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Two-stage CIC-based decimator with improved characteristics

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Abstract: A simple two-stage multiplierless cascaded–integrator–comb (CIC)-based decimator is presented. The first stage is a cascaded CIC filter whereas the second stage is a cascaded CIC filter and a second-order multiplierless compensator. The proposed decimator can be realised without filtering at high input rate by making use of the polyphase decomposition of the comb filter in the first stage. The proposed filter exhibits high aliasing attenuation and a low passband droop. The design parameters are the decimation factors, M_1 and M_2, numbers of cascaded CIC filters L and K, and parameter b of the compensator.

1 Introduction

A commonly used decimation filter is the cascaded–integrator–comb (CIC) filter [1], which performs sample rate conversion (SRC) using only additions/subtractions. It consists of two main sections: an integrator and a comb, separated by a down-sampler. Each of the main sections is a cascade of N identical filters. The transfer function of the resulting decimation filter (referred to as a comb filter) is given by

$$H_{\mathrm{CIC}}(z) = \left[\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right]^N \tag{1}$$

where M is the decimation ratio and N is the number of cascaded filters. The CIC filter is usually used at the first decimation stage, whereas a second decimator block with a decimation factor R which is smaller than that of the CIC follows the CIC filter. The factor R determines the frequency at which the worst-case aliasing occurs and also the passband edge frequency where the worst-case passband distortion occurs. For example for the case of a factor-of-R second decimation, the passband edge of interest normalised with respect to the high sampling rate is at [2]

$$\frac{\omega_c}{\pi} = \frac{\pi}{RM}\frac{1}{\pi} = \frac{1}{RM} \tag{2}$$

where M is the decimation factor of the CIC stage. Likewise the worst-case aliasing is at the frequency

$$\frac{\omega_A}{\pi} = \frac{2}{M} - \frac{1}{RM} = \frac{2R-1}{RM} \tag{3}$$

Consequently, the CIC filter must have a low passband droop in the passband determined by ω_c, and enough attenuation in the so-called folding bands, that is the bands defined around the zeros of the CIC filter

$$\left[\frac{2i}{M} - \frac{\omega_c}{\pi}; \frac{2i}{M} + \frac{\omega_c}{\pi}\right], \quad i = 1, 2, \ldots, \lfloor M/2 \rfloor \tag{4}$$

as indicated in Fig. 1. For more details and explanations see [2, 3].

However, the frequency response of the CIC filter does not satisfy the desired specifications, that is the CIC filter has a high passband droop and a low stopband attenuation. The latter can be improved by increasing the number N of the cascaded CIC filters resulting in a higher passband droop. Additionally, the integrator sections work at the higher input data rate resulting in a larger chip area and a higher power consumption especially when the decimation factor and the filter order are high. Various methods have been advanced to solve the above two problems [2, 4–18]. However, the proposed methods generally solve either the first problem or the second problem, but not both. In our

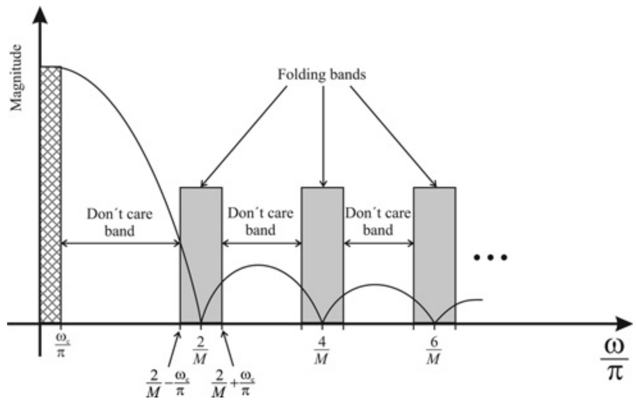

Figure 1 *Magnitude response of the CIC filter*

previous work [14] we proposed a two-stage sharpened CIC decimator structure consisting of a cascade of a CIC-based decimator and a sharpened CIC decimator. The proposed scheme allows the sharpened section to operate at a lower rate that depends on the decimation factor of the first section. Using a polyphase decomposition, the subfilters of the first section can also be operated at this lower rate.

The main idea of this paper is to modify our earlier structure [14] in order to obtain a less complex realisation that can operate at a lower sampling rate while achieving better performance. The paper is organised as follows. In Section 2 we present the modified structure and in Section 3 we discuss the choice of the design parameters. Section 4 describes the design procedure which is illustrated with an example. The last Section 5 provides the comparisons with our earlier method [14] and the original CIC filter.

2 Proposed structure

We consider here the case where $M = M_1 M_2$. We rewrite the transfer function of CIC filter as

$$H_{\mathrm{CIC}}(z) = \left[H_1(z) H_2(z^{M_1})\right]^N \tag{5}$$

where

$$\begin{aligned} H_1(z) &= \frac{1}{M_1}\frac{1 - z^{-M_1}}{1 - z^{-1}} \\ H_2(z^{M_1}) &= \frac{1}{M_2}\frac{1 - z^{-M_1 M_2}}{1 - z^{-M_1}} \end{aligned} \tag{6}$$

The corresponding magnitude responses are

$$\begin{aligned} \left|H_1(e^{j\omega})\right| &= \left|\frac{\sin(\omega M_1/2)}{M_1 \sin(\omega/2)}\right| \\ \left|H_2(e^{j\omega M_1})\right| &= \left|\frac{\sin(\omega M/2)}{M_2 \sin(\omega M_1/2)}\right| \end{aligned} \tag{7}$$

An efficient implementation of the above using the multirate identity [3] is shown in Fig. 2, where H_1 and H_2 are both

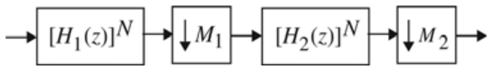

Figure 2 *Two-stage CIC filter*

CIC filters with lengths M_1 and M_2, respectively. Using a polyphase decomposition, the CIC filter of the first section can be moved to operate at a lower rate. The second section is a CIC filter of length M_2 and works at the lower rate which is M_1 times less than the high input rate. In order to arrive at the desired design parameters, we consider the general case when the numbers of cascade of the first and the second stages are different. We denote the number of the cascaded filters at the first and the second stages as L and K, respectively. The resulting decimation structure is called the modified CIC filter [14].

Next issue is to improve the characteristic of the modified CIC filter. In [14] we made use of the sharpening of the second section to arrive at much better characteristics than the equivalent CIC filter such as better alias attenuation and similar passband droop as in the sharpened comb proposed in [2]. Applying the simplest sharpening to the cascade of k comb filters $H_2(z^{M1})$, we get

$$\mathrm{Sh}\{H_2^k(z^{M1})\} = H_2^{2k}[3z^{-d} - 2H_2^k(z)] \tag{8}$$

where $\mathrm{Sh}\{H\}$ indicates the sharpening of the original filter H, and d is a delay to equalise a group delay of the second term in (8). In [14] is related the number of the cascaded comb filters at first stage, L with the number of k cascaded comb filters in (8) as

$$\frac{L}{2} \geq k \tag{9}$$

In this paper, we propose to replace the sharpening section with a simple multiplierless compensator. To this end we adopt simple multiplierless compensator filter having only three additions [7, 15],

$$G(z^M) = A[1 + Bz^{-M} + z^{-2M}] \tag{10}$$

where

$$A = -2^{-(b+2)}, \quad B = -(2^{b+2} + 2) \tag{11}$$

and b is an integer. From (10) and (11) we have the corresponding magnitude response

$$\left|G(e^{j\omega M})\right| = \left|1 + 2^{-b}\sin^2(\omega M/2)\right| \tag{12}$$

The proposed decimation filter is the cascaded modified comb filter with the compensator filter

$$H(z) = \left[H_1(z)\right]^L \left[H_2(z^{M_1})\right]^K G(z^M) \tag{13}$$

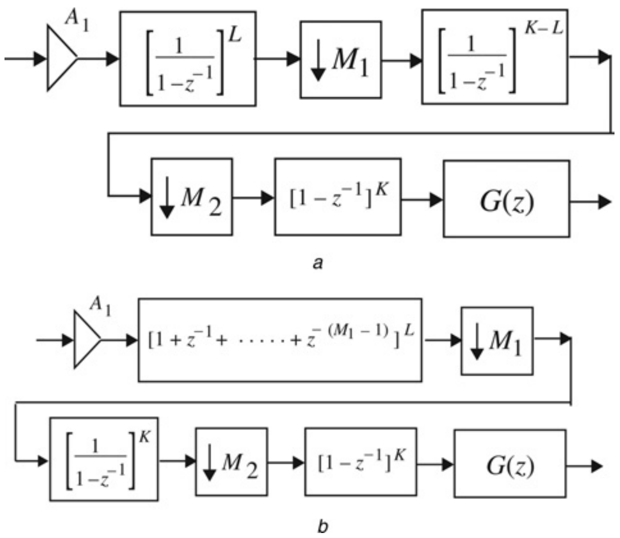

Figure 3 *Proposed structures*

From (7), (12) and (13) we have the overall magnitude response given by (see (14))

Using the multirate identity [3] and the notation

$$A_1 = \frac{A}{M_1^L M_2^K}, \quad G(z) = 1 + Bz^{-1} + z^{-2} \tag{15}$$

we arrive at two efficient multiplierless structures, depending on the realisation of the first comb filter in recursive or non-recursive form, as shown in Fig. 3.

The structure in Fig. 3*a* consists of L integrators at the high input rate, $(K - L)$ integrators operating at M_1 times lower rate, and cascaded K combs and second-order compensator operating at M times lower rate than the high input rate.

In the first stage of the structure in Fig. 3*b* there is a cascade of L comb filters in a non-recursive form followed by a factor $-M_1$ down-sampler. Applying the polyphase decomposition we can move the polyphase filters to a lower rate as explained in [3, 4]. The second section is the cascade of K CIC filters of the length M_2 followed by the compensation filter.

The design parameters are: M_1, M_2, L, K and b. Next section discusses the choice of the design parameters.

3 Choice of the design parameters

The choice of M_1 is a matter of the compromise between having less complex polyphase components in the first stage and making the filter in the second stage working at as much as possible lower rate. To this end, we propose to choose M_1 and M_2 as follows: From all possible factors of M choose two factors M_1 and M_2 that are close to each other in values with $M_1 \le M_2$.

For example for $M = 16$, we choose $M_1 = M_2 = 4$, whereas for $M = 12$ we choose $M_1 = 3$ and $M_2 = 4$.

In the ideal case where $M_1 = M_2$, we give the same load to two opposite conditions: to have a low complexity of the polyphase components and to move the second comb stage to as much possible lower rate. If this condition is not possible to satisfy like in $M = 12$ we choose the factors that are close to each other in values, for example, 3 and 4 but not 2 and 6. In that case the condition $M_1 \le M_2$ means that we prefer a slightly low complexity of the polyphase components than the lower rate of the second comb stage.

Next issue is the choice of L and K for a given M_1. The idea is to increase the number of the cascaded filters at the lower rate without increasing the complexity of the original comb filter, while improving its stopband characteristic. To this end, we use the result from [14], that is $L \ge 2k$. There is the cascade of $3k$ comb filters $H_2(z)$ [see (8)] in the structure [14], and in the proposed structure there is K cascaded comb filters $H_2(z)$. Therefore it holds the relation $K = 3k$, resulting in

$$L = \left\lceil \frac{2}{3} K \right\rceil \tag{16}$$

where $\lceil x \rceil$ denotes rounding of x to the nearest higher integer.

As an example Fig. 4 presents the gain responses of the modified comb filter for $M_1 = 3$ and $M_2 = 4$, $L = 2$ and $K = 3$, along with that of the corresponding comb filter with $M = 12$ and $N = 2$. Note that the modified comb filter has better alias rejection keeping the same complexity as the original comb filter. However the passband of interest has a higher passband droop.

In the following, we consider the choice of the parameter b for passband improvement. We consider four values of b: -1, 0, 1 and 2. Fig. 5 shows the passband droops for $R = 8$ and $L = 2$ and $K = 3$ for $M_1 = M_2 = 4$ (Fig. 5*a*), and for $M_1 = 3$, $M_2 = 4$ (Fig. 5*b*).

Similarly, Fig. 6 shows the passband details for the same values of $M_1 = M_2 = 4$, but two different values of K. In Fig. 6*a*, $L = 2$ and $K = 4$; whereas in Fig. 6*b*, $L = 5$ and $K = 7$.

$$|H(e^{j\omega})| = \left| \left[\frac{\sin(\omega M_1/2)}{M_1 \sin(\omega/2)}\right]^L \left[\frac{\sin(\omega M/2)}{M_2 \sin(\omega M_1/2)}\right]^K [1 + 2^{-b} \sin^2(\omega M/2)] \right| \tag{14}$$

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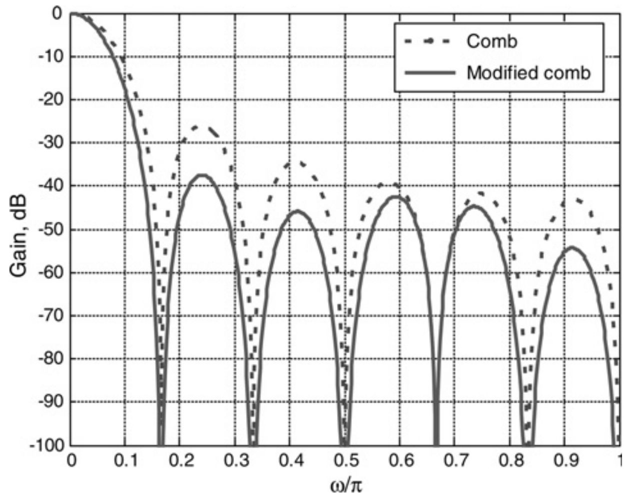
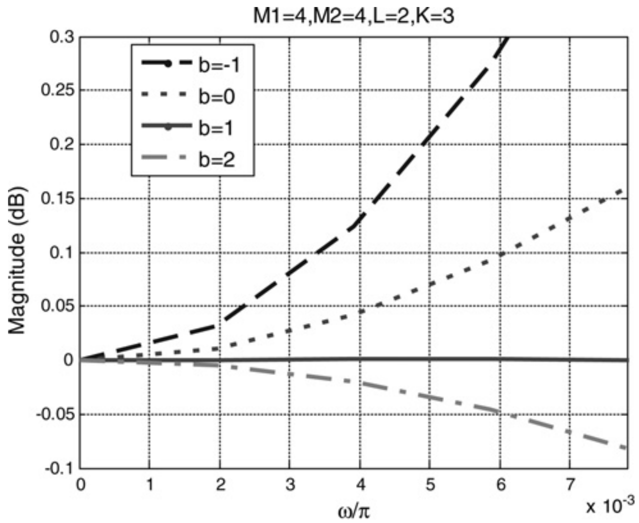
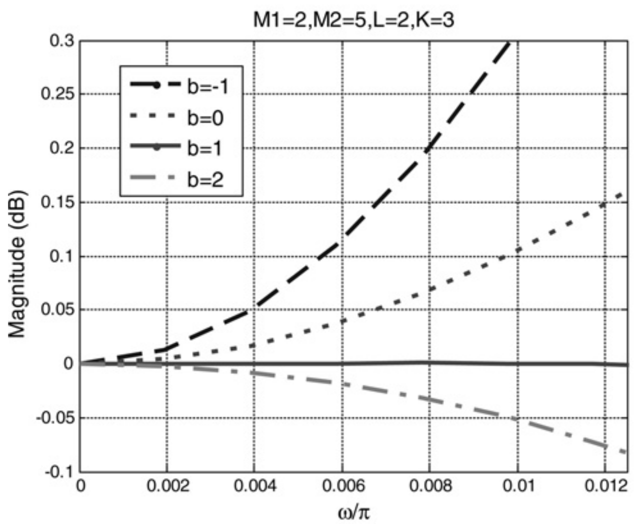


Figure 4 *Gain responses of comb ($M = 12$ and $N = 2$) and modified comb ($M_1 = 3$, $M_2 = 4$, $L = 2$ and $K = 3$)*



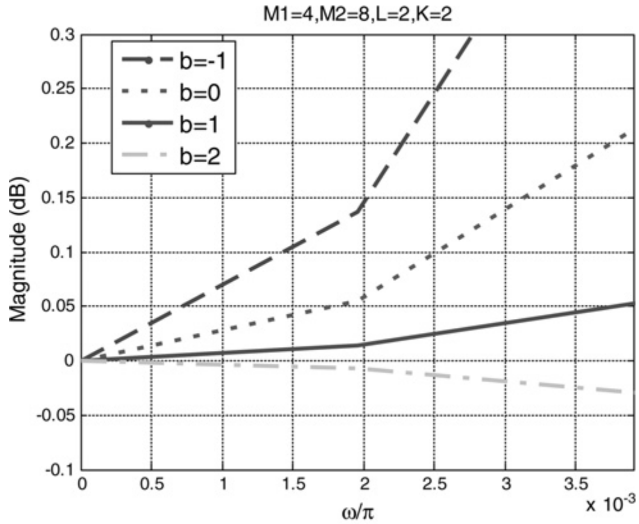
a



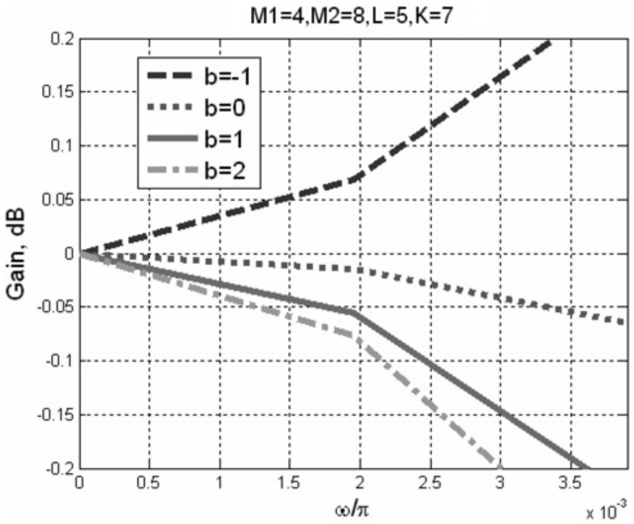
b

Figure 5 *Passband details for $L = 2$, $K = 3$ and two different values of M*

a $M_1 = M_2 = 4$
b $M_1 = 3$ and $M_2 = 4$



a



b

Figure 6 *Passband details for $M_1 = 4$, $M_2 = 8$ and two different values of K*

a $L = 2$ and $K = 2$
b $L = 5$ and $K = 7$

Table 1 The values of parameter b for $R = 8$

Parameters *K, L*	Parameter *b*
2, 2	2
3, 2	1
4, 3	1
5, 4	0
6, 4	0
7, 5	0
8, 6	0
9, 6	−1

Table 2 Passband droops for different values of M for $R = 8$

Parameters K, L	Parameter b	$M = 20$, $M_1 = 4$, $M_2 = 5$, passband	$M = 32$, $M_1 = 4$, $M_2 = 8$, droop	$M = 64$, $M_1 = 8$, $M_2 = 8$, (dB)
2, 2	2	−0.0292	−0.0294	−0.0295
3, 2	1	−0.0014	−0.0029	−0.003
4, 3	1	−0.0571	−0.0588	−0.0589
5, 4	0	0.0478	0.0461	0.0460
6, 4	0	−0.0058	−0.00089	−0.0091
7, 5	0	−0.0616	−0.0647	−0.0649
8, 6	0	−0.1173	−0.1205	−0.1208
9, 6	−1	0.14	0.1372	0.1370

Table 3 The values of worst-case aliasing rejections for different values of M and b

M	M_1	A (dB), $b = 2$	$b = 1$	$b = 0$	$b = -1$
16	4	−69.75	−72.57	−69.51	−69.19
32	4	−70.42	−70.34	−70.18	−69.87
64	8	−70.4	−70.36	−70.20	−69.89

We make the following observations:

1. For a given second-stage decimation factor R and the parameter K, the passband droop depends on the value b. For each value of K there is a particular value of b for which the passband droop is smallest. For example, $b = 1$ in Fig. 5, and $b = 2$ and 0 in Figs. 6a and b, respectively. The values of b for $R = 8$ and different values of K and L are given in Table 1.

For the chosen value b from Table 1, the passband droop does not change significantly for different values of M as shown in Table 2 for $M = 20$ ($M_1 = 4$, $M_2 = 5$), $M = 32$ ($M_1 = 4$, $M_2 = 8$) and for $M = 64$ ($M_1 = M_2 = 8$). Minimum passband droop is achieved for $K = 3$ and $b = 1$, shown in Table 2.

Next we consider the worst-case aliasing situation. We present the values of the worst-case aliasing rejections in Table 3, for $R = 8$ and values of $b = -1$, 0, 1 and 2, for $M = 16$, 32 and 64, using $L = 2$ and $K = 3$. Worst-case aliasing rejections for various values K and L, and $M_1 = M_2 = 4$ are given in Table 4.

Note the following:

The worst-case aliasing rejection does not depend significantly on the choice of the parameters b and M_1.

Table 4 Typical worst-case aliasing rejections for different values of K

Parameters K, L	A(dB)
2, 2	−46.5
3, 2	−68.75
4, 3	−92.25
5, 4	−115
6, 4	−139.34
7, 5	−160
8, 6	−184.186
9, 6	−205.75

The alias rejection is increased with an increase in the value of K.

4 Design procedure

Based on the results of Section 3 the design procedure is as follows:

For a given M choose the value M_1.

Choose the values K and L depending of the desired alias rejection (see Table 4 for tentative values).

For given K and L, choose value of b according to Table 1.

The method is illustrated in the following example.

Example 1: We design a CIC-based decimator with at least 100 dB worst-case aliasing attenuation for $M = 20$ and $R = 8$.

For $M = 20$ we choose $M_1 = 4$ and $M_2 = 5$.

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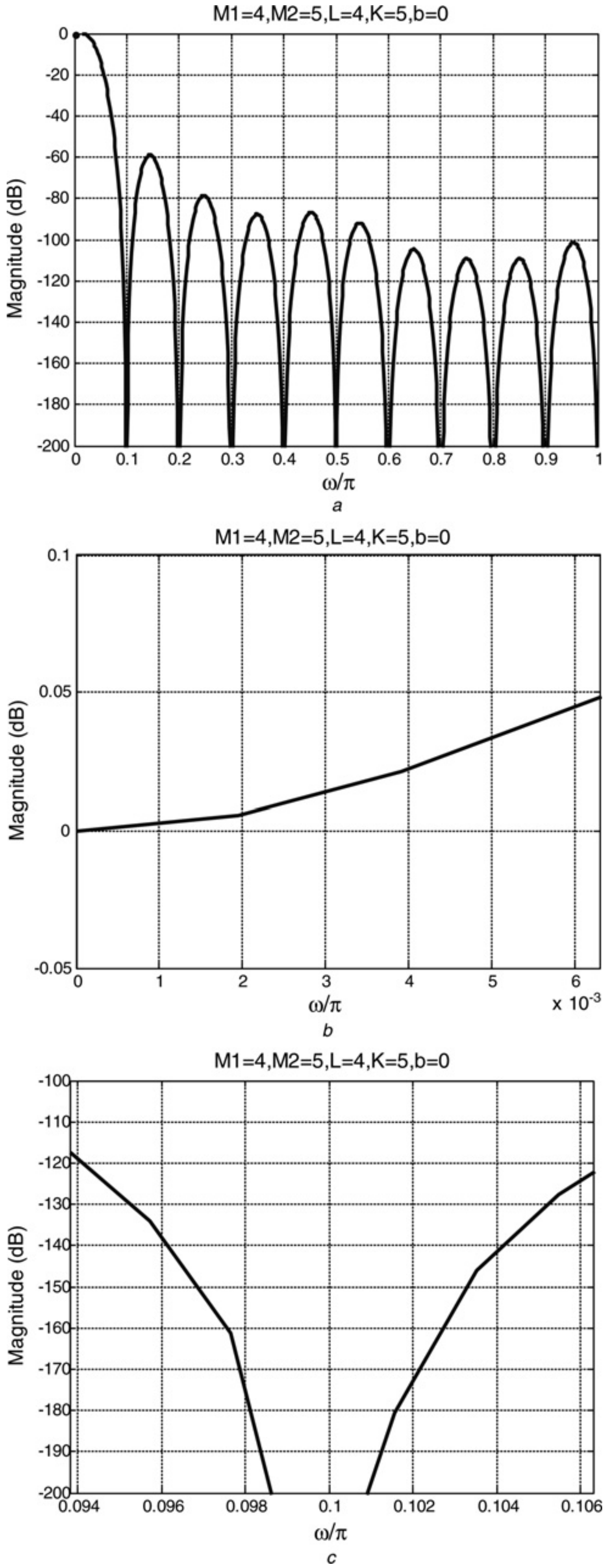

Figure 7 *Example 1*

a Overall magnitude response
b Passband zoom
c Stopband zoom

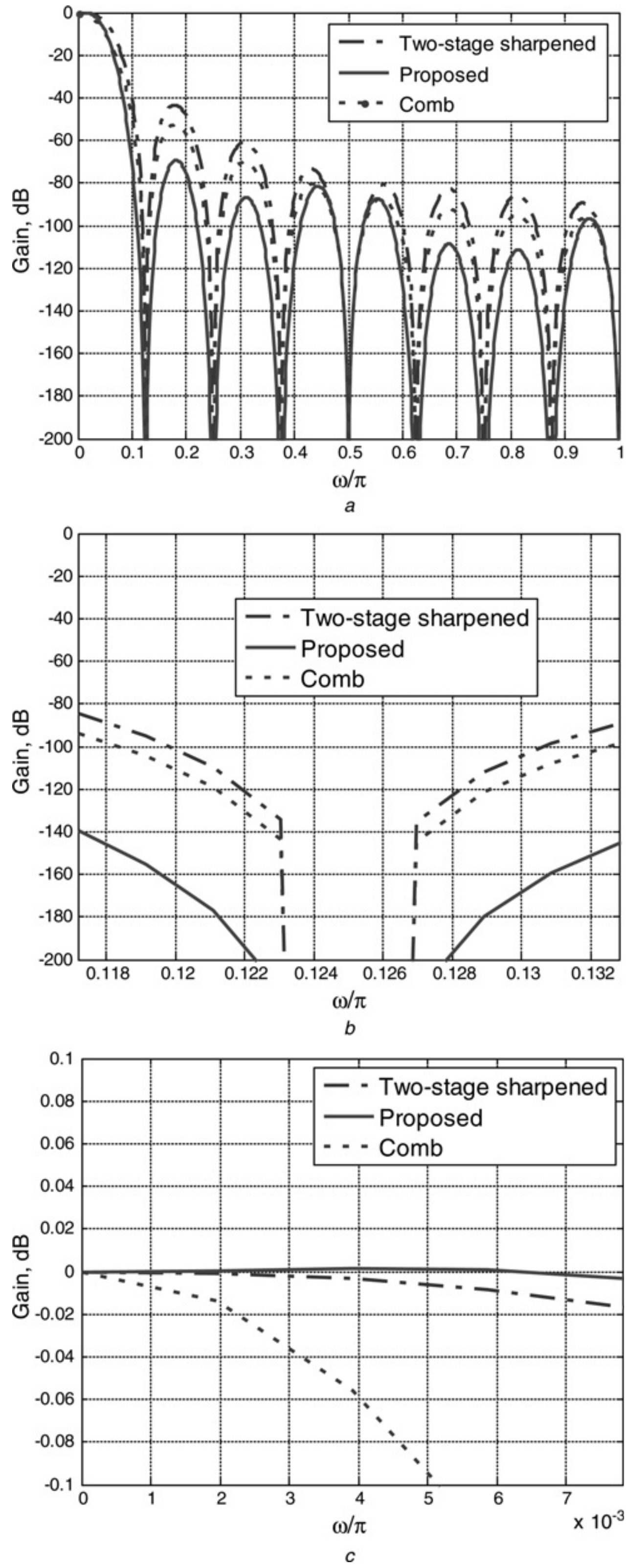

Figure 8 *Performance comparisons, $M = 16$, $R = 8$ (proposed: $M_1 = 4$, $M_2 = 4$, $L = 4$ and $K = 6$; Comb: $M = 16$ and $N = 4$; two-stage sharpened [14]: $M_1 = 4$, $M_2 = 4$, $L = 4$ and $k = 2$)*

a Overall gain responses
b Stopband details
c Passband details

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From Table 4 we have $K = 5$ and $L = 4$.

From Table 1 we have $b = 0$.

The magnitude response of the designed decimator is given in Fig. 7 along with the passband and stopband details. The worst-case alias rejection is 118 dB, while the passband deviation is 0.0478 dB.

5 Discussion of results

In this section we compare the performance of the decimator designed using the proposed method with that designed using the method outlined in [14]. We consider the decimator with $M = 16$. We choose $M_1 = 4$, $M_2 = 4$, $L = 4$ and $K = 6$. We compare the proposed design with the method of [14] where the sharpening polynomial $(3H^{2k} - 2H^{3k})$ is applied to the second stage and the cascade of two comb filters ($k = 2$) [see (8)].

Fig. 8 shows the gain responses of the designed decimator along with that of two-stage sharpened comb decimator from [14] and original comb filter with $M = 16$ and $N = 4$ for $R = 8$.

Table 5 compares the values of the worst-case passband deviations and worst-case aliasing attenuations of all three designs. Note that the proposed filter provides best worst-case alias rejection as well as the smallest passband droop.

The complexities of the proposed filter and the filter from [14] along with the conventional CIC can be compared in terms of their memory requirements and number of additions (or subtractions) per output sample (APOS) as shown in Table 6.

The proposed filter exhibits less complexity compared with the filter from [14]. The proposed structure of Fig. 3*a* and conventional CIC have similar complexity for $N = 5$ whereas the proposed structure is less complex than CIC for $N = 6$. Note that in the proposed structure there are four integrators at high input rate compared with 5 and 6, respectively, for the CIC with $N = 5$ and 6. The structure of Fig. 3*b* with the polyphase decomposition does not have the filtering at the high input rate and requires a similar number of APOS and twice the number of memory elements compared with that of the CIC with $N = 6$.

Table 5 Characteristics comparisons

Filter	Passband droop (dB)	Alias attenuations (dB)
proposed $M_1 = M_2 = 4$ $L = 4$, $K = 6$, $b = 0$	−0.003	−139.3463
method [14] $M_1 = M_2 = 4$ $L = 4$, $k = 2$	−0.0167	−84.66
CIC $M = 16$ $N = 4$	−0.227	−94.11
$N = 5$	−0.2783	−117.64
$N = 6$	−0.334	−141.17

Table 6 Complexity comparisons

Filter	Memory requirements	APOS
proposed $M_1 = M_2 = 4$ $L = 4$ $K = 6$ $b = 0$ structure 3(a)	14	81
structure 3(b)	23	101
method [14] $M_1 = M_2 = 4$ $L = 4$, $k = 2$	30	148
CIC $M = 16$ $N = 4$	8	68
$N = 5$	10	85
$N = 6$	12	102

6 Conclusions

We presented a simple method to improve the passband and stopband characteristics of the CIC decimation filter. The only restriction is that the decimation factor can be expressed as a factor of two integers $M = M_1M_2$. The proposed filter is compared with our earlier method [14] because this method provides no filtering at the high input rate while improving both passband and stopband responses. The decimator designed using the proposed method is also multiplierless and requires less APOS and memory elements than the method in [14]. Additionally, the passband and the stopband characteristics are better. The analysis in this paper is given for the second decimation factor $R = 8$. Similar analysis can be performed for another value of the second decimation factor R. However the recommended value of the second decimation factor R has to be more or equal to 8. Otherwise the compensation in the passband will not be satisfactory because the passband edge of interest according to (2) will be increased.

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