	E: 0%, M:0%	E: 0%, M: 0%	E: 0%, M: 0%	E: 0%, M: 0%
2	E: 0%, M: 0%	E: 0%, M: 0%	E: 0%, M: 0%	E: 7%, M: 5%
3	E: 0%, M: 0%	E: 0%, M: 0%	E: 0%, M: 0%	E: 11%, M: 10%
4	E: 100%, M: 100%	E: 100%, M: 100%	E: 100%, M: 100%	E: 82%, M: 85%

"E" means group E; "M" means group M.

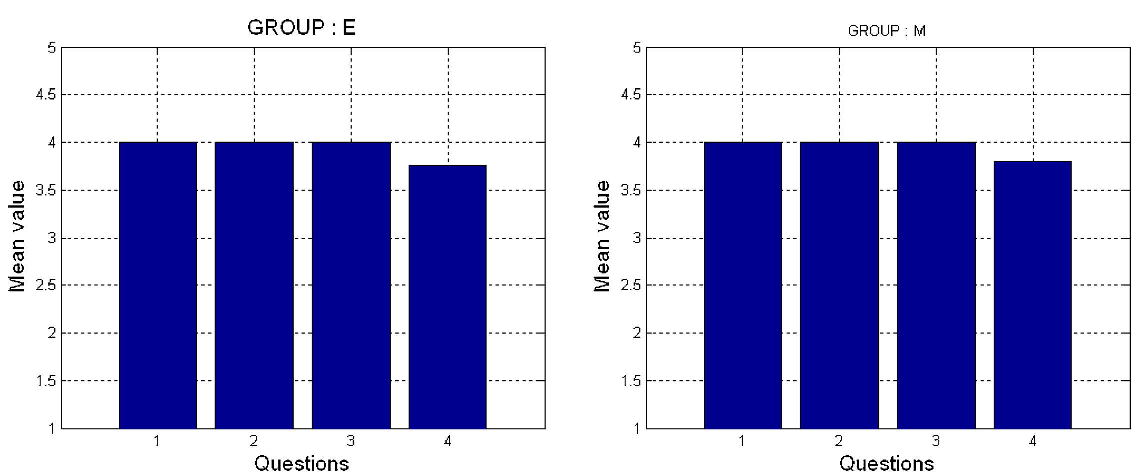

Figure 13 Rating scheme: Software and design features.

Institute INAOE [26−28], shows that the development and usage of software tools represent an effective teaching approach and increase the efficiency of student's learning.

In this article we presented the demo programs for teaching the statistical characteristics of narrowband Gaussian processes which is of interest in differential field of engineering including, communications, control, mechanical, and oceanic engineering, among others. The feedback obtained from the students has indicated generally strong support from the students to use this demo program as a complementary tool for better understanding the characteristics of narrowband Gaussian noise.

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BIOGRAPHIES

Gordana Jovanovic Dolecek received a BS degree from the Department of Electrical Engineering, University of Sarajevo, an MSc degree from University of Belgrade, and a PhD degree from the Faculty of Electrical Engineering, University of Sarajevo. She was professor at the Faculty of Electrical Engineering, University of Sarajevo until 1993, and 1993−1995 she was with the Institute Mihailo Pupin, Belgrade. In 1995 she joined Institute INAOE, Department for Electronics, Puebla, Mexico, where she works as a professor and researcher. During 2001−2002 she was at Department of Electrical & Computer Engineering, University of California, Santa Barbara, as visiting researcher on a sabbatical leave. Actually she is with SDSU San Diego on sabbatical leave. She is the author of three books, editor of one book, and author of more than 200 articles. Her research interests include digital signal processing and digital communications. She is a Senior member of IEEE, the member of Mexican Academy of Sciences, and the member of National Researcher System (SNI) Mexico.

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