

## Research Article

# Frequency-Based Optimization Design for Fractional Delay FIR Filters with Software-Defined Radio Applications

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A frequency-designed fractional delay FIR structure, which is suitable for software radio applications, is presented. The design method is based on frequency optimization of a combination of modified Farrow and mutirate structures. As a result the optimization frequency range is made only in half of desired total bandwidth. According to the obtained results the proposed fractional delay structure allows online desired fractional delay update, with a high fractional delay value resolution.

## 1. Introduction

Software-defined radio represents a major change in the design paradigm for radios, [1], in which most of the functionality is made through programmable signal processing devices, giving the radio the ability to change its operating parameters to accommodate new features and capabilities [2].

A software-defined radio platform is designed to make mobile systems more flexible with respect to the bandwidth requirements of different mobile standards. This flexibility is achieved by performing channel selection in the digital domain through sample rate conversion (SRC) with programmable digital filters. Fractional Delay (FD) filters are key components used to perform nonrational SRC [3–5].

Additionally, Software radio systems employ direct conversion receivers with asynchronous sampling such that the actual sampling instants are not synchronized with the incoming symbols. In order to evaluate the received symbols a digital symbol synchronization is implemented through FD filter structures [6, 7].

One of the key requirements for FD in software radio applications is to have the flexibility to change among different communication protocols and to be able to perform

fractional delay value update on line, known as variable fractional delay (VFD) filters [8]. Other important FD characteristics are wide bandwidth, high fractional delay resolution value, and a small number of arithmetic operations per output sample.

There are several FD design methods [9]; among them the use of a polynomial approach allows online desired fractional delay value update using a Farrow structure [10] or a modified Farrow structure [11, 12]. Both structures are composed of  $L + 1$  parallel FIR filters  $C_l(z)$ , each one with length  $N$ , where  $L$  is the chosen polynomial order, as it is shown in Figure 1. In a modified Farrow structure  $\gamma = 2\alpha - 1$ , where  $\alpha$  is the required fractional delay value,  $0 < \alpha < 1$ , and  $C_l(z)$  are linear phase filters (symmetrical coefficient values). In the original Farrow structure  $\gamma = \alpha$  and parallel filters are not symmetrical.

There are two main FD design approaches based on the polynomial approach. The first one is completely time domain design based on either Lagrange interpolation [13] or B spline functions [14, 15]. The implementation of this design approach is made through original Farrow structure having as main advantage that filter coefficients are obtained by closed form expressions. The disadvantage is the small flexibility available for this approach to meet FD filter

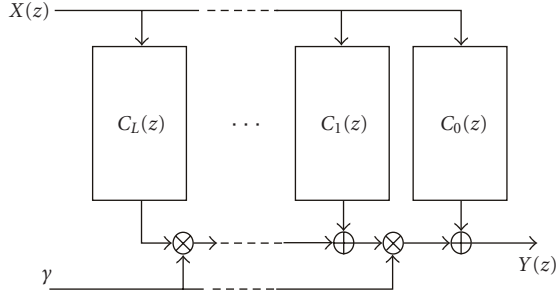


FIGURE 1: Farrow structure.

frequency domain specifications. This is because there is only one design parameter, that is, the polynomial order  $L$ . Most of the recently reported FD design methods belong to this time domain approach such as [16, 17].

The second FD design approach is made in frequency domain using optimization techniques for coefficients computing. The main advantage of this design approach is an improved control on frequency specifications. This is because three design parameters are available: polynomial order  $L$ , filter length  $N$ , and desired frequency passband  $\omega_p$ . The disadvantage of this approach is the need of an optimization method for filter coefficient computing. Several design methods have been proposed such as, for example, [18], where the FD filter is implemented in a modified Farrow structure and a Taylor approximation is achieved. Similarly in [19–21] the implementation is made using an original Farrow structure and a weighted least square optimization is accomplished.

The use of frequency domain design methods for FD filters with a wide bandwidth requires that an optimization method be applied over a large frequency range. On the other hand large filter length  $N$  and polynomial order  $L$  are obtained when high fractional delay resolution is required. Hence the design process requires high coefficients computing time and a high number of arithmetic operations per output sample in the resulting FD filter implementation.

This paper describes the use of a multirate structure in a frequency design approach in order to reduce the optimization workload in coefficients computing for FD filters with a wide bandwidth, high fractional delay resolution, and online fractional delay value update capability. In this way a flexible frequency design method with a reduced optimization workload as well as a resulting structure with a reduced number of arithmetic operations per output sample is obtained.

The used frequency design method is the modified Farrow structure [18], where each parallel filter  $C_l(z)$  is designed as a minimum least square approximation of an  $l$  order differentiator. In the same way it is possible to extend such proposal through other optimization frequency design methods.

Section 2 describes in a general way the frequency design method basis. The multirate structure is given in Section 3. In Section 4 the proposed design method is shown, which

is illustrated through one design example. Conclusions are presented in Section 5.

## 2. Frequency Design

The frequency design method in [18] is based on the following properties of the parallel digital filters  $C_l(z)$ .

- (1) FIR filters  $C_l(z)$ ,  $0 \leq l \leq L$ , in original Farrow structure are an  $L$  order Taylor approximation to the continuous-time interpolated input signal.
- (2) In the modified Farrow structure the FIR filters  $C_l(z)$  are linear phase type II for  $l$  even and type IV for  $l$  odd.

Each filter  $C_l(z)$  approximates in magnitude the function  $K_l \omega^l$ , where  $K_l$  is a constant. The ideal frequency response of an  $l$  order differentiator is  $(j\omega)^l$ ; hence the ideal response of each  $C_l(z)$  filter in the Farrow structure is an  $l$  order differentiator.

In the same way it is possible to approximate the input signal through Taylor series in a modified Farrow structure. The  $l$  order differential approximation to the continuous-time interpolated input signal is done through the branch filter  $C_l(z)$ , with a frequency response given as

$$\hat{C}_l(e^{j\omega}) = e^{-j\omega((N-1)/2)} \frac{(-j\omega)^l}{2^l l!}. \quad (1)$$

The input design parameters are the filter length  $N$ , the polynomial order  $L$ , and the desired passband frequency  $\omega_p$ .

The  $N$  coefficients of the  $L + 1$   $C_l(z)$  FIR filters are computed in such a way that the following error function is minimized in a least square sense in the frequency range  $[0, \omega_p]$ :

$$e_l(\omega) = \left[ \sum_{n=0}^{(N/2)-1} C_l\left(\frac{N}{2} - 1 - n\right) \gamma(l, n, \omega) - D(l, \omega) \right], \quad (2)$$

where

$$D(l, \omega) = \frac{(-\omega)^l}{2^l l!},$$

$$\gamma(l, n, \omega) = 2 \cos\left[\left(n + \frac{1}{2}\right)\omega\right], l \text{ even}, \quad (3)$$

$$\gamma(l, n, \omega) = 2 \sin\left[\left(n + \frac{1}{2}\right)\omega\right], l \text{ odd}.$$

Hence the objective function is given as

$$E_1 = \int_0^{\omega_p} \left[ \sum_{n=0}^{(N/2)-1} C_l\left(\frac{N}{2} - 1 - n\right) \gamma(l, n, \omega) - D(l, \omega) \right]^2 d\omega. \quad (4)$$

From this equation it can be observed that the design of a wide bandwidth FD filter requires an extensive computing workload. For high fractional delay resolution FD filters high precise differentiator approximations are required; this

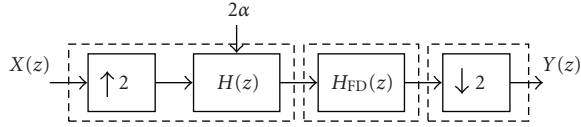


FIGURE 2: Multirate structure for FD filter.

implies high branch filters length,  $N$ , and high polynomial order,  $L$ . Hence an FD filter structure with high number of arithmetic operations per output sample is obtained.

The arithmetic complexity of the resulting implemented structure is an important factor to be considered. The comparative parameters are the following:

- (1) number of multipliers per output sample (MPS),
- (2) number of additions per output sample (APS).

In the modified Farrow structure the  $MPS_1$  and  $APS_1$  are given as

$$MPS_1 = \left(\frac{N}{2}\right)(L+1) + L, \quad (5)$$

$$APS_1 = (N-1)(L+1) + L + 1.$$

### 3. Multirate Structure

The multirate structure in [22] is proposed for designing FD filters in time domain. The input signal bandwidth is reduced by incrementing the sampling frequency. In this way Lagrange interpolation is used in filter coefficients computing for an FD filter with a wide bandwidth.

The multirate structure shown in Figure 2 is composed of three sections. The first one is an upsampler and a half band image suppressor filter  $H(z)$  for incrementing twice the input sampling frequency. The second section is the FD filter  $H_{FD}(z)$ , which is designed in time domain through Lagrange interpolation [11]. Since the signal processing frequency of filter  $H_{FD}(z)$  is two times more than the input sampling frequency, such filter can be designed to meet only half of the required bandwidth. Last section deals with a downsampler for decreasing the sampling frequency to its original value. The upsampling process is made through insertion of one zero between every two input samples. Hence for each output sample only half of the FIR filter coefficients are used. This means that in one time instant the input samples are processed through the even coefficients and next time instant through the odd coefficients of the filters  $H(z)$  and  $H_{FD}(z)$ . According to this technique and using multirate processing noble identities [23], such processing can be represented as shown in Figure 3, where filters  $H_0(z)$  and  $H_1(z)$  are the first and second polyphase components of the half band filter  $H(z)$ . In the same way  $H_{FD,0}(z)$  and  $H_{FD,1}(z)$  are the first and second polyphase components of fractional delay filter  $H_{FD}(z)$ . As can be seen the input sampling frequency is the same for all filters in the resulting structure.

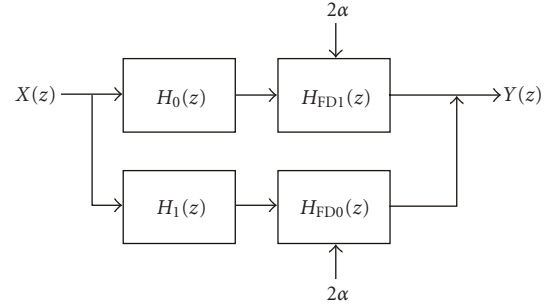


FIGURE 3: Resulting structure for FD filter.

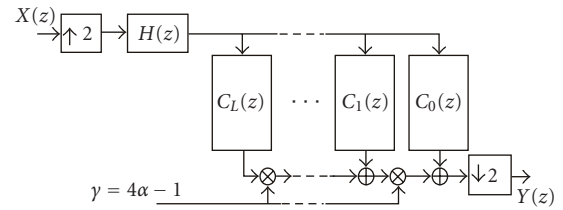


FIGURE 4: Initial structure of the proposed method.

### 4. Proposed Design Method

The proposed method for FD filter design with a wide bandwidth and high fractional delay resolution is based on a frequency domain optimization approach, described in the second section, applied to the FD multirate structure, described in last the section.

As mentioned before the maximum frequency of the FD filter in the multirate structure is half of the desired bandwidth. In this way the frequency optimization is made only on the half of the required passband. That means that the upper frequency limit in (4) is  $\omega_p/2$ . This optimization frequency range decrease allows an abrupt coefficient computing time reduction for the wide bandwidth FD filters and the resulting structure requires  $C_l(z)$  filters with smaller length  $N$ .

In Figure 4 is shown the initial structure, where a double fractional delay value is considered in the update parameter  $\gamma$ , as a result of the doubled processing sampling frequency. The resulting structure after applying noble identities is shown in Figure 5, where the filters  $C_{l,0}(z)$  and  $C_{l,1}(z)$  are the first and second polyphase components of  $C_l(z)$ , respectively.

The  $H(z)$  filter plays a key role in resulting bandwidth and fractional delay resolution of FD filter. The higher the stopband attenuation of the filter  $H(z)$ , the higher the resulting fractional delay resolution. Similarly the smaller the transition band of  $H(z)$ , the higher the resulting bandwidth of the FD filter. Both conditions imply the use of a high-order  $H(z)$  filter.

In order to reduce the total number of arithmetic operations per output sample the filter  $H(z)$  is designed as

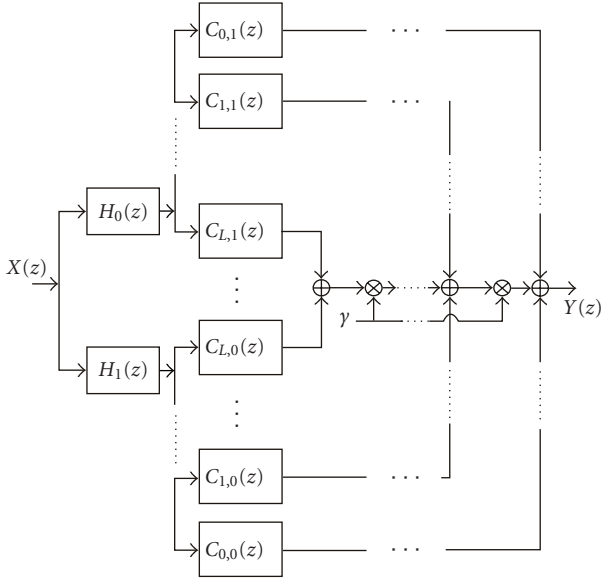


FIGURE 5: Resulting proposed structure.

a half band FIR filter. In this way the total number of MPS and APS is given as

$$\begin{aligned} \text{MPS}_2 &= (N)(L+1) + \left\lfloor \frac{N_{LP}+1}{4} \right\rfloor + L, \\ \text{APS}_2 &= (N-1)(L+1) + L + \frac{N_{LP}-1}{2}, \end{aligned} \quad (6)$$

where  $N_{LP}$  is the number of coefficients of  $H(z)$ .

## 5. Obtained Results

The design method was implemented in MATLAB. An illustrative design example is presented with an FD filter bandwidth of  $0.9\pi$  and a fractional delay resolution of  $1/10000$ .

The FD filter design using WLS method [20] results in an implementation processing arithmetic of  $\text{MPS} = 703$  and  $\text{APS} = 775$  with design parameters  $N = 87$  and  $L = 7$  and weighting functions given by

$$W_1(\omega) = \begin{cases} 1, & \omega \in [0, 0.88\pi), \\ 10, & \omega \in [0.88\pi, 0.8994\pi), \\ 0, & \omega \in [0.8994\pi, \pi], \end{cases} \quad (7)$$

$$W_2(p) = 1.$$

In the same way the resulting processing arithmetic for the implementation of the FD filter example based on variable

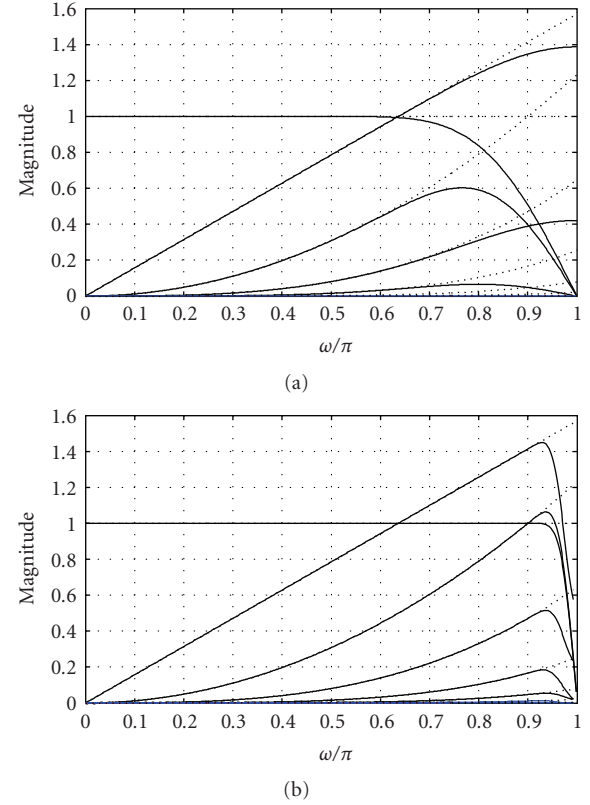


FIGURE 6: Differentiators ideal responses (dot line) and approximations (solid lines). (a) Proposed method  $\omega_p = 0.45\pi$ ,  $N = 12$ , and  $L = 14$ . (b) Direct all band optimization  $\omega_p = 0.9\pi$ ,  $N = 104$ , and  $L = 12$ .

FD design method of [21] is  $\text{MPS} = 543$  and  $\text{APS} = 535$  with design parameters  $N = 67$  and  $L = 7$  and next weighting function:

$$W_1(\omega) = \begin{cases} 1 & \omega \in [0, 0.88\pi), \\ 3 & \omega \in [0.88\pi, 0.8994\pi), \\ 0 & \omega \in [0.8994\pi, \pi]. \end{cases} \quad (8)$$

The direct use of the frequency domain method [18] with design parameters of  $L = 12$  and  $N = 104$  results in a total number of  $\text{MPS} = 688$  and  $\text{APS} = 1352$ .

For the proposed design method an interpolator filter  $H(z)$  with 241 coefficients was used, designed with a Dolph-Chebishev window having a stopband attenuation of 140 dBs. The design parameters are  $L = 12$  and  $N = 14$  with a resulting processing arithmetic of  $\text{MPS} = 254$  and  $\text{APS} = 242$ .

In the proposed method the frequency optimization is applied up to  $\omega_p = 0.45\pi$  and in the direct method to  $\omega_p = 0.9\pi$ , as is depicted in Figure 6, where the first seven differentiator approximations are shown for both methods. This half frequency range optimization implies a notable

TABLE 1: Comparison of approximation errors for several methods.

Methods	$e_{\max}$ (dBs)	$e_{\text{rms}}$
WLS design [19]	-100.0088	$2.9107 \times 10^{-6}$
Improved WLS [24]	-100.7215	$2.7706 \times 10^{-6}$
Discretization-free [20]	-99.9208	$4.931 \times 10^{-4}$
Variable Fractional Delay [21]	-99.3669	$2.8119 \times 10^{-6}$
	-100.9278	$2.8497 \times 10^{-6}$
Direct Taylor approximation [18]	-93.69	$4.81 \times 10^{-4}$
Proposed method	-86.17	$2.78 \times 10^{-4}$

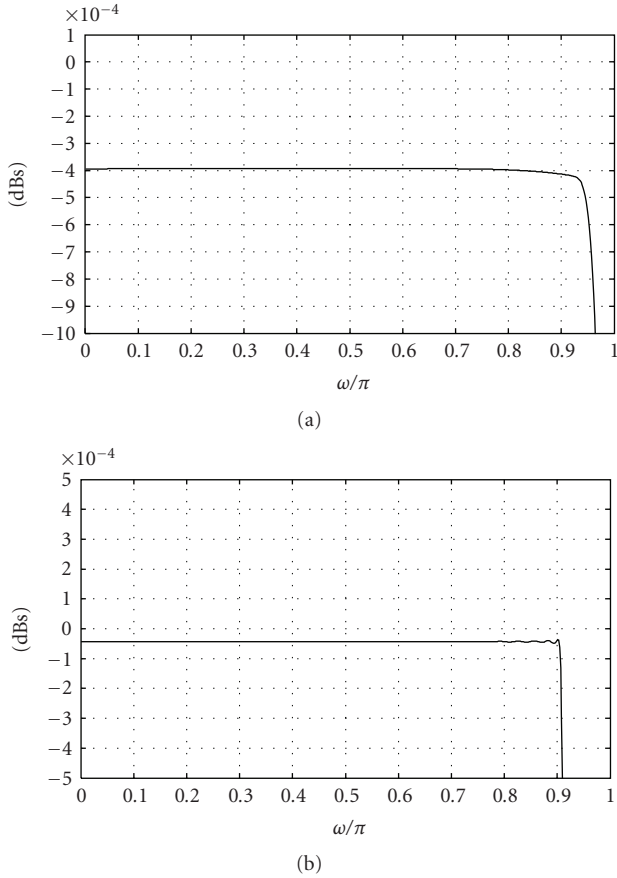


FIGURE 7: All band magnitude responses for (a) proposed method and (b) direct all band optimization.

computing workload reduction. The MATLAB computing time in a PC running at 2 GHz for the proposed method is 6.95 seconds and for direct frequency optimization of [18] is 322 seconds.

The all band magnitude responses and group delays for fractional delay values range from 0.0080 to 0.0090 using the direct frequency FD design method and the proposed method results are shown in Figures 7 and 8, respectively.

According to obtained results the proposed method has smaller number of operations per output sample. In order to compare the achieved proposed method approximation with the one obtained with existing methods, the frequency

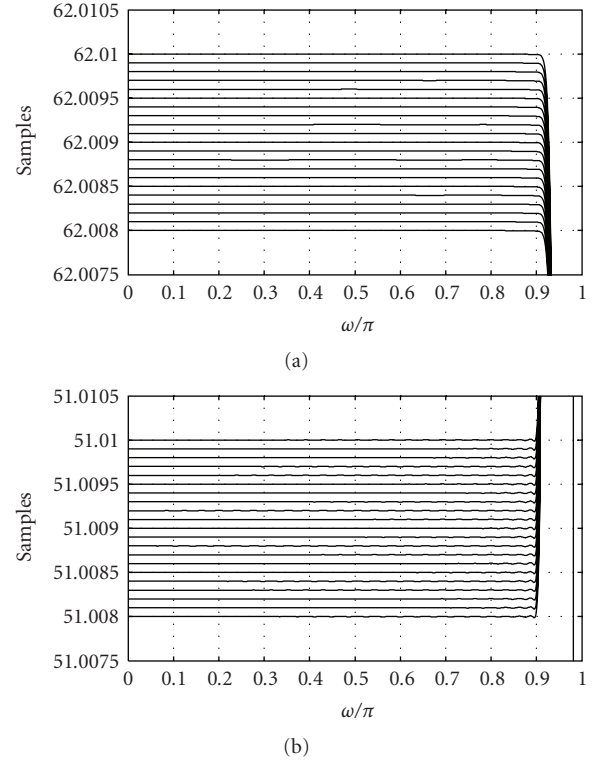


FIGURE 8: All band fractional delay responses for (a) proposed method and (b) direct all band optimization.

domain error  $e(\omega, \alpha)$ , the maximum absolute error  $e_{\max}$ , and the root mean square error  $e_{\text{rms}}$  are defined, like in [21], as

$$e(\omega, \alpha) = |H(\omega, \alpha) - H_d(\omega, \alpha)|,$$

$$e_{\max} = \max\{e(\omega, \alpha), 0 \leq \omega \leq \omega_p, 0 \leq \alpha \leq 1\}, \quad (9)$$

$$e_{\text{rms}} = \left[ \int_0^{\omega_p} \int_0^1 e^2(\omega, \alpha) d\alpha d\omega \right]^{1/2},$$

where  $H(\omega, \alpha)$  and  $H_d(\omega, \alpha)$  are the frequency response of the designed and ideal FD filters, respectively, and  $\omega_p$  is passband frequency of the FD filter.

The obtained maximum absolute error and the root mean square error are presented in Table 1; for comparison purpose the results obtained by using the approaches in [18–21, 24] are also presented.

## 6. Conclusions

A frequency optimization design approach for wide bandwidth and high fractional value resolution FD filters has been proposed. These specifications coupled with the capability of updating the fractional delay value in real-time make the resulting structure suitable to perform important physical layer functions for software-defined radio applications, such as digital symbol synchronization and sample rate conversion.

The obtained results show that the design method notably reduces the coefficients computing workload. The resulting structure allows fractional delay values of  $1/10000$  of sample and a bandwidth of  $0 \leq \omega \leq 0.9\pi$ , with a reduced number of arithmetic operations per output sample. The described method is based on a least mean square frequency optimization for coefficients computation. In a future work we will consider the use of other optimization methods in the same proposed approach.

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