Glauber–Fock photonic lattices

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We show that classical analogs to quantum coherent and displaced Fock states can emerge in one-dimensional semi-infinite photonic lattices having a square root law for the coupling coefficients. Beam dynamics in these fully integrable structures is described in closed form, irrespective of the site of excitation. The trajectories of these beams are closely examined, and pertinent examples are provided for their realization. © 2010 Optical Society of America

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Light propagation in waveguide lattices has been the subject of considerable interest over the past few years. Such array structures provide a versatile platform upon which one can observe a host of processes, such as optical Bloch oscillations, Zener tunneling, Rabi oscillations, and Talbot revivals, to mention a few [1–3]. The discrete diffraction properties of such configurations can mold the flow of light in a predictable manner, hence providing altogether new opportunities for applications. Quite recently, light propagation in random and quasi-random arrays has also been considered—ranging from ballistic to the Anderson localization regime [4]. In addition, quantum correlations in regular lattice structures have been investigated for both classical and completely quantum states [5,6]. Yet, despite all the efforts put forth in this area, only a few of the reported lattices are known to have closed-form solutions [7,8]. In fact, integrable discrete systems are rather rare, and any new addition to this class will further facilitate such fundamental studies.

In this Letter we show that a semi-infinite array having a square root law distribution for the coupling constants can admit classical analogs to quantum coherent and displaced Fock states. The linear impulse response in these Glauber–Fock photonic lattices can be described in a closed form and are shown to be markedly different from those occurring in other classes of optical arrays. The proposed lattices can be established by judiciously adjusting the separation distance between successive identical single-mode waveguide elements, in such a manner that the coupling constants vary as $\sqrt{n}$ (see the inset of Fig. 1). This can be readily accomplished, given that the coupling constant between the waveguides depends exponentially on the separation distance d_n, $\kappa_{n,n+1} \sim \exp(-\gamma d_n)$ [9]. The self-bending beam trajectories in such structures are analytically examined.

In the proposed system, the normalized modal field amplitudes obey a discrete linear Schrödinger-like equation:

$$i\frac{dE_n}{dZ} + \sqrt{n+1}E_{n+1} + \sqrt{n}E_{n-1} = 0, \qquad (1)$$

where $n \geq 0$. The normalized coordinate Z is given by $Z = \kappa_1 z$, where z is the actual propagation distance and κ_1 stands for the coupling coefficient between sites 0 and 1. We begin by exploring the stationary wave states

allowed in this system. To do so, we assume the solution $E_n = a_n \exp(i\mu Z)$ in Eq. (1), where μ is a propagation eigenvalue. In this case, the resulting linear difference equation has the form

$$-\sqrt{n+1}\,a_{n+1} + \mu a_n - \sqrt{n}a_{n-1} = 0. \qquad (2)$$

A direct calculation indicates that an eigensolution in these arrays is given by $a_n = H_n(\mu/\sqrt{2})/2^{n/2}\sqrt{n!}$, where $H_n(x)$ represents a Hermite polynomial.

To obtain the impulse response of the Glauber–Fock lattice (e.g., when only one site is excited) we consider the following virtual x representation:

$$i\frac{d\psi(x,Z)}{dZ} = -(a + a^\dagger)\psi(x,Z), \qquad (3)$$

where $a^{(\dagger)} = (1/\sqrt{2})(x + (-)d/dx)$ are the "annihilation (creation)" operators [10]. These operators satisfy the relations $a\psi_n(x) = \sqrt{n}\psi_{n-1}(x)$ and $a^\dagger\psi_n(x) = \sqrt{n+1}\psi_{n+1}(x)$, where the system eigenfunctions are given by Gauss–Hermite functions $\psi_n(x) = \left(1/\sqrt{\pi^{1/2}2^n n!}\right)\exp(-x^2/2)H_n(x)$.
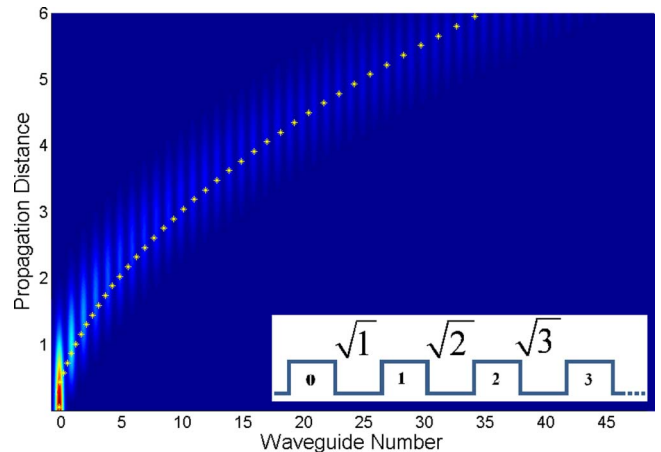

Fig. 1. (Color online) Propagation dynamics when the first waveguide site is excited. The dashed curve indicates the trajectory where the energy is highest during propagation. The inset depicts a semi-infinite waveguide array where $\sqrt{n}$ is the corresponding coupling constant.

By using separation of variables, we can express $\psi(x, Z)$ as a superposition of $\psi_n(x)$,

$$\psi(x, Z) = \sum_{n=0}^{\infty} E_n(Z)\psi_n(x). \qquad (4)$$

To introduce the Dirac notation, we use a correspondence between $\psi_n(x)$ and $|n\rangle$, ($\psi_n(x) \to |n\rangle$), then by substituting Eq. (4) into Eq. (3) yields

$$i\sum_{n=0}^{\infty} \frac{dE_n(Z)}{dZ}\Big|n\Big\rangle$$
$$= -\sum_{n=0}^{\infty}\left(\sqrt{n}\,E_n(Z)\Big|n-1\Big\rangle + \sqrt{n+1}E_n(Z)\Big|n+1\Big\rangle\right). \qquad (5)$$

Using the orthonormality properties of the eigenfunctions $|n\rangle$, we finally derive Eq. (1). Thus, by eliminating the virtual x dependence in Eq. (3), we have effectively obtained the equations governing light evolution in the system under investigation. Equation (3) can be readily solved using the evolution operator

$$\Big|\psi(Z)\Big\rangle = \exp\left(iZ\Big(a + a^\dagger\Big)\right)\Big|\psi(0)\Big\rangle. \qquad (6)$$

Note that $\hat{D}(iZ) = \exp(iZ(a + a^\dagger))$ represents the so-called Glauber displacement operator [11]. Because the operators a and $a^\dagger$ satisfy the commutation relations $[a, a^\dagger] = 1$ and $[a, [a, a^\dagger]] = [a^\dagger, [a, a^\dagger]] = 0$, we can factorize $\hat{D}(iZ)$ using the Baker–Hausdorff formula, $\hat{D}(iZ) = \exp(-Z^2/2)\exp(iZa^\dagger)\exp(iZa)$. The initial condition $|\psi(0)\rangle$ can be described in general via a linear superposition of "states," depending on which waveguides are excited. The aim here is to analyze the case when light is launched into a single lattice site at position k from the edge. In this case, $|\psi(0)\rangle = |k\rangle$, and Eq. (6) becomes $|\psi(Z)\rangle = \exp(-Z^2/2)\exp(iZa^\dagger)\exp(iZa)|k\rangle$. To evaluate the field distribution in the mth waveguide, we take the inner product $E_m = \langle m|\psi(Z)\rangle$, hence

$$E_m = \exp(-Z^2/2)\Big\langle m\Big|\exp(iZa^\dagger)\exp(iZa)\Big|k\Big\rangle. \qquad (7)$$

To develop an analytical expression for the solution of Eq. (1), a Taylor series expansion for the exponentials of the $a, a^\dagger$ operators in Eq. (7) is used. For $m = k + s$ (where $s = 0, 1, 2, \ldots$), i.e., for sites $m \geq k$ (on the right of the excited waveguide k), the field at a distance Z is given by

$$E_{k+s}(Z) = \exp(-Z^2/2)(iZ)^s\sqrt{\frac{k!}{(k+s)!}}L_k^s(Z^2), \qquad (8)$$

where $L_k^s(Z^2)$ are generalized Laguerre polynomials of degree k [12]. Conversely, if $m = k - s$, the field at any position to the left side of the excited waveguide is

$$E_{k-s}(Z) = \exp(-Z^2/2)(iZ)^s\sqrt{\frac{(k-s)!}{k!}}L_{k-s}^s(Z^2). \qquad (9)$$

When $k = 0$, i.e., when light is injected into the first waveguide, Eq. (8) readily reduces to the field redistribution:

$$E_m(Z) = \exp(-Z^2/2)\frac{(iZ)^m}{\sqrt{m!}}. \qquad (10)$$

Figure 1 depicts the intensity evolution among waveguide sites in this Glauber–Fock lattice when the first element is initially excited. Notably, the expression for $E_m(Z)$ in Eq. (10) is identical in form with the complex amplitudes involved between Fock states and coherent Glauber states in quantum optics [13]. The resulting intensity distribution $I_m = |E_m|^2$ at any distance $Z \neq 0$ is, in this case, Poissonian with m resembling the probability of finding a quantum harmonic oscillator at an energy level m, if a measurement is made when the oscillator is in a coherent state. In other words, for this particular case, we can think of the entire propagating lattice field as the classical analog of a quantum coherent state evaluated on the imaginary axis, i.e.,

$$\Big|\alpha = iZ\Big\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty} \frac{(\alpha)^n}{\sqrt{n!}}\Big|n\Big\rangle, \qquad (11)$$

whereas the vacuum state itself corresponds to the incoming field into the zeroth waveguide. The role of the translation operator, the generator of coherent states, is played by the lattice itself on the field.

As light propagates along the waveguides, the energy spreads from the left to the right (Fig. 1). In the case of $k = 0$, the trajectory where the intensity is a maximum can be exactly described by the function

$$Z = f(n) = \exp\left(\frac{1}{2}\left[-\gamma + \sum_{k=1}^{n} \frac{1}{k}\right]\right), \qquad (12)$$

where γ is the Euler–Mascheroni constant ($\gamma = 0.57721$).
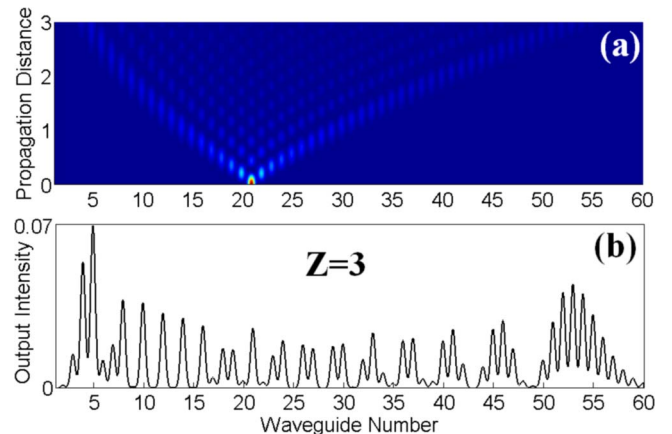

Fig. 2. (Color online) (a) Intensity evolution of the twentieth classical displaced state and (b) its corresponding output intensity profile at $Z = 3$.
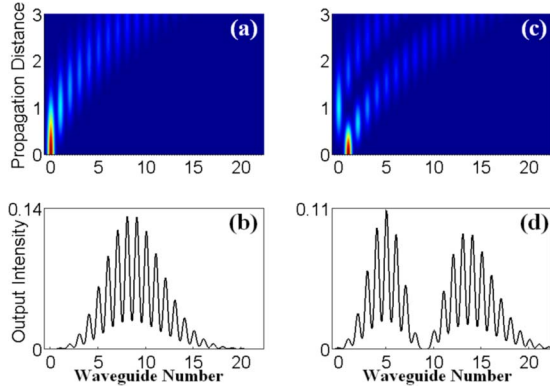
Fig. 3.   (Color online) (a) Intensity evolution of the classical coherent state and (b) its corresponding output intensity distribution at $Z = 3$. Similarly, for the (c) first classical displaced state and (d) its intensity profile at $Z = 3$.

Direct numerical simulations (dashed curve in Fig. 1) are in excellent agreement with Eq. (12).

Along similar lines, a classical number state $|k\rangle$ corresponds to an initial excitation of the kth waveguide site in this lattice. In this case, field distributions, given by Eqs. (8) and (9), represent the matrix elements of the Glauber displacement operator $\langle m|\hat{D}(iZ)|k\rangle$ in a Fock base representation. Figure 2 illustrates the intensity evolution when light is launched in the twentieth site. Notice that there is a marked difference between discrete diffraction occurring in regular waveguide arrays with that expected in a semi-infinite Glauber–Fock lattice [8]. In the Glauber–Fock lattice, the patterns are always tilted and accelerated toward the high coupling regions, a direct outcome of the imposed $\sqrt{n}$ coupling law. Figure 3 compares the resulting intensity distributions in the same structure when different sites are excited. Figures 3(a) and 3(b) were obtained for $k = 0$, while Figs. 3(c) and 3(d) were obtained for $k = 1$. The double-humped intensity profile of the first displaced number state resulting from the boundary reflection is evident in Fig. 3(d).

Glauber–Fock waveguide arrays can be fabricated on semiconductor wafers by implantation/etching techniques or in bulk silica by laser-direct writing [9,14].

The square root law required for the couplings between identical elements can be imposed by judiciously varying the distance between waveguides. Given the range of coupling constants available [14], Glauber–Fock arrays involving up to 100 elements should be feasible.

In conclusion, we have demonstrated that classical analogs to quantum coherent and displaced Fock states can emerge in one-dimensional semi-infinite photonic lattices having a square root law for the coupling coefficients. Such fully integrable configurations can provide new opportunities in the studies of optical discrete systems. Of particular interest will be the study of quantum and classical correlations in such waveguide lattices—a topic of great interest in quantum optics.

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