Unveiling a Truncated Optical Lattice Associated with a Triangular Aperture Using Light's Orbital Angular Momentum

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We show that the orbital angular momentum can be used to unveil lattice properties hidden in diffraction patterns of a simple triangular aperture. Depending on the orbital angular momentum of the incident beam, the far field diffraction pattern reveals a truncated optical lattice associated with the illuminated aperture. This effect can be used to measure the topological charge of light beams.

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Light beams possessing an azimuthal phase structure $\exp(im\phi)$ carry orbital angular momentum (OAM) of $m\hbar$ per photon, where m is an integer number [1], referred to as the topological charge. Fundamentally, OAM in light-matter interaction was investigated in several processes, for instance, second harmonic generation [2], spontaneous [3] and stimulated [4] parametric down-conversion, and wave mixing [5]. Applications of light's OAM range from optical manipulation [6] to quantum communication [7]. Just recently [8], a proposal to generate brilliant x-ray radiation with OAM has appeared, which may open new exploration venues in matter characterization.

A seminal work [1] by Allen *et al.* published in 1992 showed that light can carry a well-defined OAM associated with an helicoidal wave front. As a consequence of this spatial structure, light beams propagate with the Poynting vector twisting like a corkscrew around the propagation axis. High-order Laguerre-Gauss (LG) [1] and high-order Bessel [9] beams are examples of beams carrying OAM. Analogous to spin and two-level systems, OAM states can be decomposed in terms of orthogonal light beams, and geometric representations equivalent to Poincaré and Bloch spheres have been constructed [10,11]. On the other hand, wave fields with helicoidal phase structure give rise to optical vortices that have also been extensively studied in nonlinear periodic optical lattices [12–14]. We should mention that the rich relationship between the phase of light with OAM and diffraction phenomena has been exploited before [15–17].

In this Letter, we will show how the diffraction of helical beams by triangular apertures produces in the far field plane a surprising triangular lattice correlated with the topological charge m. We present measurements, computer simulations, and a heuristic argument that explains the observations. We will also show that a powerful application can be derived from our idea: a new direct way to measure the topological charge of a light beam.

Initially, we will briefly discuss the orbital angular momentum of a beam. The total angular momentum density of a beam is given by [18]

$$\mathbf{j} = \mathbf{r}_\perp \times \mathbf{p}, \tag{1}$$

where $\mathbf{p}$ is the linear momentum vector and $\mathbf{r}_\perp$ is the transverse coordinate at the aperture plane. The OAM's z component can be written as $j_z = r_\perp p_\phi$, where p_ϕ is the azimuthal component of the linear momentum $\mathbf{p}$. For such an incident beam, the z component of the total OAM is given by $J_z = m\hbar$ per photon.

Now, our goal is to determine the Fraunhofer diffraction pattern in the far field region of a beam carrying OAM by a triangular aperture. If we are interested only in relative intensities at a fixed plane placed at the position $z = z_0$, the diffracted field E_d is given by the integral [19]

$$E_d(\mathbf{k}_\perp) = \int \tau(\mathbf{r}_\perp) E_i(\mathbf{r}_\perp) e^{-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} d\mathbf{r}_\perp. \tag{2}$$

In this integral, the far field distribution $E_d(\mathbf{k}_\perp)$ is obtained from the Fourier transform of the product of the function describing the aperture $\tau(\mathbf{r}_\perp)$ and the incident field $E_i(\mathbf{k}_\perp)$. Note that the transverse wave vector $\mathbf{k}_\perp$ can be associated with the coordinate system of the far field region playing the role of reciprocal space. When the incident beam is a plane wave, the integral usually can be evaluated, being a two-dimensional Fourier transform of the function $\tau(\mathbf{r}_\perp)$.

In fact, the problem of the diffraction of a plane wave by a triangular aperture was solved a long time ago, and even the existence of a lattice of points of zero intensity in the Fourier plane was reported (see, for example, Sillitto and Sillitto [20] and references therein). However, we should point out that a lattice of intensity maxima has not so far been theoretically predicted or experimentally observed, for a triangular aperture. Moreover, when the incident field has the azimuthal phase structure of a beam carrying orbital angular momentum, the exact integral has not been analytically solved for polygonal apertures.

Let us start by numerically solving the Fraunhofer diffraction pattern for an equilateral triangular aperture. The integral shown in Eq. (2) can be numerically evaluated by using high-order LG beams as the initial condition for the electric field. For LG beams with $p = 0$, the Fraunhofer diffraction patterns for different values of the topological charge m ranging from one to three is shown at the top of Fig. 1. Observe that the number of maxima increases with

m. Also notice that the triangle is rotated by 30° in relation to the triangular aperture, shown in the inset.

We have experimentally confirmed our numerical results by using a very simple experimental setup. The light beams possessing OAM were prepared by using an argon laser operating at 514 nm illuminating a computer-generated hologram [21]. Different orders diffracted by the hologram produced high-order LG beams with different topological charges, which were selected by using a pinhole. An equilateral triangular aperture with side length of 1.75 mm was placed in the propagation path. Using a 30 cm focal length lens immediately after the aperture, we obtained the far field diffraction pattern at the lens's focal plane. The intensity distribution of the diffraction patterns were recorded by a CCD (charge-coupled device) camera. The experimental results are also shown at the bottom of Fig. 1.

One important point about the experimental setup is that the phase singularity must fall in the center of the aperture and the edges of the aperture should be illuminated by the inner border of the incident beam. In fact, there is a rich relationship between the relative phases on the wave front around the phase singularity, becoming even richer for high values of m. However, it is easy to center the beam in the aperture just by checking the obtained CCD image during the alignment.

It is well known that the diffraction pattern of an optical field by an aperture is the result of the interference between the edge waves. To have a physical insight, we use a simple qualitative argument to analyze the effect of the azimuthal phase over the edge, taking each edge separately. By using some trigonometric relations, for a vortex $\exp(im\varphi)$, the phase along any of the edges can be written as

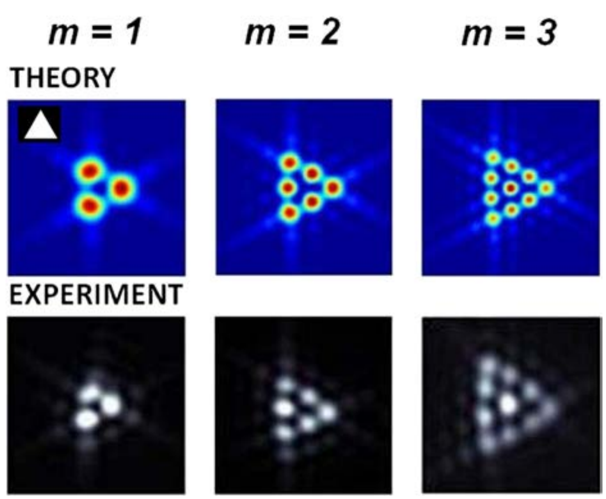

FIG. 1 (color online). Results for the diffraction of a light beam possessing OAM by a triangular aperture. Numerical (upper part) and experimental (lower part) results for m ranging from one to three. For the numerical results, light intensity increases from blue to red; for the experimental ones, from black to white. The inset in the $m = 1$ numerical result shows the orientation of the triangular aperture.

$$\varphi(s) = \arcsin\left(\frac{s}{\sqrt{(a/12)^2 + s^2}}\right), \tag{3}$$

where s is a coordinate with origin at the middle of the slit and in the range $[-\frac{a}{2}, \frac{a}{2}]$, and a is the size of each side of the triangle. We approximate this expression by the first term of the Taylor expansion: $\varphi(s) \approx 2\sqrt{3}s/a$. We now approximate the triangle's edges by infinitesimally thin slits, ignoring the variation of the field amplitude. We choose the slit width to be so narrow that we can treat it as a Dirac delta. If we consider a infinite horizontal slit, and replace x for s in the expression for $\varphi(s)$, we have the Fourier transform for the electric field E_{slit} that can be solved easily [22]:

$$\begin{aligned} E_{\mathrm{slit}}(k_x, k_y) &\sim \int \delta(y) e^{i2\sqrt{3}mx/a} e^{-i\mathbf{k}_\perp \cdot \mathbf{r}_\perp} dx dy \\ &= \delta(k_x - 2\sqrt{3}m/a), \end{aligned} \tag{4}$$

where x and y are the transverse Cartesian coordinates in the plane of the aperture; k_x and k_y are the transverse coordinates in the Fourier plane; and δ is the delta of Dirac. To solve Eq. (4), we also used the two-dimensional shift theorem [22]. As expected, we can see from this result that an infinite slit along x produces a pattern that is displaced in the k_x direction (i.e., the center of the pattern is displaced) by an amount proportional to m. The effect of the azimuthal phase over this diffraction pattern is to produce a shift proportional to the amount of OAM. We

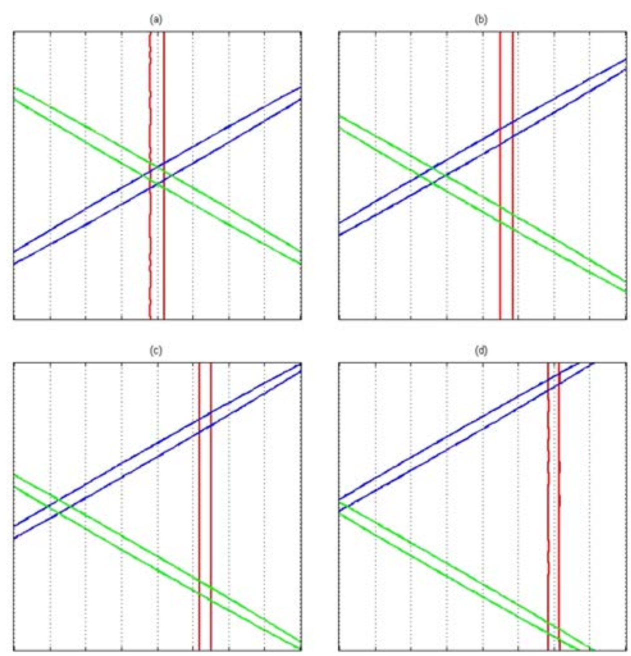

FIG. 2 (color online). Contour lines for the central fringe for the light diffracted separately by each edge forming the triangle for the topological charge m being equal to (a) 0, (b) 1, (c) 2, and (d) 3. The triangular aperture is in the same position as in Fig. 1.

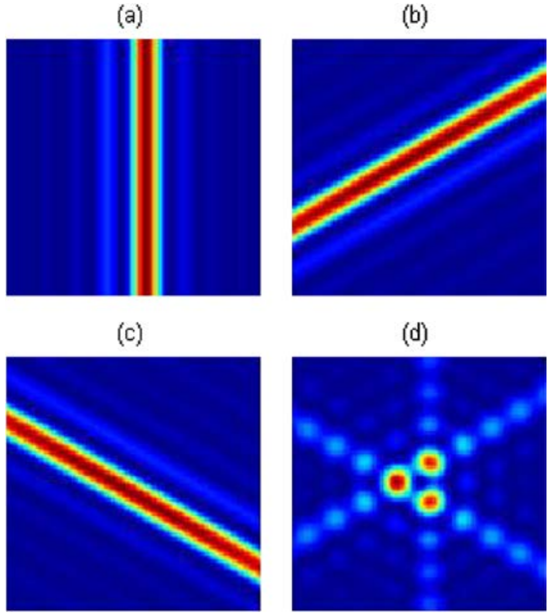

FIG. 3 (color online). Contour plot showing the diffraction pattern produced by the horizontal (a), right (b), and left (c) edge of the triangle. Also the intensity plot obtained by summing the field diffracted by each edge (d) producing the lattice for $m = 1$. The intensity scale is the same as in Fig. 1. The triangular aperture is in the same position as in Fig. 1.

also see a dependence of the shift proportional to the inverse of the size of the triangle's side.

Finally, we will corroborate the previous reasoning, numerically evaluating Eq. (2) for each edge separately. The contour lines corresponding to the initially central fringe for Laguerre-Gauss beams diffracted by each edge for m changing from zero to three are shown in Fig. 2. We can see that the effect of the azimuthal phase is to shift each of the rays of the starlike diffraction pattern for the case without OAM. Each pattern is shifted by a linear amount proportional to the topological charge m, and it delimits the lattice in the reciprocal space. In Figs. 3(a)–3(c), we show intensity contour plots to each edge for the case of $m = 1$. When the diffraction patterns corresponding to each edge in the triangle are superposed, i.e., they are allowed to interfere, one recovers the truncated lattice corresponding to $m = 1$, as seen in Fig. 3(d).

For the case where the topological charge of the incident field is negative ($m < 0$), the spatial shift will be in the opposite direction and the diffraction triangular pattern will be rotated by 180° in relation to the former. Figure 4 illustrates this effect by comparing the diffraction pattern in this case of Bessel beams with $m = -7$ (a) and $m = 7$ (b), showing the 180° rotation.

A practical application emerges from our results: a simple way to determine the magnitude and sign of a beam's topological charge. So far, mainly the azimuthal phase structure of light beams with OAM has been exploited to obtain the value of the topological charge.

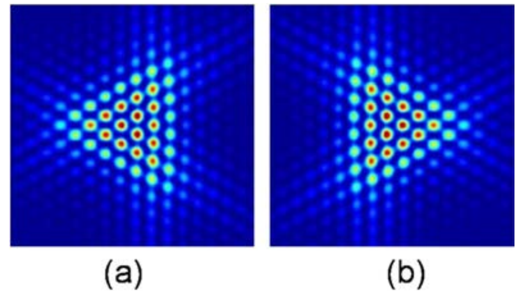

FIG. 4 (color online). Effect of changing the sign of the topological charge in the diffraction patterns by a triangular aperture. (a) Incident beam with $m = -7$. (b) Incident beam with $m = 7$. The intensity scale is the same as in Fig. 1. The triangular aperture is in the same position as in Fig. 1.

Different techniques to measure the light's OAM have been developed so far, many of them based on interference [16,23,24] and others, for example, related with direct measurement of the wave front [25] and with whispering gallery resonators [26]. From Fig. 1, we can observe that the value of m is directly related to external points of the lattice forming a triangle. The total charge is given by $m = N - 1$, where N is the number of points on any side of the triangle.

In conclusion, we numerically and experimentally demonstrated the generation of a truncated triangular optical lattice just diffracting beams with OAM through a equilateral triangular aperture. The shift effect caused by the OAM reveals a concealed lattice, which is not observed for beams without OAM. The obtained optical lattices are constrained by the topological charge of the incident beam. From a practical point of view, we can use the diffraction patterns to determine the topological charge of a light beam in an easy and direct way, just by counting the number of external points of a triangular lattice.

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[1] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Phys. Rev. A **45**, 8185 (1992).
[2] J. Courtial, K. Dholakia, L. Allen, and M. J. Padgett, Phys. Rev. A **56**, 4193 (1997).
[3] A. Mair, A. Vaziri, G. Weihs, and A. Zeilinger, Nature (London) **412**, 313 (2001).

[4] D. P. Caetano *et al.*, Phys. Rev. A **66**, 041801(R) (2002).
[5] J. W. R. Tabosa and D. V. Petrov, Phys. Rev. Lett. **83**, 4967 (1999).
[6] D. G. A. Grier, Nature (London) **424**, 810 (2003).
[7] G. Molina-Terriza, J. P. Torres, and L. Torner, Nature Phys. **3**, 305 (2007).
[8] S. Sasaki and I. McNulty, Phys. Rev. Lett. **100**, 124801 (2008).
[9] K. Volke-Sepulveda, V. Garcés-Chéz, S. Chávez-Cerda, J. Arlt, and K. Dholakia, J. Opt. B **4**, S82 (2002).
[10] M. J. Padgett and J. Courtial, Opt. Lett. **24**, 430 (1999).
[11] W. C. Soares, D. P. Caetano, and J. M. Hickmann, Opt. Express **14**, 4577 (2006).
[12] T. J. Alexander, A. A. Sukhorukov, and Y. S. Kivshar, Phys. Rev. Lett. **93**, 063901 (2004).
[13] D. N. Neshev *et al.*, Phys. Rev. Lett. **92**, 123903 (2004).
[14] T. J. Alexander, A. S. Desyatnikov, and Y. S. Kivshar, Opt. Lett. **32**, 1293 (2007).
[15] J. Arlt, J. Mod. Opt. **50**, 1573 (2003).
[16] I. Sztul and R. R. Alfano, Opt. Lett. **31**, 999 (2006).
[17] G. C. G. Berkhout and M. W. Beijersbergen, Phys. Rev. Lett. **101**, 100801 (2008).
[18] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1999), 3rd ed.
[19] J. W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 1996), 2nd ed.
[20] R. M. Sillitto and W. Sillitto, Opt. Acta **22**, 999 (1975).
[21] J. Arlt, K. Dholakia, L. Allen, and M. J. Padgett, J. Mod. Opt. **45**, 1231 (1998).
[22] R. Bracewell, *The Fourier Transform and Its Applications* (McGraw-Hill, New York, 1999), 3rd ed.
[23] N. R. Heckenmberg, R. McDuff, C. P. Smith, and A. G. White, Opt. Lett. **17**, 221 (1992).
[24] M. Harris, C. A. Hill, and J. M. Vaughan, Opt. Commun. **106**, 161 (1994).
[25] J. Leach, S. Keen, M. J. Padgett, C. Saunter, and G. D. Love, Opt. Express **14**, 11 919 (2006).
[26] A. B. Matsko, A. A. Savchenkov, D. Strekalov, and L. Maleki, Phys. Rev. Lett. **95**, 143904 (2005).