



CHAOTIC COMMUNICATION SYSTEM USING CHUA'S OSCILLATORS REALIZED WITH CCII+s

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This work shows the experimental implementation of a chaotic communication system based on two Chua's oscillators which are synchronized by Hamiltonian forms and observer approach. The chaotic communication scheme is realized by using the commercially available positive-type second generation current conveyor (CCII+), which is included into the AD844 device. As a result, experimental measurements are provided to demonstrate the suitability of the CCII+ to implement chaotic communication systems.

Keywords: Chua's oscillator; circuit realization; Hamiltonian forms approach; current conveyor; secure communication system; chaos.

1. Introduction

An undoubtedly relevant application of synchronization of chaotic oscillators [Cruz-Hernández, 2001; Khan & Singh, 2008; Kilinc *et al.*, 2008; Nijmeijer & Mareels, 1997; Tsay *et al.*, 2005] is in the field of secure/private communications; see e.g. [Alvarez & Li, 2006; Cruz-Hernández *et al.*, 2005; Cruz-Hernández & Romero-Haros, 2008; Materassi & Basso, 2008; Trejo-Guerra *et al.*, 2008]. Although many kinds of chaotic oscillators have been introduced, in electronics, Chua's circuit has been widely used because it allows the development of new designs and applications [Barboza & Chua, 2008; Bilotta *et al.*, 2007; Cafagna & Grassi,

2004; Caponetto *et al.*, 2005; Chua, 1994; Demirkol *et al.*, 2008; Kilic, 2003; Tlelo-Cuautle & Muñoz-Pacheco, 2007; Yu *et al.*, 2007]. Basically, it generates the double-scroll attractor [Cruz & Chua, 1993; Shil'nikov, 1993; Tlelo-Cuautle *et al.*, 2006], and up to now many kinds of implementations have been reported, some at the integrated circuit level, see e.g. [Cruz & Chua, 1993; Tlelo-Cuautle *et al.*, 2006]; by using field programmable analog array [Caponetto *et al.*, 2005], and by using current-feedback operational amplifiers (CFOAs) [Elwakil & Kennedy, 2000; Kilic, 2004; Sánchez-López *et al.*, 2008; Senani & Gupta, 1998].

In Chua's chaotic system, circuit designers proposed new topologies to implement the Chua's

diode and inductor elements [Kilic, 2003]. Besides, since the CFOA¹ provides higher bandwidth compared to conventional operational amplifiers, it is a good candidate to enhance Chua's system [Elwakil & Kennedy, 2000; Kilic, 2004; Sánchez-López et al., 2008; Senani & Gupta, 1998]. However, since the CFOA consists of a positive-type second generation current conveyor (CCII+) in cascade connection with a voltage follower [Tlelo-Cuautle et al., 2006], then this work is oriented to show the usefulness of the CCII+ to realize a chaotic communication system based on two synchronized Chua's oscillators. Henceforth, Chua's diode is realized by using two CCII+s as shown in [Senani & Gupta, 1998], while the simulated inductance is realized by using four CCII+s.

The design of the CCII+ at the transistor level of abstraction can be revised in [Fakhfakh et al., 2007; Sedra & Smith, 1970]. In this manner, the main goal is to offer a guideline for the synchronization of two Chua's CCII+ based oscillators under Hamiltonian forms and observer approach. This process is verified experimentally by implementing a master-slave communication system.

The rest of the paper is organized as follows: in Sec. 2, Chua's oscillator is described by their state variable equations. These equations are arranged to apply Hamiltonian approach [Sira-Ramírez & Cruz-Hernández, 2001] to design the observer in Sec. 3. The chaotic communication system by using only CCII+s is shown in Sec. 4, while the synchronization and signal transmission results are given in Sec. 5. Finally, Sec. 6 summarizes some important remarks.

2. Chua's Oscillator

As a difference from the so-called Chua's circuit, *Chua's oscillator* only takes an extra linear resistor

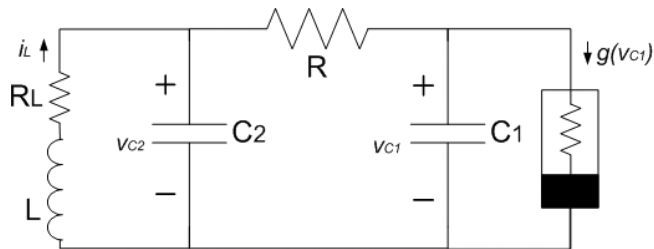


Fig. 1. Chua's oscillator.

in series with the inductor [Chua et al., 1993], as shown in Fig. 1. In this work, the resistance R_L will be considered as a parasitic effect belonging to the simulated inductance which is described in Sec. 4. By taking the nodal currents, the following system of equations arise

$$\begin{aligned} C_1 \frac{dv_{C1}}{dt} &= G(v_{C2} - v_{C1}) - g(v_{C1}) \\ C_2 \frac{dv_{C2}}{dt} &= G(v_{C1} - v_{C2}) + i_L \\ L \frac{di_L}{dt} &= -v_{C2} - i_L R_L \end{aligned} \quad (1a)$$

where $G = 1/R$ and the nonlinear characteristic of Chua's diode is given by

$$\begin{aligned} g(v_{C1}) &= G_b v_{C1} + \frac{1}{2}(G_a - G_b) \\ &\quad \times (|v_{C1} + E| - |v_{C1} - E|) \end{aligned} \quad (1b)$$

The normalized set of equations is given by the state space form

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1 - f(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 - \gamma x_3 \end{aligned} \quad (2a)$$

where the nonlinear function is described as

$$f(x_1) = b x_1 + \frac{1}{2}(a - b)(|x_1 + 1| - |x_1 - 1|) \quad (2b)$$

With solutions given by x_1 , x_2 and x_3 ; the selection of $x_1 = v_{C1}/E$, $x_2 = v_{C2}/E$, and $x_3 = i_L R/E$ implies that the parameters have also to be defined as $\alpha = C_2/C_1$, $\beta = R^2 C_2/L$ and $\gamma = R R_L C_2/L$. Thus, the main elements of the system are related to the bifurcation parameters α , β and γ . These parameters allow the chaotic regime due to the influence on the eigenvalues of the system. A set of known parameters is

$$\alpha = 9, \quad \beta = \frac{100}{7}, \quad a = -\frac{8}{7}, \quad b = -\frac{5}{7} \quad (3)$$

Parameter γ can be chosen $\gamma < 3$. The function $f(x_1)$ is described by the parameters $a = R G_a$ and $b = R G_b$ which allocate its slopes and the normalized break point $E = 1$, related to the signal amplitude and it is arbitrarily selected to enlarge/contract the attractor.

¹CFOA Datasheet available in: http://www.analog.com/static/imported-files/data_sheets/AD844.pdf

3. Synchronization by Hamiltonian Forms Approach

According to [Cruz-Hernández *et al.*, 2005], Chua's system can be synchronized by Hamiltonian forms approach. First, the system is expressed in a Hamiltonian form with destabilizing vector field F , as described by (4), with integers $n \geq m$.

$$\begin{aligned} \dot{x} &= J(y) \frac{\partial H}{\partial x} + (I + S) \frac{\partial H}{\partial x} + F(y), \quad x \in \mathfrak{R}^n, \\ y &= C \frac{\partial H}{\partial x}, \quad y \in \mathfrak{R}^m \end{aligned} \tag{4}$$

Here, the energy conservative part is related to the skew symmetric matrix J , while the non-conservative part is, in general, represented by the symmetric matrix S ; I is a constant skew symmetric matrix; $H(x)$ denotes a smooth energy function associated to the system and y , the lineal output mapping. Thus, the normalized Chua's oscillator (2) in Hamiltonian form is given by (5), with the gradient vector given by (6).

$$\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \beta \\ 0 & -\beta & 0 \end{pmatrix} \frac{\partial H}{\partial x} \\ &+ \begin{pmatrix} -\alpha^2 & \alpha & 0 \\ \alpha & -1 & 0 \\ 0 & 0 & -\beta\gamma \end{pmatrix} \frac{\partial H}{\partial x} + \begin{pmatrix} -\alpha f(x_1) \\ 0 \\ 0 \end{pmatrix} \end{aligned} \tag{5}$$

$$\frac{\partial H}{\partial x} = \begin{pmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\beta} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{6}$$

An observer for (4) is described by (7), where K represents the observer's gain.

$$\begin{aligned} \dot{\hat{x}} &= J(y) \frac{\partial H}{\partial \hat{x}} + (I + S) \frac{\partial H}{\partial \hat{x}} \\ &+ F(y) + K(y - \eta), \quad \hat{x} \in \mathfrak{R}^n, \tag{7} \\ \eta &= C \frac{\partial H}{\partial \hat{x}}, \quad \eta \in \mathfrak{R}^m \end{aligned}$$

According to [Cruz-Hernández *et al.*, 2005; Sira-Ramírez & Cruz-Hernández, 2001], in a general case, once the system is expressed in this way, matrices C and $W = I + S$ are found to be observable. But the condition for the states x of the system

can be globally, exponentially, asymptotically estimated by the states \hat{x} with an adequate selection of the observer gain K to fulfill the matrix $2(S - (1/2)(KC + C^T K^T))$, which is negative definite and here denoted as M .

For the particular system described by (5), such observer is given by (8), where $e_y = y - \eta$ represents the output estimation error [Sira-Ramírez & Cruz-Hernández, 2001].

$$\begin{aligned} \begin{pmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \end{pmatrix} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \beta \\ 0 & -\beta & 0 \end{pmatrix} \frac{\partial H}{\partial \hat{x}} \\ &+ \begin{pmatrix} -\alpha^2 & \alpha & 0 \\ \alpha & -1 & 0 \\ 0 & 0 & -\beta\gamma \end{pmatrix} \frac{\partial H}{\partial \hat{x}} \\ &+ \begin{pmatrix} -\alpha f(y) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} e_y \end{aligned} \tag{8}$$

Matrix M is calculated to accomplish

$$\begin{aligned} 2 \left(S - \frac{1}{2}(KC + C^T K^T) \right) &= \begin{pmatrix} -2\alpha(\alpha + k_1) & \alpha(2 - k_2) & -\alpha k_3 \\ \alpha(2 - k_2) & -2 & 0 \\ -\alpha k_3 & 0 & -2\beta\gamma \end{pmatrix} < 0 \end{aligned} \tag{9}$$

By means of the Sylvester's Theorem [Ogata, 1995], and by solving the required conditions for the selection of a non-negative observer's gain K , one achieves:

$$\begin{aligned} k_1 &> 0 \\ 0 &\leq k_2 \leq 4 \\ 0 &\leq k_3 < 2 \left| \sqrt{\frac{\beta\gamma k_1}{\alpha}} \right| \end{aligned} \tag{10}$$

If we assume for simplicity $k_2 = k_3 = 0$, it is clear that for all $k_1 > 0$, M is then negative definite matrix. However, in the experiment, the coupling stage was realized by setting $k_3 = 0$, while k_1 and k_2 are positive constants as shown in the next section.

4. Circuit Construction

In this section, the mixed mode characteristic of the CCII+ is exploited to implement the Chua's diode and the simulated inductor in straight form.

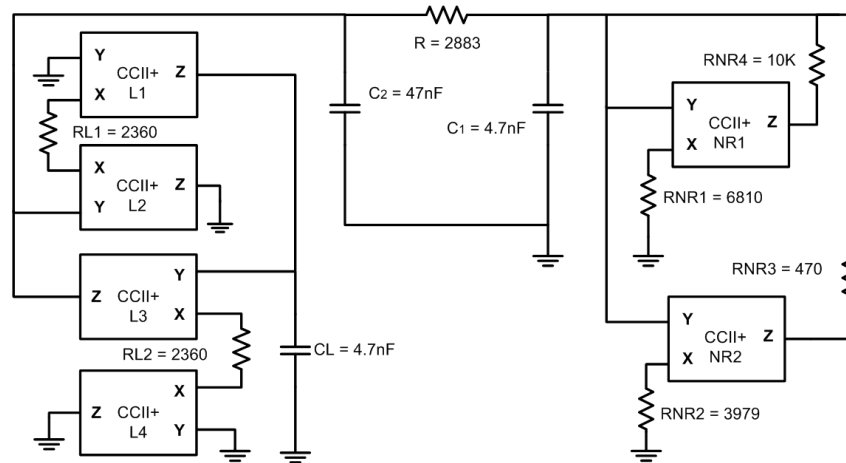


Fig. 2. Current conveyor based realization of Chua's oscillator.

An important issue is that this realization allows integrated circuit design. The construction of the chaotic circuit is shown in Fig. 2, where the simulated inductance is implemented by four CCII+s (L1 to L4), two resistors ($RL1$ and $RL2$) and one capacitor (CL).

The CCII+ is included into the commercially available CFOA AD844. Its main parasitic effects are associated to the parasitic resistance in terminal X (R_X), and the finite resistances presented at terminals Y and Z. From Fig. 2, by applying the symbolic method introduced by Tlelo-Cuautle *et al.* [2009], the equivalent analytic inductance obtained by this arrangement is

$$Z(s) = \frac{sCL + G_y}{sG_yCL + G_y^2 + G_1G_2} \quad (11)$$

The term G_y represents the conductance at terminal Y, while $G_1 = 1/(RL1 + 2R_X)$ and $G_2 = 1/(RL2 + 2R_X)$, and CL is an external capacitor. In this manner, in the ideal case, when $G_y = 0$ and $R_X = 0$, (11) becomes $Z(s) = s \cdot CL \cdot RL1 \cdot RL2$. Equation (11) embeds the resistance denoted by R_L in Fig. 1, which is a parasitic effect to the CCII+ based simulated inductance. An important thing is that R_L simplifies the synchronization procedure by turning negative all roots of M .

The design of Chua's diode is shown on the right side of Fig. 2, it consists of two CCII+s (NR1 and NR2) and four resistors (RNR1 to RNR4) [Senani & Gupta, 1998]. This cell is saturated by controlling the bias supply as well as the voltage saturation in terminal Z of each CCII+. The attractor limiting is obtained by the generated breakpoint. Further, the schematic of the proposed private communication system is shown in Fig. 3. The coupling is realized by

using two CCII+s arranged to perform a subtraction with a gain equal to the inverse of the resistance seen between terminals X and given by (12).

$$k_i = \frac{1}{2R_X} \quad (12)$$

In Fig. 3, the gain is only determined by the parasitic resistances of CCII+s 1K2, 2K2, 1K1 and 2K1; thus since R_X in the AD844 is approximately 50Ω , then the value of the gains are approximated to $k_1 = k_2 = 1/100$. The remaining CCII+s (A1, A2 and S1, S2) are the additive and subtractive parts of the signal masking technique (see Fig. 6).

5. Experimental Results

5.1. Synchronization results

Figures 4 and 5 detail synchronization results obtained with the proposed scheme, both in time and in phase planes, respectively.

5.2. Information transmission

Once the circuit synchrony has been observed, the transmission process can be carried out by several strategies. The main ones are chaotic masking, shift keying and chaotic modulation. If it is preferred to work the devices in the linear region in this part, one can keep using the CCII+ to implement the chaotic masking scheme shown in Fig. 6.

The signal S_1 represents the information to be encrypted as signal S_2 , and finally recovered as signal S_3 . Due to the mixed nature of the CCII+, one can consider the possibility of using either voltage or current mode signals for the transmission. Figure 7 shows the required connections in each

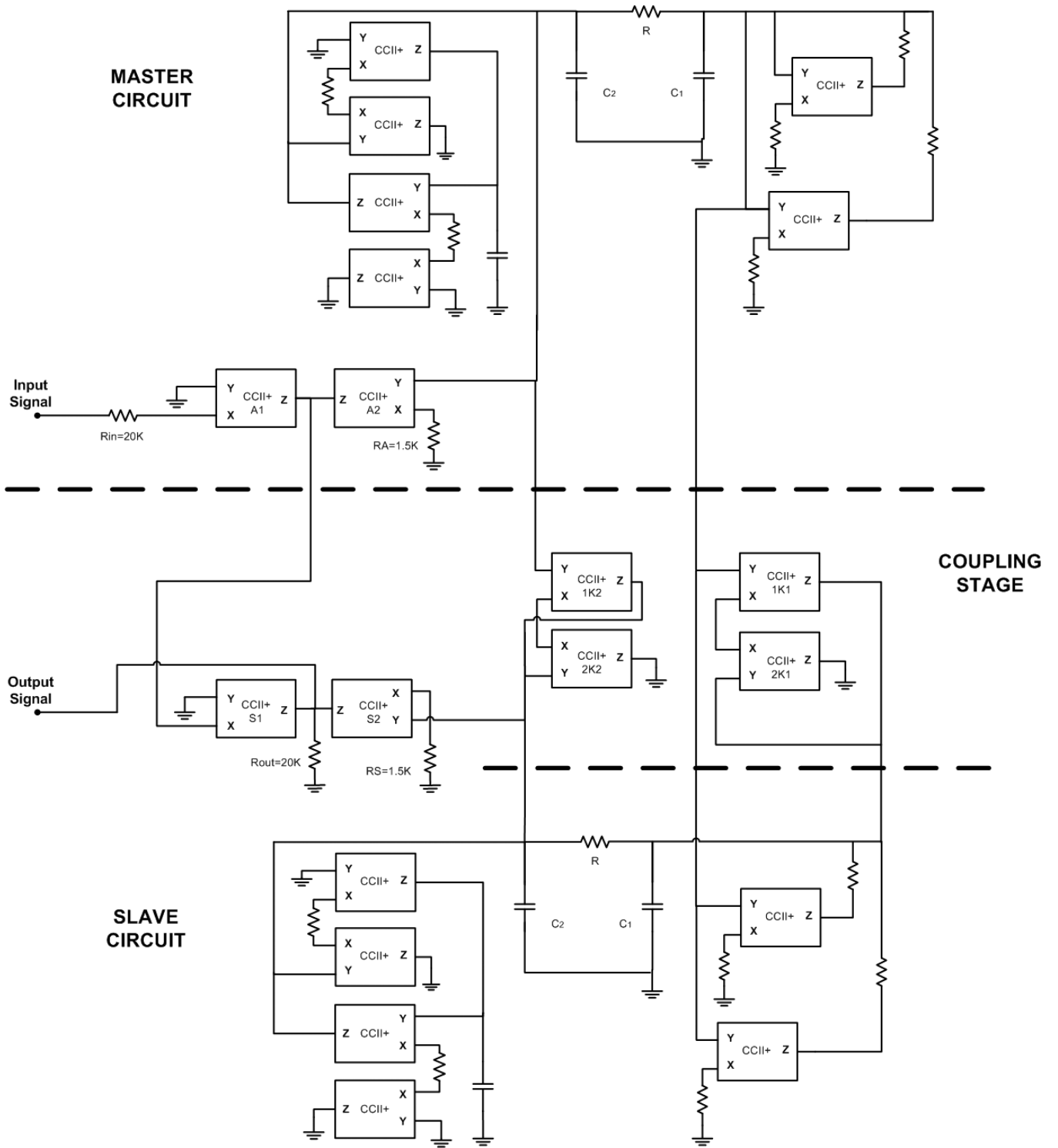
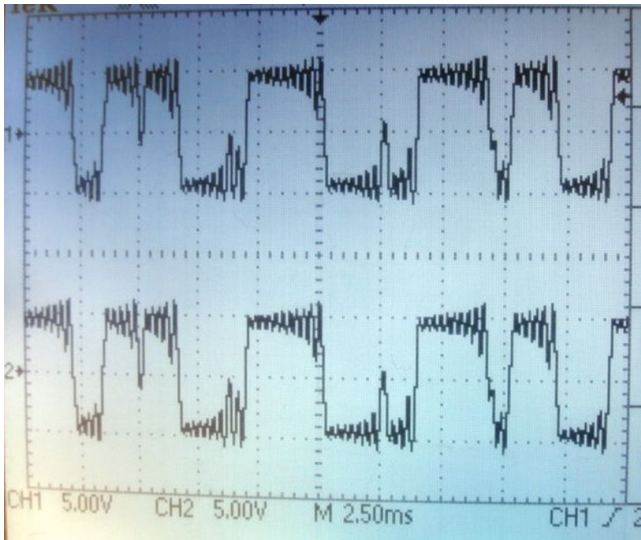
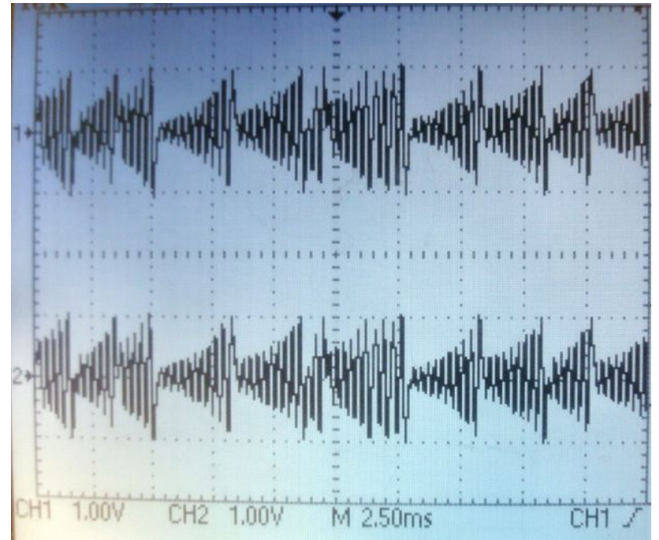


Fig. 3. Complete transmitter/receiver circuits using only current conveyors.

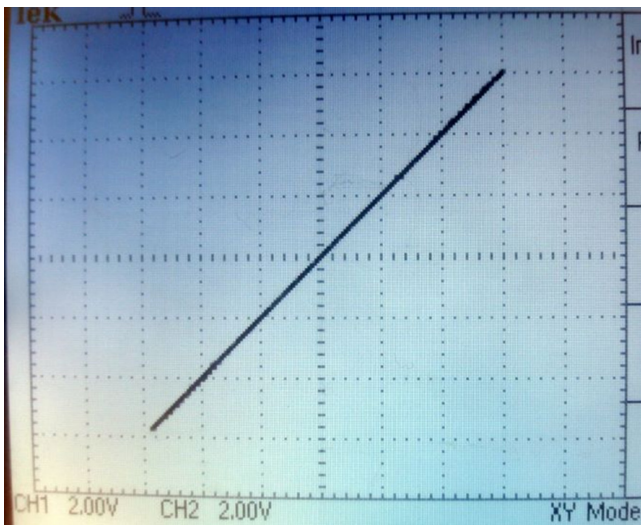


(a)

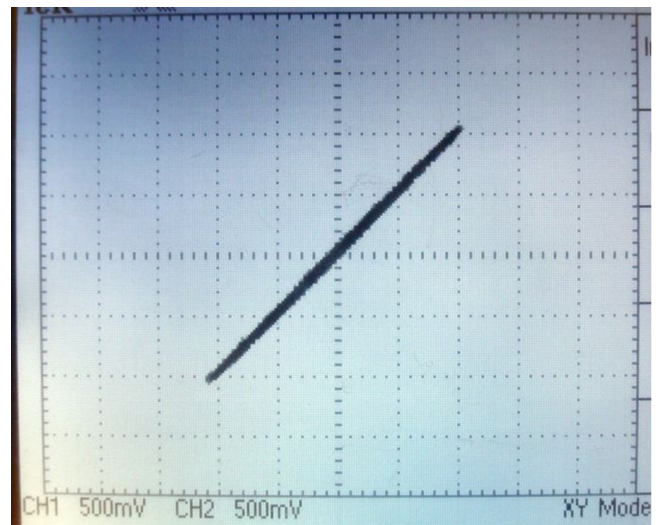


(b)

Fig. 4. Signals (a) channel 1: x_1 , Y-scale: 5V/div; channel 2: \hat{x}_1 , Y-scale: 5V/div, (b) channel 1: x_2 , Y-scale: 5V/div; channel 2: \hat{x}_2 , Y-scale: 5V/div, synchronized in time.



(a)



(b)

Fig. 5. Signals (a) x_1 and \hat{x}_1 , X-scale and Y-scale: 2V/div, (b) x_2 and \hat{x}_2 , X-scale and Y-scale: 0.5V/div, synchronized in phase plane.

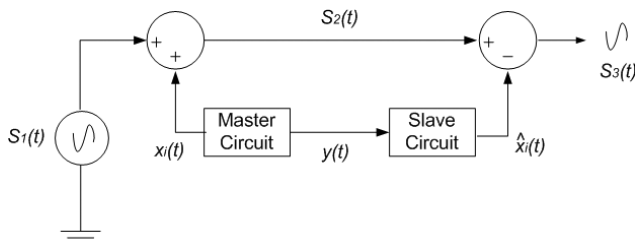


Fig. 6. Chaotic masking, block diagram encryption.

case. The constant D represents a reduction factor of signal S_1 compared to S_2 .

When considering the high impedance in terminals Y and Z of the CCII+ respect to X [Fakhfakh *et al.*, 2007], R_X should be taken into account because it generates tracking errors known as voltage-gain (A_v) between Y and X, and current-gain (A_i) between X and Z. In the ideal case, $A_v = A_i = 1$, however, in the real case these tracking errors affect the acquisition of the signal S_3

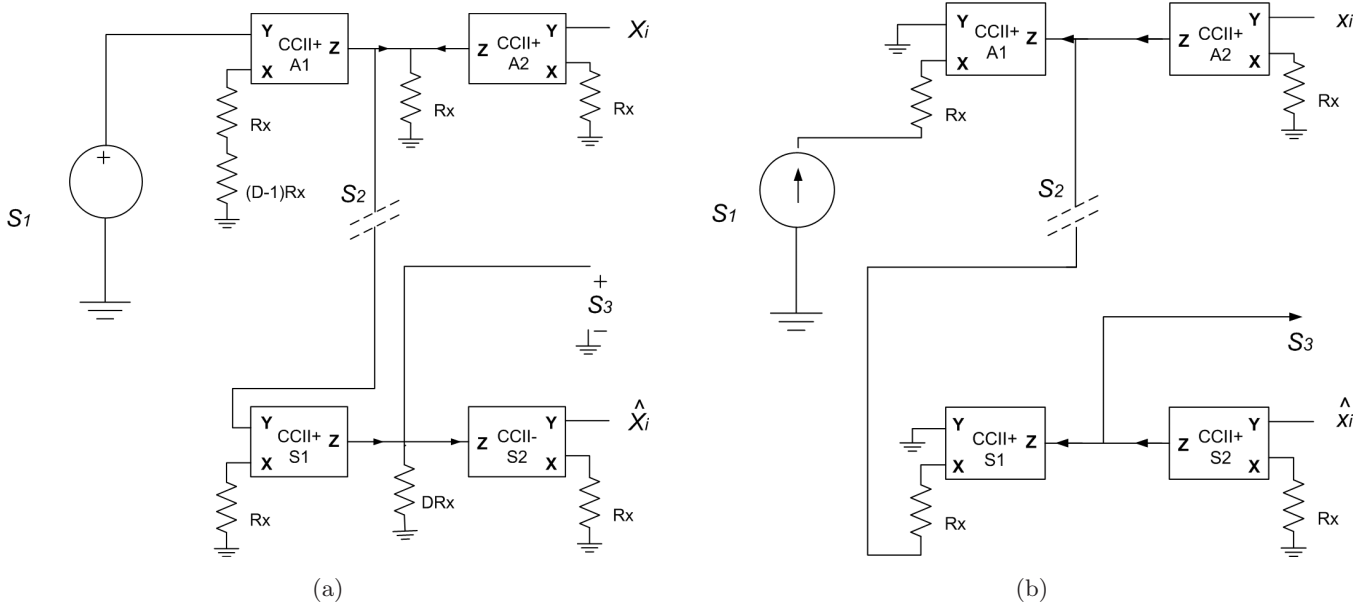


Fig. 7. Current conveyor based chaotic masking scheme for (a) voltage and (b) current mode.

according to

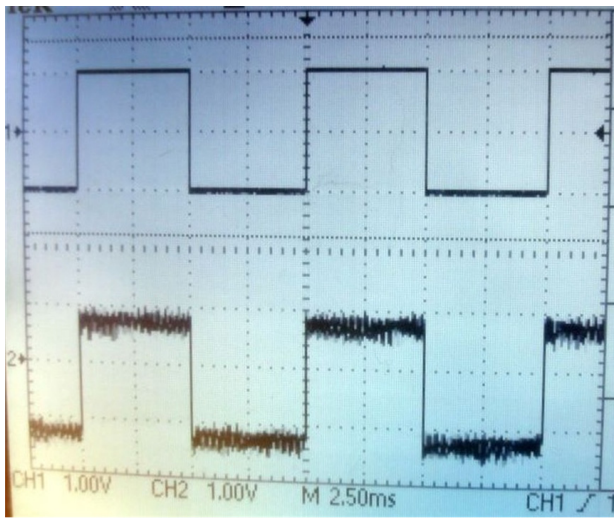
$$S_3 = A_v^2 A_i^2 S_1 + A_v A_i D (A_v A_i x_i - \hat{x}_i) \quad (13a)$$

for the voltage mode circuit, and

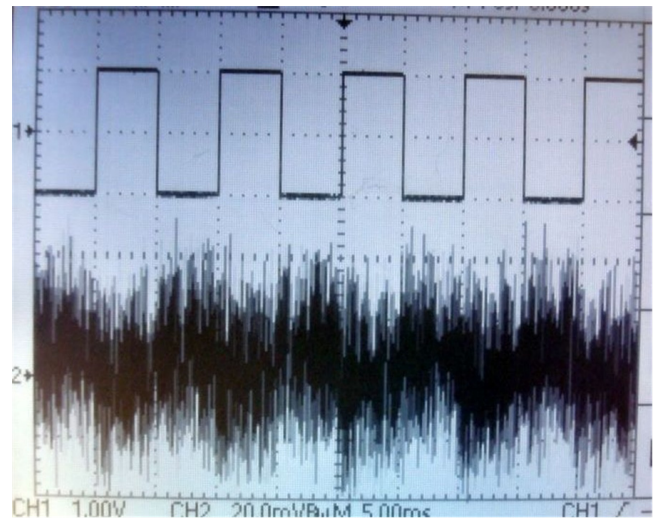
$$S_3 = A_i^2 S_1 + A_v A_i \left(\frac{\hat{x}_i - A_i x_i}{R_X} \right) \quad (13b)$$

for the current mode one. It is clear that the current approach is not just more accurate, it also allows the determination of the exact value for R_X .

Once this consideration has been made, the complete design with the discussed current mode approach is shown in Fig. 3. R_S and R_A are chosen to adjust the chaotic signal magnitude, while R_{in} and R_{out} are used to manipulate the external signal as voltage. State x_2 has been taken as the encryption signal x_i . Figure 8 shows the recovered signals for several frequencies of S_1 , as well as S_2 sequences.

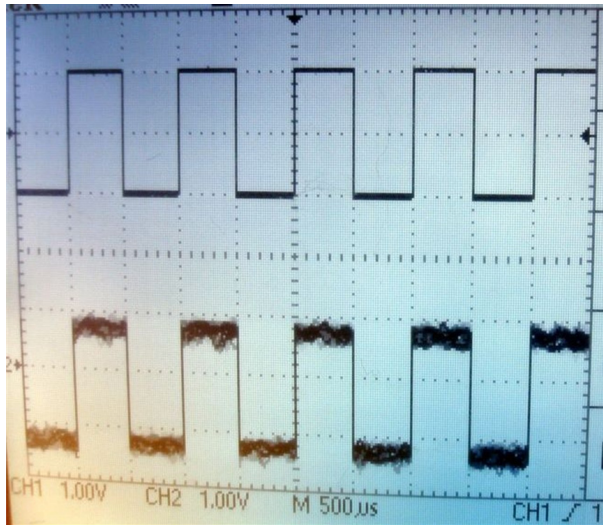


(a)

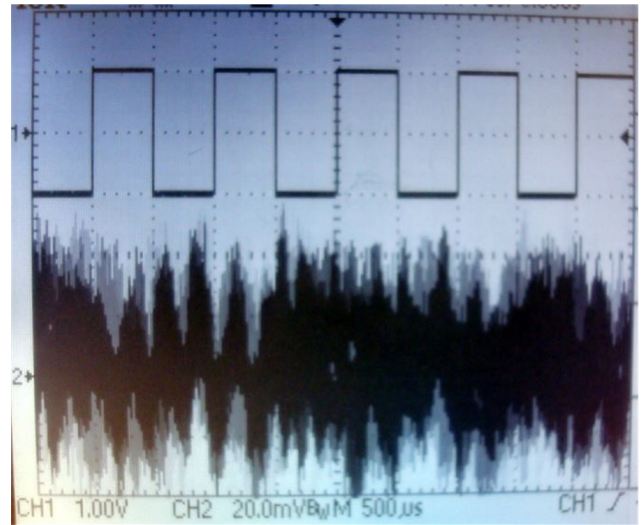


(b)

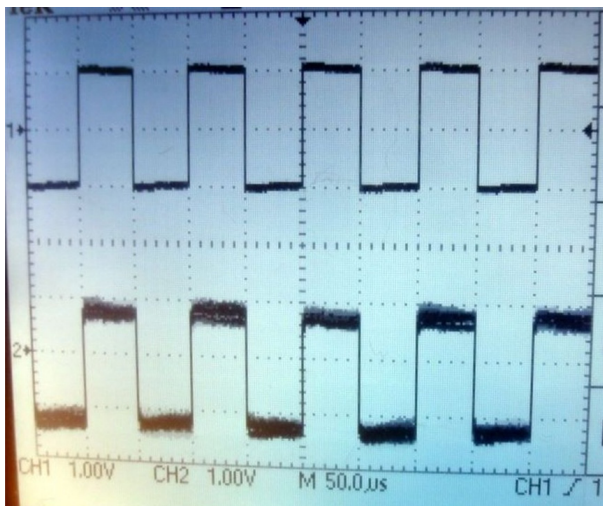
Fig. 8. (a), (c), (e) and (g) Comparison between the transmitted (channel 1) and received (channel 2) signals, Y-scale channel 1 and channel 2: 1 V/div; and (b), (d), (f) and (h) transmitted (channel 1) and encrypted (channel 2) signals, Y-scale channel 1: 1 V/div; Y-scale channel 2: 20 mV/div; for: (a) and (b) 100 Hz, (c) and (d) 1 KHz, (e) and (f) 10 KHz, and (g) and (h) 50 KHz.



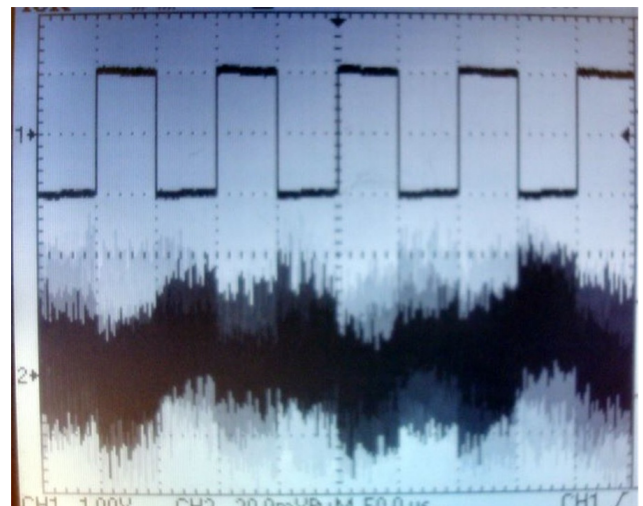
(c)



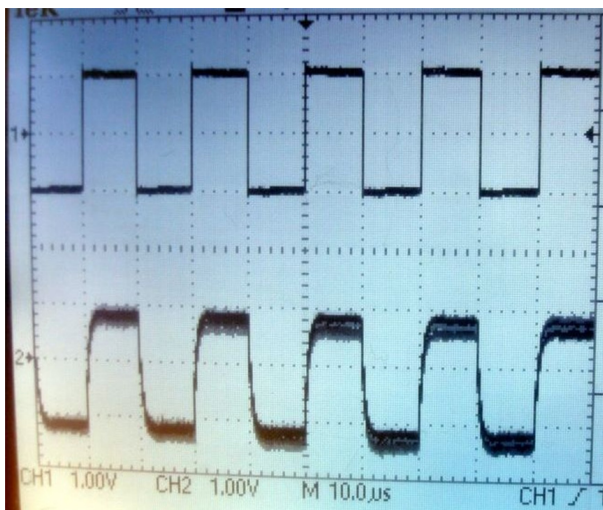
(d)



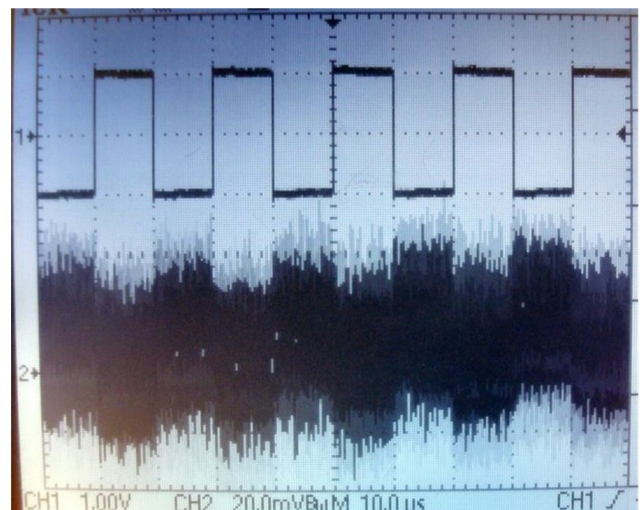
(e)



(f)



(g)



(h)

Fig. 8. (Continued)

6. Conclusions

An experimental implementation of Chua's oscillator has been presented using only CCII+s. The circuit performance has been reviewed experimentally using a commercial CCII+ included into the AD844 from Analog Devices.

Parasitic effects of the implementation are analyzed according to the general data provided by the manufacturer to explain the most susceptible points of the design despite assistance. Furthermore, the synchronization of two Chua's oscillators is shown by applying the Hamiltonian approach, in order to implement a chaotic communication system.

On the other hand, since chaotic masking is one of the potential forms to solve the transmission problem of encrypted information, then the versatility of the CCII+ suggests different approaches to provide some solutions in this area. As a result, a chaotic communication system realized with only CCII+s has been proposed and verified experimentally.

Acknowledgments

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