Compensated CIC-Cosine Decimation Filter

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ABSTRACT

This paper presents efficient modification of the CICcosine decimation filter. The second order compensator filter is introduced at low rate in order to improve the passband of interest of the overall filter. The coefficients of the compensator filter are presented in a canonical signed digits (CSD) form, and can be implemented using only adders and shifts. Consequently, the resulting filter is a multiplierfree filter and exhibits a high attenuation in the stopband, as well as a low passband droop.

Keywords: Decimation filter, Compensator, CIC filter, Cosine filter.

1. INTRODUCTION

The process of converting the given rate of a signal into a different rate is called sampling rate conversion (SRC). The reduction of a sampling rate is called decimation, and is accomplished in two stages, filtering and downsampling. Hogenauer [1] introduced a very simple decimation filter called cascaded integrator comb (CIC) filter, which consists of cascaded integrators and differentiators separated by a downsampler, as shown in Fig. 1. The transfer function of the resulting decimation filter is given as

$$H(z) = \left[\frac{1}{M}\left(\frac{1-z^{-M}}{1-z^{-1}}\right)\right]^{K}. \qquad (1)$$

where M is the decimation factor and K represents the number of cascaded filters.

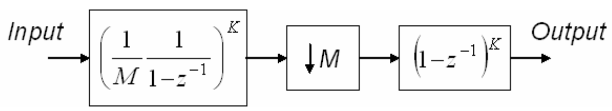

Fig.1: *CIC decimation filter.*

The above decimation filter is attractive in many applications because of its very low complexity. It is usually used at the first stage of decimation [1]. However, this decimation filter exhibits several drawbacks. The integrator section works at the high input data rate resulting in a larger chip area and higher power consumption especially when the decimation factor is high. In order to resolve this problem the non-recursive structure of (1) can be used [2], [3]. More details on a comparison of the performances of the recursive and nonrecursive implementations are given in [2]. Additionally, the magnitude response of the CIC filter has a high droop in the desired passband and low stopband attenuation. The decimation factor R of the next decimation stage determines the frequency ω_c, where the worst passband distortion occurs, and the frequency ω_A, where the worst case aliasing occurs [4]. The normalized frequencies ω_c/π and ω_A/π are at

$$\frac{\omega_c}{\pi} = \frac{\pi}{MR}.\frac{1}{\pi} = \frac{1}{MR} \quad \frac{\omega_{AI}}{\pi} = \frac{2}{M} - \frac{1}{MR} = \frac{2R\text{-}1}{MR}. \qquad (2)$$

Several structures have been proposed to design CIC filters with improved frequency response as for example [4-11]. Usually methods deal to improve either passband [5-6], or stopband [7-10].

The goal of this work is to propose a simple multiplierless decimation filter with no filtering at high input rate, and with a desired high stopband attenuation and a desired low passband droop. To this end we propose to compensate the passband droop of the cascaded CIC-cosine decimation filter [10], introducing a simple compensator at the low rate. The rest of the paper is organized as follows. In Section 2 we propose the new second order compensator filter which is illustrated with two examples. Section 3 briefly introduces the cascaded CIC-cosine decimation filter from [10]. The proposed modified compensated CIC-cosine decimation filter is described in section 3, and illustrated with one example.

2. PROPOSED COMPENSATION FILTER

The transfer function of the proposed compensation filter is given by

$$H_{COMP}\left(z^{M}\right) = a + bz^{-M} + az^{-2M}. \qquad (3)$$

where a and b are real valued constants, and M is decimation factor. Using a multirate identity this filter can be moved to a low rate which is M times less than high input rate becoming a second order filter. The magnitude response of (3) is expressed as

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$$\left|H_{COMP}\left(e^{jM\omega}\right)\right| = |2a\cos(M\omega) + b|\,. \qquad (4)$$

Next issue is to define the coefficients a and b. To this end we consider magnitude response of Eq. (4) in two frequency points ω=0, and $\omega = \omega_c$, where the worst passband distortion occurs, defined in Eq. (2). In ω=0 the desired magnitude characteristic has the value 1, resulting in the following equation

$$2a + b = 1. \qquad (5)$$

In order to compensate the passband droop δ_c at the frequency ω_c, we have from Eq. (4)

$$2a\cos(M\omega_c) + b = 1/\delta_c. \qquad (6)$$

Denoting

$$\delta_{comp} = 1/\delta_c. \qquad (7)$$

we have

$$\begin{bmatrix} 2 & 1 \\ 2\cos(M_c) & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \delta_{comp} \end{bmatrix}. \qquad (8)$$

Solving (8) we get the coefficients a and b as

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{2(\cos(M\omega_c)-1)} & \dfrac{1}{2(\cos(M\omega_c)-1)} \\ \dfrac{\cos(M\omega_c)}{\cos(M\omega_c)-1} & -\dfrac{1}{\cos(M\omega_c)-1} \end{bmatrix} \begin{bmatrix} 1 \\ \delta_{comp} \end{bmatrix}. \qquad (9)$$

In following, the coefficients a and b from (9) are quantized as shown in the next relations

$$a_q = 2^{-k}\left\lfloor a/2^{-k}\right\rfloor \quad b_q = 2^{-k}\left\lfloor b/2^{-k}\right\rfloor. \qquad (10)$$

the integer part of x. The relation (5) must also hold for the quantized values of a and b resulting in the condition

$$2a_q + b_q = 1. \qquad (11)$$

Starting with k=2 the values of k are increased until the condition (11) is satisfied. If the passband droop is within the desired limit, the corresponding quantized coefficients a_q and b_q are presented in canonical signed digit (CSD) form [12-15].

$$H_{COMP\text{-}CSD}\left(z^M\right) = a_{CSD} + b_{CSD}z^{-M} + a_{CSD}z^{-2M}. \qquad (12)$$

where a_{CSD}, and b_{CSD} are the CSD representations of the quantized coefficients a_q, b_q of the proposed compensation filter (3).

Otherwise, the procedure is continued until the desired passband compensation is obtained. This procedure is programmed in MATLAB, and the corresponding block diagram is shown in Fig. 2.

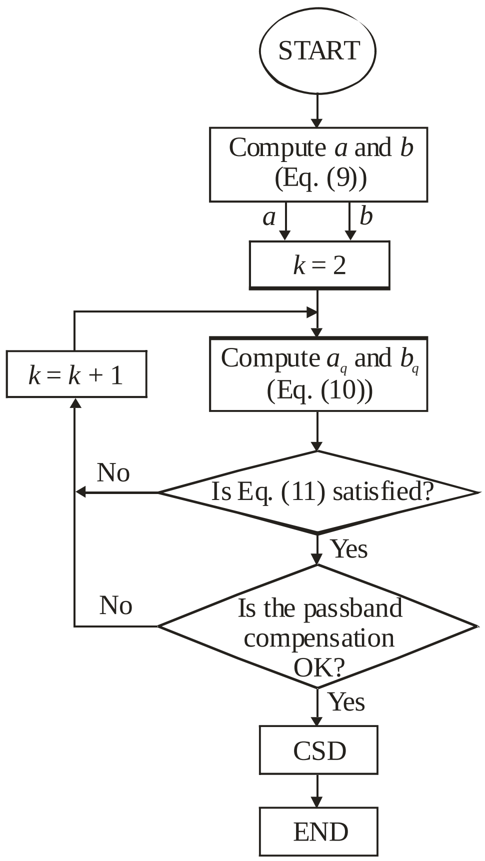


Fig.2: *Block diagram of the compensator filter design.*

The method is illustrated in the following example.

Example 1: Consider the design of compensator for cascaded CIC decimation filter with M=4, K=4, and decimation factor of second stage is R= 8. The passband droop of CIC filter at normalized frequency ω_c/π is 0.2095792dB.

However, we want to have a resulting passband droop less than 0.01dB.

From Eq. (7), it follows

$$\delta_{comp} = 1.02442214823189. \tag{13}$$

From (9) we have

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -0.16041774990093 \\ 1.32083549980185 \end{bmatrix}. \tag{14}$$

Starting with k=2 we find the quantized values shown in Table I.

The condition (11) is satisfied only for the values of k=3 and k=5, shown bold in Table I.

However, for k=3 the passband droop does not satisfy the desired specifications. For k =5 the desired passband specification is satisfied. Therefore using k =5 we find the corresponding CSD terms for quantized coefficients and the transfer function is given by

$$H_{COMP\text{-}CSD}\left(z^4\right) = \left(-2^{-3} - 2^{-5}\right)\left(1 + z^{-8}\right) + \left(2^0 + 2^{-2} + 2^{-4}\right) z^{-4}. \tag{15}$$

The passband details of the CIC and the proposed compensated CIC filters are shown in Fig. 3.

There exists a trade-off between the desired passband droop and the number of the CSD terms of coefficients a and b.

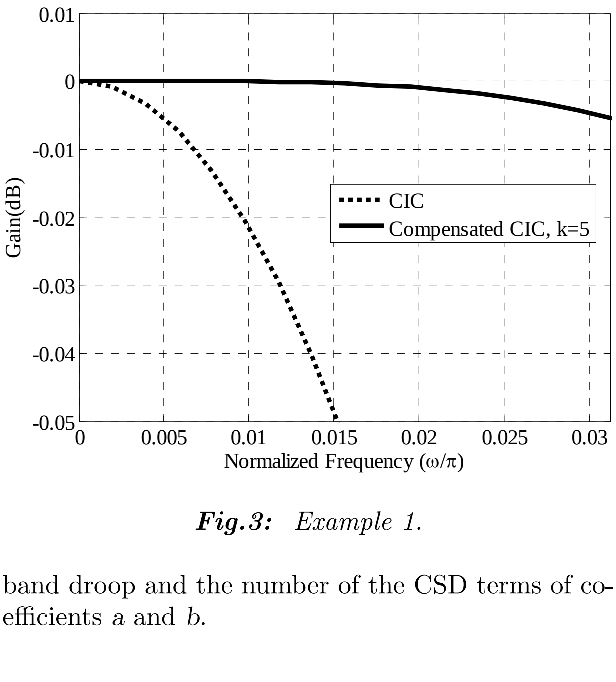


Fig.3: *Example 1.*

Next example compares the proposed compensation filter with that recently proposed in literature [5].

Example 2: Consider the design of a CIC decimation filter for the design parameters M=11, K=5, [5].

From [5], the normalized passband and stopband frequencies of interest are ω_c/π=0.035455 and ω_{AI}/π=0.146364. The corresponding passband droop and stopband attenuation of the CIC filter are -2.7288dB and A_{AI}=-63.9441dB, respectively.

The compensation filter from [5] has the corresponding passband droop 0.01597dB, and stopband attenuation - 61.199338dB, respectively.

Using the proposed compensator for the quantized factor k=8, we got the passband droop of -0.01475dB and the stopband attenuation of -61.215311dB. The compensation filter with the corresponding CSD terms of quantized coefficients is given by

$$H_{COMP\text{-}CSD}\left(z^{11}\right) = \left(-2^{-2} - 2^{-5} + 2^{-8}\right)\left(1 + z^{-22}\right) + \left(2^1 - 2^{-1} + 2^{-4} - 2^{-7}\right) z^{-11}. \tag{16}$$

The passband details for CIC filter, the proposed compensated CIC, and compensated CIC from [5] are given in Fig.4.

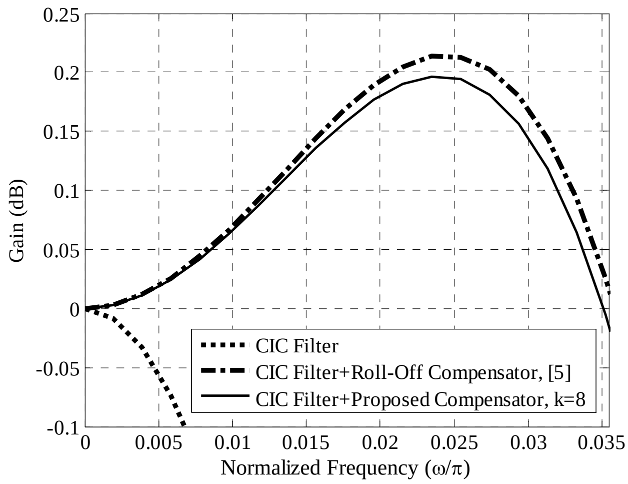


Fig.4: *Example 2.*

Note that the proposed compensator results in a slightly better passband characteristic.

3. CIC-COSINE DECIMATION FILTER

Considering the case when the down-sampling factor can be expressed as

$$M = M_1 M_2 M_3 ... M_N. \tag{17}$$

we can rewrite Eq. (1) as

$$H(z) = \left[\prod_{i=1}^{N} H_i \left(z^{\prod_{j=0}^{i-1} M_j} \right) \right]^K . \quad (18)$$

$$H_i\left(z^{\prod_{j=0}^{i-1} M_j}\right) = \frac{1}{M_i}\left(\frac{1-z^{-\prod_{j=1}^{i} M_j}}{1-z^{-\prod_{j=0}^{i-1} M_j}}\right);\ M_0 = 1 . \quad (19)$$

Using Eqs. (18) and (19) we express the modified CIC filter Hm(z) as, [10]

$$H_m(z) = H_1^{k_1}(z) .H_2^{k_2}\left(z^{M_1}\right) ...H_N^{k_N}\left(z^{M_1...M_{N-1}}\right) . \quad (20)$$

where ki is the number of the cascaded filters Hi.

We use the cosine prefilters introduced in [16] to improve the frequency characteristic of the modified CIC filter (20).

$$H_{CCOS}(z) = \prod_{i=1}^{K} K_{COS}\left(z^{N^i}\right) . \quad (21)$$

where [16].

$$H_{COS}\left(z^N\right) = 0.125\left(1 + z^{-2N}\right)\left(1 + z^{-N}\right)^2 . \quad (22)$$

In general we have [10]

$$N_i = \frac{M}{2^{i+1}} . \quad (23)$$

The transfer function of the cascaded modified CIC and cosine prefilters is

$$H_{mCCOS}(z) = H_m(z) \prod_{i=1}^{K} H_{COS}^{n_i}\left(z^{N_i}\right) . \quad (24)$$

where n_i is the number of cascaded cosine prefilters, and $N_K = M_1$.

For more details see [10].

4. PROPOSED DECIMATION FILTER

The proposed filter is the cascade of the modified CICcosine filter $H_{mCCOS}(z)$ [10] and the compensator filter introduced in Section 2.

$$G(z) = H_{mCCOS}(z) H_{COMP\text{-}CSD}\left(z^M\right) . \quad (25)$$

At first we design the modified CIC-cosine filter $H_{mCCOS}(z)$ [10] in order to satisfy the desired stopband attenuation. Next, we find the passband droop δ_c in the frequency of interest $\omega = \omega_c$ of the designed modified CIC-cosine filter. Using (7) we compute δ_{comp} and solving equations (8) we get the coefficients a and b of the compensator. Finally, the coefficients a and b are expressed in CSD form as explained in Section 2.

The method is illustrated in Example 3.

Example 3: We design decimation filter for M=32, which has passband droop at the frequency of interest less than 0.002dB, and stopband attenuation in the frequency of interest at least of 110 dB. First we design CIC-cosine filter choosing M_1=2, M_2=4 and M_3=4 yielding

$$H_1(z) = \frac{1}{2}\left(\frac{1-z^{-2}}{1-z^{-1}}\right),\ H_2\left(z^2\right) = \frac{1}{4}\left(\frac{1-z^{-8}}{1-z^{-2}}\right),$$
$$H_3\left(z^8\right) = \frac{1}{4}\left(\frac{1-z^{-32}}{1-z^{-8}}\right) . \quad (26)$$

In the next, we cascade filters $H_1(z)$, $H_2(z_2)$ and $H_3(z_8)$ as shown in the following equation

$$H_m(z) = H_1^4(z) H_2^2\left(z^2\right) H_3^2\left(z^8\right) . \quad (27)$$

The stopband attenuation of this filter is further improved introducing cascade of the expanded cosine filters

$$H_{CCOS}(z) = H_{COS}^2\left(z^8\right) H_{COS}^4\left(z^4\right) H_{COS}^4\left(z^2\right) . \quad (28)$$

where

$$H_{COS}\left(z^N\right) = 0.125\left(1 + z^{-2N}\right)\left(1 + z^{-N}\right)^2 . \quad (29)$$

From (2) using R=8 the passband and stopband frequencies of interest are

$$\omega_c/\pi = 0.00390625$$
$$\omega_A/\pi = 0.05859375. \quad (30)$$

Stopband attenuation in the frequency of interest is A_{Al}=-116.7993dB, i.e., it is satisfied. However the passband droop is -0.3162dB. In order to improve passband, we design compensation filter as described in Section 2.

Solving (9) we get the coefficients

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -0.2435389959475 \\ 1.487077991895 \end{bmatrix} . \quad (31)$$

Table II shows the corresponding quantized values a_q and b_q. The condition (11) is satisfied only for

Table 1: *Quantized Factors and Coefficients.*

k	a_q	b_q
2	0	1.25
3	-0.125	1.375
4	-0.1875	1.4375
5	-0.21875	1.46875
6	-0.234375	1.484375
7	**-0.2421875**	**1.484375**

k=7 resulting in the corresponding passband droop of -0.001723dB, which satisfies the desired specification. Consequently, we present quantized coefficients for k=7 in CSD terms

Using multirate identities we have efficient structure shown in Fig. 5.

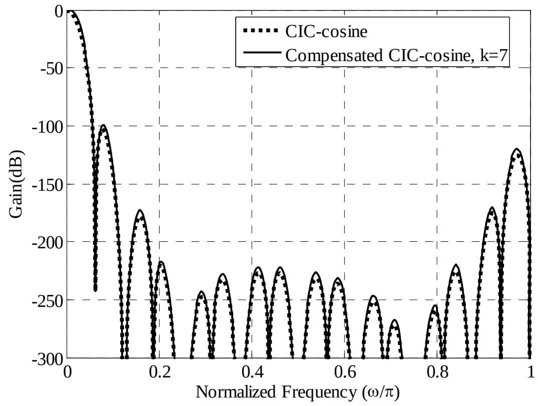
Overall magnitude response is shown in Fig. 6.a along with that of the CIC-cosine filter.

Corresponding passband zoom is shown in Fig. 6.b.

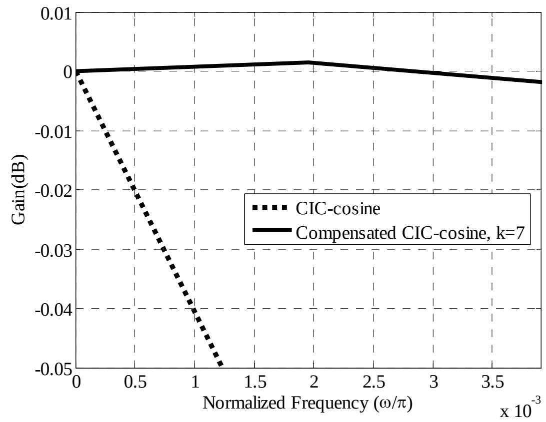
Fig. 6.c illustrates attenuation in the neighborhood of the first zero.

5. CONCLUSIONS

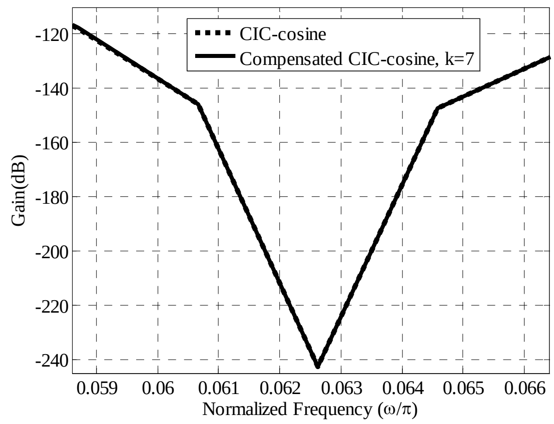
We proposed here the multistage decimation filter based on the CIC-cosine decimation filter and the symmetric second order compensator. The coefficients of the compensator are obtained solving two simple equations, and expressed in CSD terms which can be implemented using only adders and shifts. As a result the proposed filter is a multiplier free filter. There is a trade-off between the desired compensation of the passband droop, and the complexity of the CSD representation. Number of adders may be further reduced using sub-expression elimination (CSE) for the CSD represented coefficients [15]. In contrast to methods [5-6], the proposed compensation filter design is simpler, and the filter coefficients can control the desired passband droop of the overall decimation filter. The overall filter exhibits a low passband droop, and high stopband attenuation at the frequencies of interest. Additionally, using the polyphase decomposition, the filters at the first stage can be moved at the lower rate. Therefore, there is no filtering at high input rate.



a. Overall gain responses.



b. Passband zooms.



c. *Stopbands.*

Fig.6: *Example 3.*

5.1 ACKNOWLEDGEMENT

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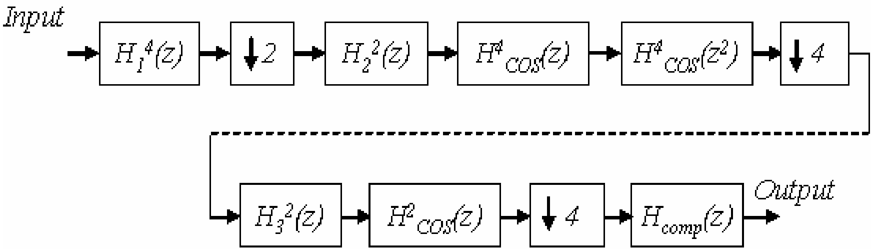

Fig.5: *Efficient structure for Example 3.*

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