Modified CIC Filter for Rational Sample Rate Conversion

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***Abstract*— The modification of the conventional CIC (cascaded–integrator-comb) filter for rational sample rate conversion (SRC) in software radio (SWR) systems is presented here. The conversion factor is a ratio of two mutually prime numbers, where the decimation factor *M* can be expressed as a product of two integers. The overall filter realization is based on a stepped triangular form of the CIC impulse response, the corresponding expanded cosine filter, and sine-based compensation filter. This filter performs sampling rate conversion efficiently by using only additions/subtractions making it attractive for software radio (SWR) applications.**

I. INTRODUCTION

The transfer function of the cascaded-integrator-comb (CIC) filter for sampling rate conversion by a factor *M/L,* shown in Fig. 1, is given by [1]

$$H(z)=\frac{(1-z^{-DM})^{K_1}(1-z^{-DL})^{K_2}}{(1-z^{-1})^{K}}, \tag{1}$$

where D is the delay of each comb stage and $K=K_1+K_2$. This filter is very simple and uses only additions/subtractions. However, it has a limited number of tuning parameters, has an high passband droop, and does not provide enough attenuation in the region of interest in the stopband. Various methods have been proposed to improve the characteristics of the conventional CIC filter to make them suitable for software radio (SWR) applications, [2]-[8]. In [2] is proposed a modified CIC filter of the form

$$H_m(z)=\frac{(1-z^{-D_1})(1-z^{-D_2})\cdots(1-z^{-D_N})}{(1-z^{-1})^{K}}, \tag{2}$$

where $D_1, D_2, \ldots, D_N$ is a set of comb delays. It has been shown that the modified CIC filter provides higher image attenuation than the conventional CIC filter. Additionally, the modified CIC filter provides higher SNR, where the SNR is defined as the power ratio after lowpass filtering of the lowest power level in the desired signal to the highest power level in the images [2]. In [8] is presented one alternative modification of the CIC filter based on stepped triangular form of the CIC impulse response and expanded cosine filters. The filter exhibits lower complexity and better performances than the modified CIC filter introduced in [2]. The main objective of this work is to propose an alternate modification of the CIC filter from [8] in order to achieve a higher attenuation of the images and an improved SNR than in [8]. Specially, we propose to cascade the simple second order compensation multiplierless filter to the modified CIC filter from [8]. The rest of the paper is organized in the following way. Next section introduces the stepped triangular (ST) and cosine based CIC filter. Next section presents the compensation filter. The proposed structure is given in Section 4 and illustrated with one example.

II. ST AND COSINE-BASED CIC FILTER

We develop next the relation between the stepped triangular impulse response filter and the CIC filter of the order *M*,

$$G(z)=\frac{1}{M}\frac{(1-z^{-M})}{(1-z^{-1})}. \tag{3}$$

In general, the transfer function of ST CIC filter is given by

$$H_{ST}(z)=\left(\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right)\left(\frac{1}{N_2}\frac{1-z^{-N_1N_2}}{1-z^{-N_1}}\right). \tag{4}$$

Note that $N_{1,2}=N_1N_2$ can be either equal to or different than M. We consider the case where the decimation factor M is a product of two integers, i.e.,

$$M=M_1M_2. \tag{5}$$

Using Eq. (5), Eq. (4) can be rewritten as

$$H_{ST}(z)=\left(\frac{1}{M_2}\frac{1-z^{-M}}{1-z^{-M_1}}\right)\left(\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right)\left(\frac{1}{N_2}\frac{1-z^{-N_1N_2}}{1-z^{-N_1}}\right). \tag{6}$$

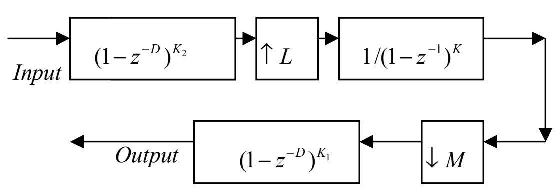

Fig. 1. Conventional CIC filter for STC by *L*/*M*.

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In [8] is proposed to have

$$H_{ST}(z)=\begin{cases}\dfrac{1}{M_2}\dfrac{1-z^{-M}}{1-z^{-M_1}}\left[\dfrac{1}{M_1}\dfrac{1-z^{-M_1}}{1-z^{-1}}\right]^2 & \text{for } M_1>M_2,\\ \left[\dfrac{1}{M_2}\dfrac{1-z^{-M}}{1-z^{-M_1}}\right]^2\dfrac{1}{M_1}\dfrac{1-z^{-M_1}}{1-z^{-1}} & \text{for } M_1<M_2.\end{cases} \quad (7)$$

Additionally, it is proposed to cascade an expanded cosine filter with the ST CIC filter of Eq. (7) to introduce an additional zero in the frequency band where the worst case of aliasing occurs so as to increase the SNR. The expanded cosine filter is given by [8]

$$H_{\cos 1}(z)=(1+z^{-R_1})/2 \quad (8)$$

where

$$R_{\min}\le R_1\le M \quad . \quad (9)$$

This value is computed as using

$$R_{\min}=\mathrm{int}[1/\omega_1] \quad (10)$$

where int[.] means integer part of [.] and

$$\omega_1=\frac{2}{M}-\omega_p,\omega_p=\frac{3}{4M} \quad . \quad (11)$$

Using Eqs. (7) and (8) we arrive at the transfer function of the decimation filter [8]

$$H_{STCOS}(z)=\left[\frac{1}{M_2}\frac{1-z^{-M}}{1-z^{-M_1}}\right]^{k_1} \times\left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^{k_2}\left[\frac{1+z^{-R_1}}{2}\right]^{k_3}, \quad (12)$$

where

$k_2\ge 2k_1$ for $M_1>M_2$,
$k_1\ge 2k_2$ for $M_1<M_2$.

III. COMPENSATION FILTER

Consider the filter with the magnitude response

$$\left|G(e^{j\omega M})\right|=\left|1+2^{-b}\sin^2(\omega M/2)\right|^{k_4}. \quad (13)$$

Using the well known relation

$$\sin^2\alpha=(1-\cos 2\alpha)/2, \quad (14)$$

the corresponding transfer function is given as

$$G(z^M)=B[1+Az^{-M}+z^{-2M}]^{k_4} \quad , \quad (15)$$

where
B is the scaling factor, and

$$A=-[2^{(b+2)}+2]. \quad (16)$$

Two principal characteristics of this filter are:

- The transfer function is a function of z^{-M} and using the multirate identity [9], the filter can be moved after the downsampling, and
- The compensator filter is multiplier-free and has only one coefficient A which can be realized using only additions and shifts.

Figure 2 illustrates the magnitude responses of the compensator filters for three different values of b along with the magnitude responses of the CIC filter with $M = 8$.

To decrease side lobes of the compensated CIC filter we propose to use the following cosine filter

$$H_{\cos 2}(z)=[(1+z^{-R_2})/2]^{k_5} \quad , \quad (17)$$

where

$$R_2<R_{\min} \quad . \quad (18)$$

IV. PROPOSED FILTER

We propose to cascade the STCIC filter from Eq.(12) with the compensated filter (15) followed by expanded cosine filter from Eq. (17).
Using Eqs (12), (15) and (17) we have the proposed filter

$$H_p(z)=H_{STCOS}(z)G(z^M)H_{\cos 2}(z), \quad (19)$$

or

$$H_p(z)=\left[\frac{1}{M_2}\frac{1-z^{-M}}{1-z^{-M_1}}\right]^{k_1} \times\left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^{k_2}\left[\frac{1+z^{-R_1}}{2}\right]^{k_3} \times B\left[1+Az^{-M}+z^{-2M}\right]^{k_4}\left[\frac{1+z^{-R_2}}{2}\right]^{k_5}.$$

(20)

Denoting

$$H_{p1}(z)=\left(\frac{1}{1-z^{-1}}\right)^{k_2}(1+z^{-R_1})^{k_3}(1+z^{-R_2})^{k_5}, \quad (21)$$

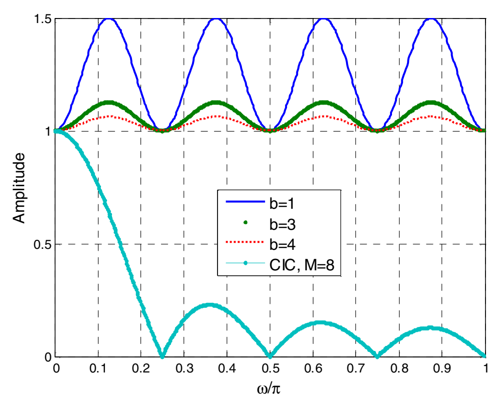


Fig.2. CIC and compensation filters.

$$H_{p2} = (1 - z^{-1})^{k_2 - k_1}, \quad (22)$$

$$H_{p3} = (1 - z^{-1})^{k_1} (1 + Az^{-1} + z^{-2})^{k_4}, \quad (23)$$

and making use of the multirate identity [9] we have the final structure shown in Fig. 3(a). The structure consists of three stages. In general, the first stage is a cascade of $k_3 + k_5$ combs with the delays R_1 and R_2, and k_2 integrators. The next stage is a cascade of $(k_2 - k_1)$ combs or integrators as indicated in Eq. (22). Finally, the last stage is a cascade of k_1 combs and a second order filter. The complexity of the proposed filter, presented in terms of their memory requirements and number of additions (or subtractions) per output sample (APOS), is given in Table I.

TABLE I.

Memory	**APOS**
$2k_2+R_1k_3+R_2k_5+2k_4$	$2k_4+k_1+\mid k_2-k_1 \mid M_2$ $+(k_3+k_2+k_5)M$

Example 1: We consider the design of a sampling rate converter for a conversion factor of 9/10 [2]. From Eqs. (10) and (11) we have

$$\omega_1 = \frac{2}{10} - \frac{3}{4} \times \frac{1}{10} = 0.125$$

and $R_{\min} = 8$. We choose $M_1 = 2$ and $M_2 = 5$, and R_1= 8, and R_2=4. Using $k_3 = 1, k_1 = 8$, $k_2 = 4$, k_4=4, k_5=2, b=1, from Eq.. (20) we have

$$H(z) = \left[\frac{1}{5}\frac{1 - z^{-10}}{1 - z^{-2}}\right]^8 \left[\frac{1}{2}\frac{1 - z^{-2}}{1 - z^{-1}}\right]^4 \left[\frac{1 - z^{-8}}{2}\right]\left[\frac{1 - z^{-4}}{2}\right]^2$$
$$\times \left[1 - (2^3 + 2)z^{-1} + z^{-2}\right]^4$$

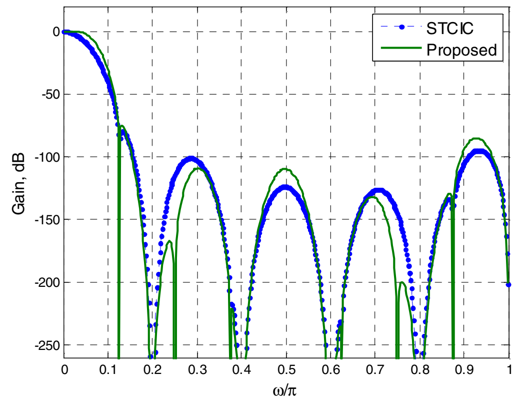
The corresponding gain response is shown in Fig. 4 along with that of the proposed filter from [8]. The passband zoom illustrates the improvement in the passband. The proposed filter has SNR of 64.44 dB while the filters from [8] and [2] have SNRs of 58.9dB, and 50dB, respectively.

The price for the improved SNR is the slight increase of memory elements and APOS.

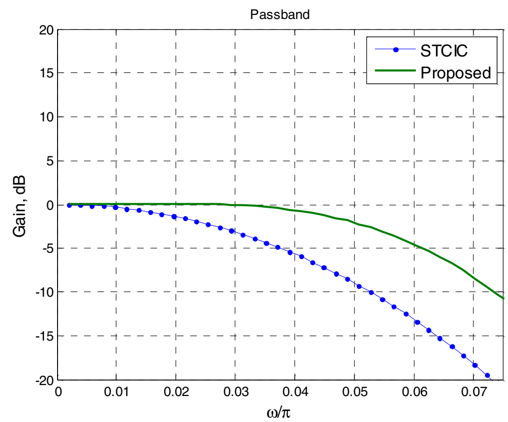
The proposed filter requires 32 memory elements, comparing with the number of 20 for the filter [8] and 56 for the filter [2]. The number of APOS for the proposed, and filters from [8], and [2] are 106, 73 and 80, respectively. The corresponding structure is shown in Fig. 3. b.

Conclusion

This paper has presented a new decimation filter for a rational conversion factor where interpolation and decimation factors are mutually prime numbers and the decimation factor is the product of two simple factors M_1 and M_2. The proposed filter is based on the stepped triangular CIC filter, expanded cosine filters and the sine-based compensation filter. The compensation filter is a simple second order filter with one multiplier per output sample which can be realized by shift and add operations. The proposed filter has an improved passband leading to the improved SNR. The expense of the frequency improvement is the slight increase of memory elements and APOS. The resulting proposed structure is a multiplierfree structure.



a. Overall magnitude responses.



b. Passband zoom.

Fig. 4. Example 1.

Acknowledgment

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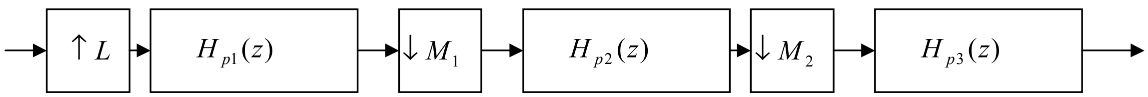
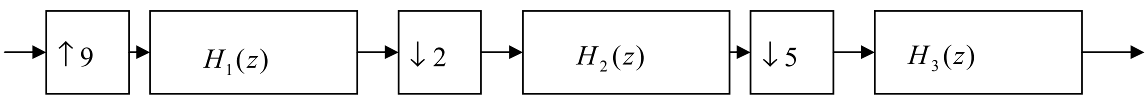


a. General structure.



b. Example 1: SRC=9/10, M_1=2, M_2=5.

Legend:

$$H_1(z) = \left[\frac{1}{1-z^{-1}}\right]^4; H_2(z) = \left[1+z^{-2}\right]^2\left[1+z^{-4}\right]\left[\frac{1}{1-z^{-1}}\right]^4; H_3(z) = \left[1-z^{-1}\right]^8\left[1-(2^3+2)z^{-1}+z^{-2}\right]^4;$$

Fig.3. Proposed structure.