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N-scroll chaotic attractors from saturated function series employing CCII+s

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Abstract The generation of n-scroll chaotic attractors by using saturated nonlinear function series (SNFS) realized with positive-type second generation current conveyors (CCII+s), is introduced. The nonlinear dynamical system is expressed by a third-order differential equation and to carry out numerical simulations, SNFS are ideally modeled by using staircase functions. Therefore, numerical simulations are introduced to approximate the swings, widths, breakpoints and equilibrium points of the n-scroll attractors by considering, as input variables: the dynamic range associated to active devices, gain of the nonlinear system and the number of scrolls. Therefore, its dynamical behavior is investigated in the state space. Besides, the CCII± is a versatile analog building block and it has been demonstrated to be very useful in several linear and nonlinear applications, since CCII-based implementations offer better performances that Opamps-based implementations in terms of accuracy and bandwidth. Therefore, the nonlinear system is synthesized with CCII+s to generate 3- and 4-scrolls. HSPICE simulations and experimental results are shown to verify the agreement on the behavior of the proposed circuit and the numerical simulations.

Keywords Multiscroll chaotic attractor · Saturated function series · Current conveyor

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1 Introduction

It is well known that chaotic attractors can be used in several possible commercial applications, such as data encryption, medicine, biology and secure communication systems [12, 17, 30, 34, 35]. Among all the chaotic oscillators that have been proposed in the literature, Chua's circuit is the most widely studied and used in several engineering applications, because it is an extremely simple circuit, it can easily be built, simulated and exhibits a rich variety of phenomena which have been experimentally confirmed

[5, 7–9, 25]. Chua's circuit was generalized to obtain n-scroll attractors by adding multiple breakpoints to the nonlinear resistor [2, 26, 31, 37, 40, 42], or by replacing the original nonlinear resistor function by other nonlinear functions, such as: a cubic function [4, 42], a sinusoidal function [32] or a piecewise-quadratic function [41]. More recently, the generation of n-scroll attractors has been widely investigated and realized with other kinds of circuits. Several novel approximations are, for example: Jerk circuit [17, 39], Brockett system [3], nonlinear transconductor method [23], step function approach [38], hysteresis series method [14, 18] and SNFS approach [17, 19, 20]. These oscillators have been developed not only to generate chaotic attractors at one direction (1D) but also at two directions (2D) and three directions (3D) [18–22]. The approximations mentioned above have been experimentally tested and they have been mainly built with Opamps, Current Feedback Operational Amplifiers (CFOAs), Operational Transconductance Amplifiers (OTAs), CCII±s, or specific integrated circuits (IC) to design the special functions [13, 17]. However, it is well known that the Dynamic Range (*DR*) associated with active devices is the main obstacle to be able to generate physically n-scroll chaotic attractors. Therefore, if the nonlinear system parameters are known—such as: number of scroll (*N*) and the gain of the nonlinear system (*a*)—along with real physical active device parameters—such as *DR*—then some n-scroll attractor parameters—such as: the Swings (*S*), Widths (*W*), Equilibrium points (*Ep*) and Breakpoints (*Bp*)—can be approximated [17, 39].

Particularly, in the fields of secure communication and data encryption systems, n-scroll attractors are preferred over the double-scroll attractors because they offer more dynamical complexity [4, 9, 31, 40]. Also, to be able to transmit high-speed data, the chaotic attractors should operate at high frequency [30]. It is worth to mention that simplest designs will have better performance in speed since they will present less dependence of parasitic effects [15, 25, 26]. Hence, not only the generation of n-scroll in 1D, 2D or 3D by using simplest designs are required, but also their operation at high frequency as well as low bias voltages are necessary to overcome the portability requirements. Among the n-scroll chaotic circuits proposed, Opamp-based chaotic attractors are less suitable for high speed since they not only have limitations in slew-rate and non-ideal phase characteristics, but also the chaotic circuits proposed are more complex and therefore, they use several ICs [12, 17, 20–22]. For instance in [17, 19–22, 39, 40], n-scroll chaotic attractors in 1D, 2D and 3D have mainly been synthesized with Opamps, but however, the frequency response is still low. On the other hand, a CFOA is built internally by a CCII+ connected in cascade with a voltage follower and it has become a versatile cell providing high speed and high slew-rate. Besides, although n-scroll chaotic attractors have been designed with CFOAs or CCII±s [6, 10, 15], still the design of these cells by using CMOS technology is subject of research.

This paper also focusses on the n-scroll chaotic attractors generation by using CCII+s, but the proposed circuit requires few active and passive elements. The third-order system along with SNFS is implemented with the AD844 in configuration of CCII+ for frequencies up to 40 kHz [1]. However, higher frequencies can easily be achieved by designing the CCII+s with CMOS

$$f(x) = A_1\left[\sum_{i=0}^{(N-3)/2}\big(sgn\big(x+(2i+1)A_1\big)+sgn\big(x-(2i+1)A_1\big)\big)\right] \quad \text{for } N\geq 3,\ N\text{-odd scrolls} \tag{1}$$

or

$$f(x) = A_1\left[-sgn(x)+\sum_{i=0}^{(N-2)/2}\big(sgn(x+2iA_1)+sgn(x-2iA_1)\big)\right] \quad \text{for } N\geq 2,\ N\text{-even scrolls} \tag{2}$$

where $A_1 > 0$ and

$$sgn(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

technology. Therefore, chaotic communication systems, as the ones proposed in [12, 34, 35], can be improved. The rest of the paper is organized as follows: Sect. 2 describes briefly the nonlinear system and its dynamical behaviors are studied by means of numerical simulations, where the SNFS are approximated with staircase functions [40]. Here, n-scroll attractor parameters—such as: S, W, Ep_j and principally Bp_j—are approximated by using nonlinear system parameters—such as: a and N along with the DR associated with active devices. Later in Sect. 3, the nonlinear dynamical system is synthesized by using CCII+s. Hspice simulation results are also discussed. Afterwards, in Sect. 4 we present experimental tests in both time and frequency domains. Conclusions are finally drawn in Sect. 5.

2 Nonlinear dynamical system

The third-order continuous time differential equation given by (3), has been early proposed by [9, 11] and has been widely used to synthesize multiscroll chaotic attractors in 1D, 2D and 3D by using different active devices [1, 3, 9, 11, 14, 17–24, 30, 38, 39, 41, 42].

$$-\dddot{x} = a\big[\ddot{x} + \dot{x} + x - f(x, \dot{x}, \ddot{x})\big] \tag{3}$$

Here, a is one parameter that can be varied to alter the dynamics of the system. In this case, the nonlinear function can be chosen arbitrarily, but in general it depends on the three states of the system. Several nonlinear functions have been proposed in the literature and these can be grouped in: $sgn(x)$ functions [17, 22, 38], staircase functions [40], hysteresis series functions [14, 18], SNFS piecewise-linear [17, 19, 21] and sawtooth functions [36]. It is worth to mention that SNFS are continuous functions while staircase functions are not continuous at all switching points [17, 19, 20]. However, SNFS can ideally be modeled as staircase functions by considering that the slope is infinity and the switching points are zero [40]. Therefore, in this work SNFS are ideally modeled as staircase functions given by (1) or (2) [40]. By introducing new variables, $\dot{x} = y$ and $\ddot{x} = z$, (3) can be expressed as:

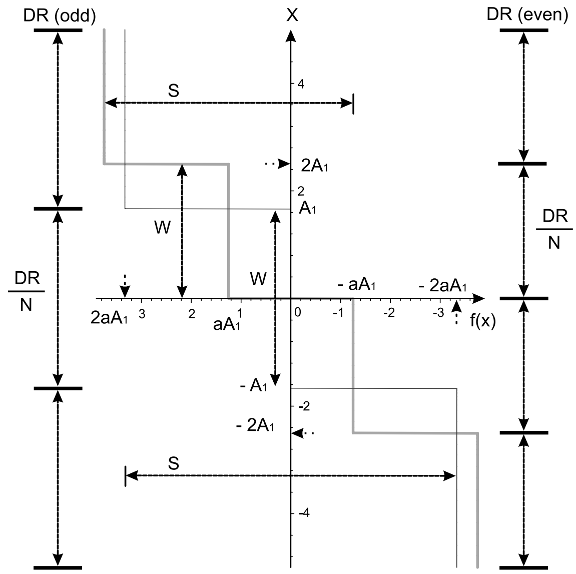

Fig. 1 Staircase functions for: $N = 3$ (*thin line*) by using (1) and $N = 4$ (*thick line*) by using (2)

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -ax - by - cz + df(x) \end{aligned} \tag{4}$$

For $b = c = d = 1$, (4) is reduced to the system given in [29]. On the contrary, if $b = c = d = a$, then (4) is reduced to the system introduced in [9, 11, 38]. This work deals with the last case. Staircase waveforms for (1) and (2) are shown in Fig. 1. As it was described above, n-scroll chaotic attractor parameters should be estimated by using nonlinear system parameters along with real physical active device parameters, for a later hardware implementation. This way, as can be seen from Fig. 1, the width of each scroll can be estimated by $W = DR/N$ (except the outside edge scroll) and since the even or odd period of the staircase functions is given by $W = 2A_1$, then $A_1 = DR/2N$. Note that $f(x)$ in (4) is multiplied by a, therefore, the swings of the scrolls are estimated as $S = 4aA_1 = 2aDR/N$. Also, one can derive the recursive formulas for Bp and Ep as follows:

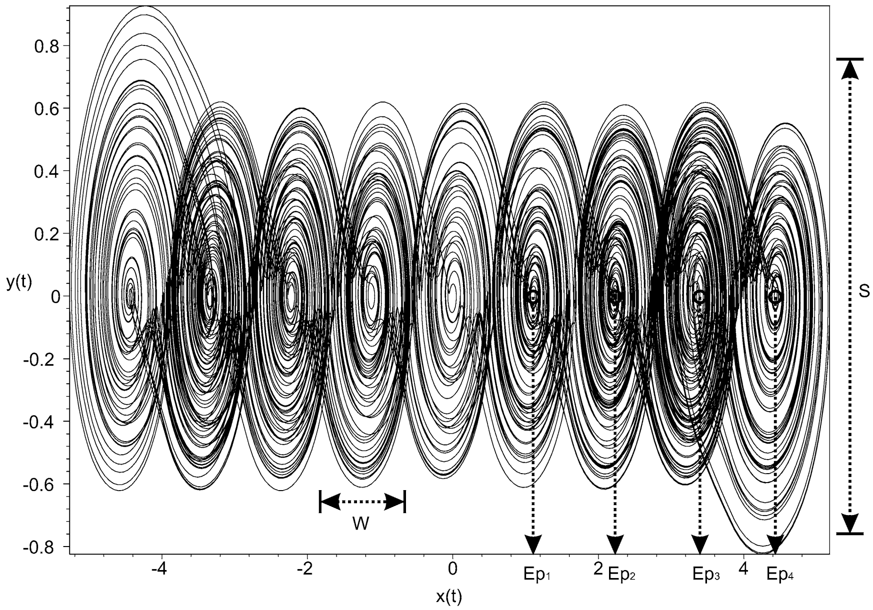

Fig. 2 9-scroll generated by (1), (4) and (5)

(1) *For odd chaotic attractors,* $N \geq 3$

$$\pm Bp_{j+1} = \pm \sum_{j=0}^{\frac{N-3}{2}} (2j+1)A_1$$
$$\pm Ep_{j+1} = \pm \sum_{j=0}^{\frac{N-3}{2}} (Ep_j + 2A_1), \qquad Ep_0 = 0 \tag{5}$$

(2) *For even chaotic attractors,* $N \geq 2$

$$\pm Bp_{j+1} = \pm \sum_{j=0}^{\frac{N-2}{2}} 2jA_1$$
$$\pm Ep_{j+1} = \pm \sum_{j=0}^{\frac{N-2}{2}} (Ep_j + 2A_1), \qquad Ep_0 = -A_1 \tag{6}$$

Let $DR = 10$ V, $N = 9$ and $a = 0.7$, then by using (1), (4) and (5), a 9-scroll chaotic attractor is generated as shown in Fig. 2, and W, S, Bp and Ep are given in Table 1. Similarly, if $N = 8$ and now by using (2) and (4) along with the recursive formulas given by (6), chaotic attractor parameters are approximated and given in Table 1, therefore, an 8-scroll chaotic attractor is generated, as shown in Fig. 3.

Table 1 Chaotic attractor parameters

9-scroll		8-scroll	
$W = 1.111$	$S = 1.555$	$W = 1.25$	$S = 1.75$
$\pm Bp_j$	$\pm Ep_j$	$\pm Bp_j$	$\pm Ep_j$
	0		
±0.555	±1.111	0	±0.625
±1.666	±2.222	±1.25	±1.875
±2.777	±3.333	±2.5	±3.125
±3.888	±4.444	±3.75	±4.375

3 Circuit implementation

The current conveyor has been considered as the first building block designed for current-mode signal processing [16, 27, 28]. CCII±s are the most versatile among all current conveyors and they are widely used to design linear and nonlinear circuits [6, 15, 16, 28]. However, only the CCII+ is commercially available as CFOA [1], which includes in their internal structure a CCII+ in cascade connection with a voltage follower. On the other hand, the nonlinear dynamical system given by (4) has been synthesized by using different active devices, such as: Opamps [17, 20, 40], OTAs [11, 24] and CFOAs [38]. For this case, (4) is synthesized with the AD844 in configuration of CCII+, as shown in Fig. 4 [1]. In this way, by considering parasitic elements associated with the input–output termi-

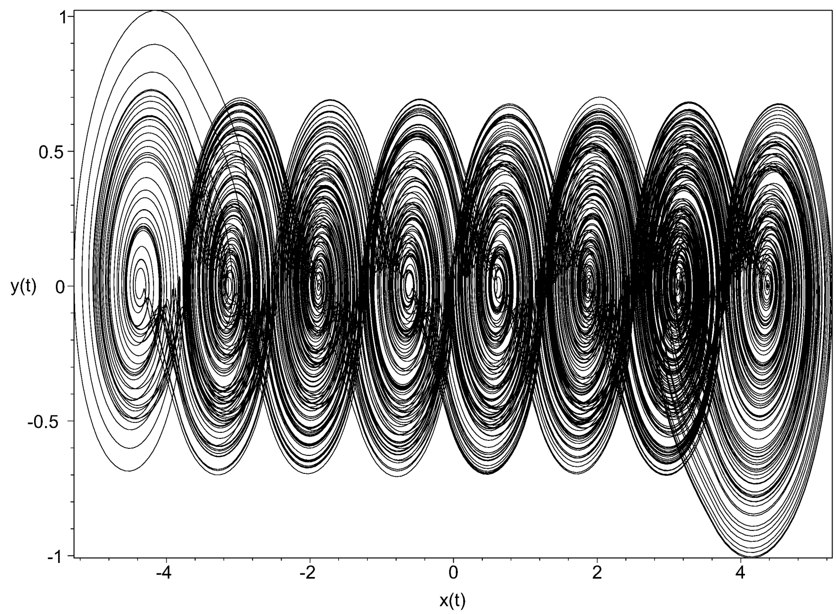

Fig. 3 8-scroll generated by (2), (4) and (6)

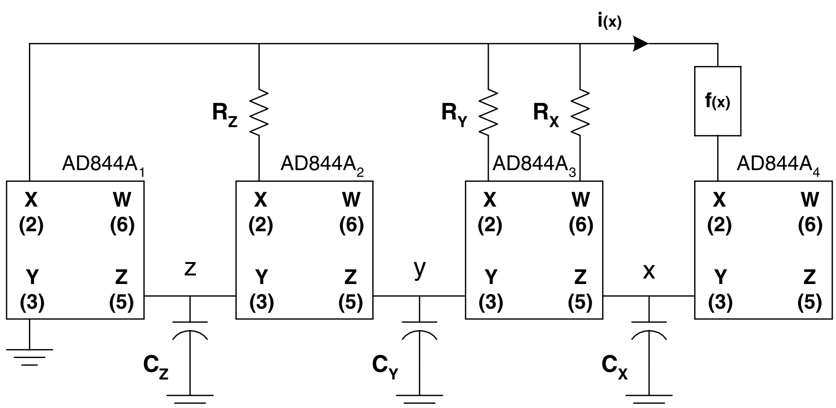

Fig. 4 Circuit diagram to synthesize (4) with CCII+s

nals of the AD844, the nonlinear system is given by:

$$\begin{aligned}
\dot{x} &= \frac{y}{(R_y + R_{x3})(C_x + C_{z3} + C_{y4})} \\
\dot{y} &= \frac{z}{(R_z + R_{x2})(C_y + C_{z2} + C_{y3})} \\
\dot{z} &= -\frac{x}{R_x(C_z + C_{z1} + C_{y2})} \\
&\quad - \frac{y}{(R_y + R_{x3})(C_z + C_{z1} + C_{y2})} \\
&\quad - \frac{z}{(R_z + R_{x2})(C_z + C_{z1} + C_{y2})} \\
&\quad + \frac{i(x)}{C_z + C_{z1} + C_{y2}}
\end{aligned} \tag{7}$$

For simplicity, the parasitic resistances R_{x1} and R_{x4} are not considered. By comparing (4) with (7), the following relations are obtained:

$$\begin{aligned}
C_x + C_{z3} + C_{y4} &= a(C_z + C_{z1} + C_{y2}) \\
C_y + C_{z2} + C_{y3} &= a(C_z + C_{z1} + C_{y2}) \\
R_x = R_y + R_{x3} = R_z + R_{x2} &= \frac{1}{a(C_z + C_{z1} + C_{y2})}
\end{aligned} \tag{8}$$

Staircase functions given by (1) and (2) cannot be designed with real active devices, but however, they can be approached with SNFS. Contrary to [17, 19–22, 38], the SNFS can be synthesized by using a high-gain voltage amplifier designed with CCII+s, as shown in Fig. 5(a) and the proposed synthesis method still is valid. If parasitic elements are considered in Fig. 5(a), the voltage gain is given by:

$$V_{\text{sat}} = -\frac{R_b R_{z5}(x - E_1)}{(R_b + R_{z5})(R_a + R_{x5})} = -A_v(x - E_1) \quad (9)$$

Figure 5(c) shows the voltage saturated function $f(x)$ and Fig. 5(d) shows the shifted voltage saturation function $f(x - E_1)$, where k is the slope, and $\pm p$ are the switching points. The voltage saturated behavior is determined by the saturation voltage of the AD844 and the saturation current by R_c, such that: $i(x) \approx \frac{|V_{\text{sat}}|}{R_c}$. This way, by connecting several basic cells in parallel, as shown in Fig. 5(b), (1) and (2) can be implemented. In Fig. 4, two voltage followers are used to separate R_x and $f(x)$ from x-state variable and one voltage follower is used between the Z and W terminals from Fig. 5(a), to transform the saturation voltage to saturation current.

3.1 3-scrolls attractor synthesis

In order to synthesize 3-scroll attractors, we assumed the following input variables: $DR = 7$ V, $N = 3$ and $a = 0.7$. Attractor parameters are calculated by using (1), (4), (5) and are given in Table 2. Afterwards, by using (7), (8) and by considering parasitic elements, the numerical values of the passive elements from Fig. 4 are approximated and given in Table 3. Typical values of the parasitic elements associated with each terminal of the AD844 are: $R_{yj} = 10$ MΩ, $R_{xj} = 50$ Ω, $R_{zj} = 3$ MΩ, $C_{yj} = 2$ pF, $C_{zj} = 4.5$ pF [1]. It is worthy to mention that to minimize the effects of the parasitic resistors R_{xj}, the numerical values of R_x, R_y and R_z are chosen to be large. To synthesize (1), two basic cells connected in parallel should be used. Also, to minimize the effect of R_{x5} and by considering R_{z5}, we have fixed $R_a = 1$ kΩ and $R_b = 1$ MΩ. SNFS parameters are given in Table 3. To reach the behavior in frequency, the discrete capacitors are scaled by a factor $q = C^{\text{old}}_{x,y,z}/C^{\text{new}}_{x,y,z}$ but under the condition: $C^{\text{new}}_{x,y,z} > 10(C_{z1,z2,z3} + C_{y2,y3,y4})$ [33]. Hspice simulation result in the state space is shown in Fig. 6 for $q = 1$ M.

Table 2 3- and 4-scroll chaotic attractor parameters

3-scroll	Bias ±5 V,	4-scroll	Bias ±5.5 V
$W = 2.333$ V	$S = 3.266$ V	$W = 2$ V	$S = 2.8$ V
$\pm E_j = \pm Bp_j$	$\pm Ep_j$	$\pm E_j = \pm Bp_j$	$\pm Ep_j$
	0	0	± 1 V
±1.166 V	±2.333 V	±2 V	±3 V

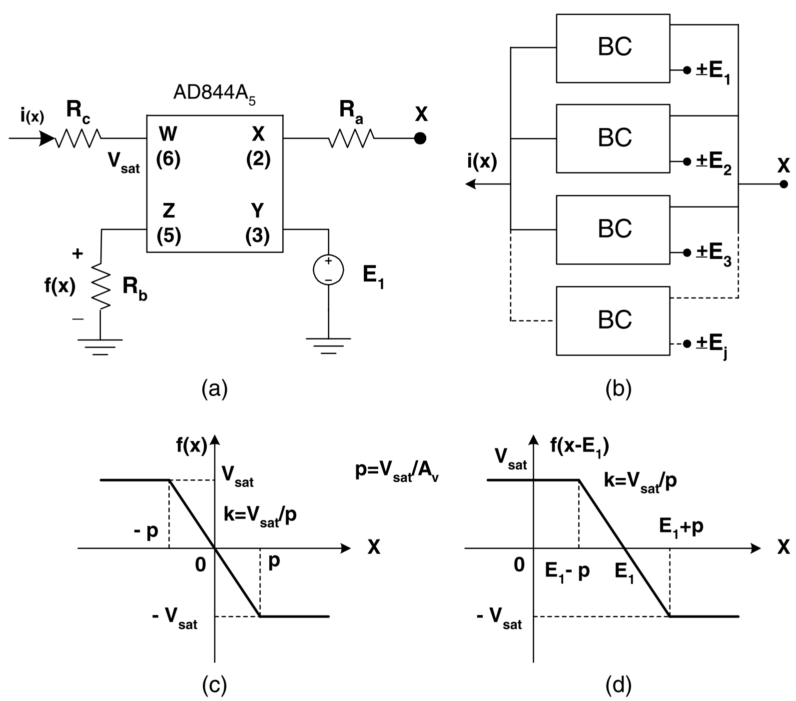

Fig. 5 (**a**) Basic cell to design SNFS. (**b**) General structure to synthesize (1) and (2). (**c**) Voltage saturation function $f(x)$. (**d**) Shifted voltage saturated function $f(x - E_1)$

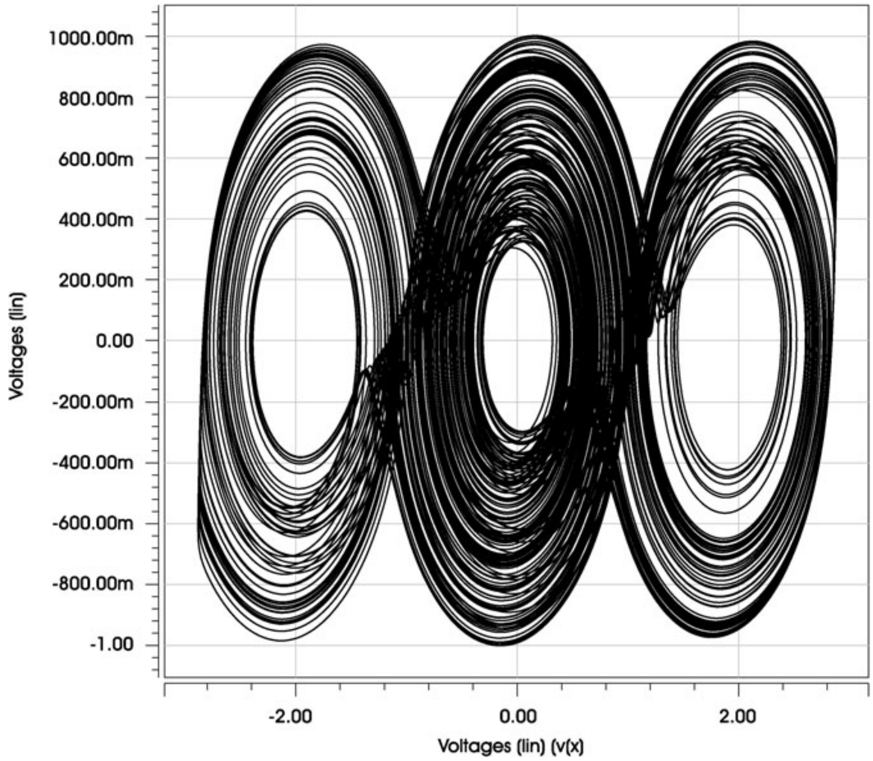

Fig. 6 3-scroll generation, $x(t)$–$y(t)$ plane

Table 3 Numerical values of passive elements for Fig. 4

Passive elements	SNFS parameters 3-scroll	4-scroll
$C_x \approx 100\ \mu F$	$\lvert V_{sat}\rvert = 4$ V	$\lvert V_{sat}\rvert = 4.5$ V
$C_y \approx 100\ \mu F$	$k = A_v = 57.07$ dB	$k = A_v = 57.07$ dB
$C_z \approx 143\ \mu F$	$p = \pm 5.6$ mV	$p = \pm 6.3$ mV
$R_x \approx 10\ k\Omega$	$R_{c,j} = 30\ k\Omega$	$R_{c,j} = 37.1\ k\Omega$
$R_y \approx 10\ k\Omega$		
$R_z \approx 10\ k\Omega$		

3.2 4-scrolls attractor synthesis

To verify the methodology on the n-scroll chaotic attractors generation by considering system and active device parameters, the supply voltage is modified to ±5.5 V, then, we assumed $DR = 8$ V. Then, if $N = 4$ and $a = 0.7$, 4-scroll should be generated and as a consequence, three basic cells connected in parallel should be used. Attractor parameters are computed by using (2), (4), (6) and are given in Table 2. The numerical values of the passive elements introduced in Table 3 are again used in Hspice simulation, but the capacitors are scaled with $q = 100k$. Hspice simulation result in the state space is shown in Fig. 7.

4 Experimental results

The proposed circuit shown in Fig. 4 is experimentally tested by using discrete components with the same numerical values used in simulation, but with the following slight modifications.

4.1 Experimental verification of 3-scrolls

Discrete capacitors from Table 3 are scaled in frequency with $q = 300k$ and by modifying $R_{c,j} = 32\ k\Omega$. Figure 8 shows the experimental result corresponding to $x(t)$–$y(t)$ plane and Table 4 shows the measured attractor parameters.

4.2 Experimental verification of 4-scrolls

In the same manner, the discrete capacitors are scaled by using $q = 300k$ and now $R_{c,j} = 36\ k\Omega$. Figure 9

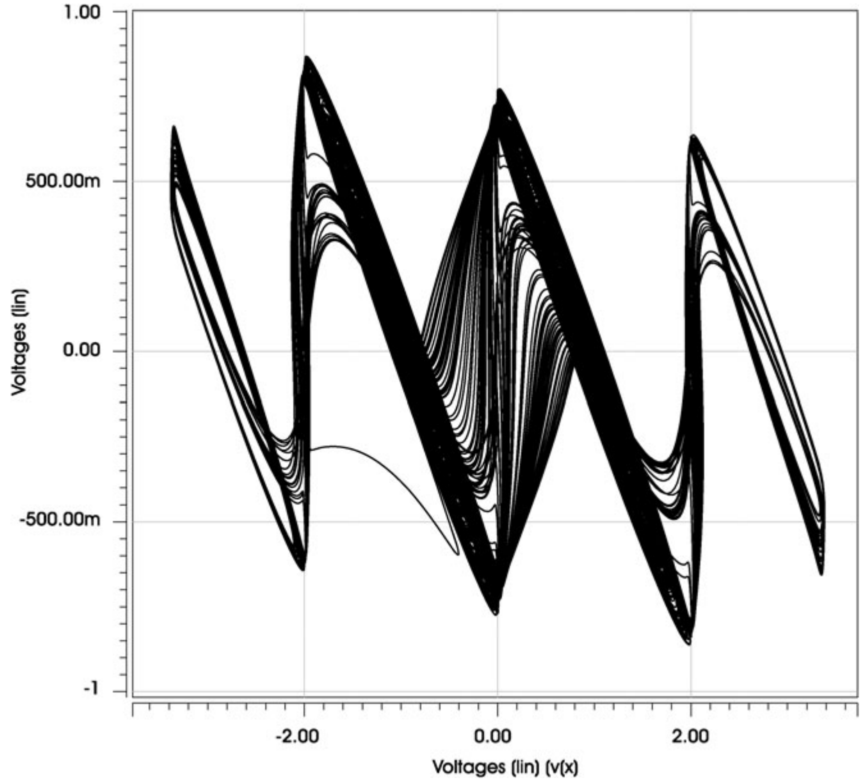

Fig. 7 4-scroll generation, $x(t)$–$z(t)$ plane

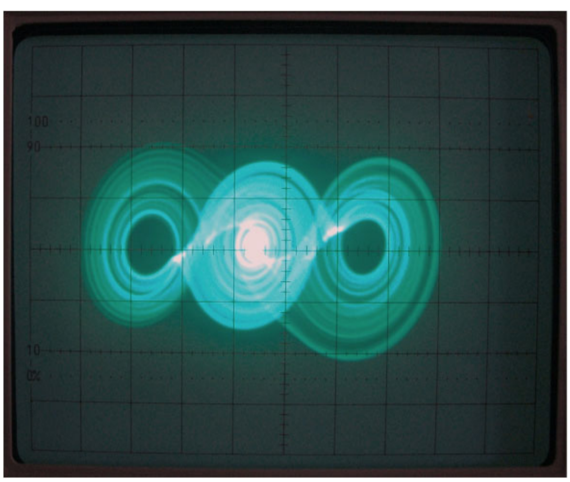

Fig. 8 Experimental verification of 3-scroll attractor; horizontal-axes: 1 V/div and vertical-axes: 0.1 V/div

shows the experimental result corresponding to $x(t)$–$z(t)$ plane and Table 4 also gives the measured attractor parameters. After several experimental tests, we observe that the behavior in frequency of the 4-scroll chaotic attractors is lower than that of the 3-scroll chaotic attractors. Here only the 4-scroll chaotic frequency spectrum is shown in Fig. 10 and it is centered to 43 kHz. In general, there is a trade-off between the number of scroll and the frequency of operation and it depends on the behavior in frequency of the cells used to design the SNFS.

Table 4 Measured attractor parameters

3-scroll	Bias ±5 V,	4-scroll	Bias ±5.5 V
$W = 2.4$ V	$S = 2.2$ V	$W = 2.1$ V	$S = 1.9$ V
$\pm E_j = \pm Bp_j$	$\pm Ep_j$	$\pm E_j = \pm Bp_j$	$\pm Ep_j$
	0	0	±1.3 V
±1	±2.1 V	±2.3 V	±3.6 V

4.3 Remarks

The synthesis of n-scroll chaotic attractors by using SNFS was first proposed in [17, 19, 20]. More recently, the synthesis of n-scroll attractors from high-level simulation was proposed in [21, 22]. Although in these papers, n-scroll chaotic attractors have mainly been synthesized, simulated and experimentally tested by using Opamps, still the DRs of real Opamps are not taken into account in the synthesis process. In fact,

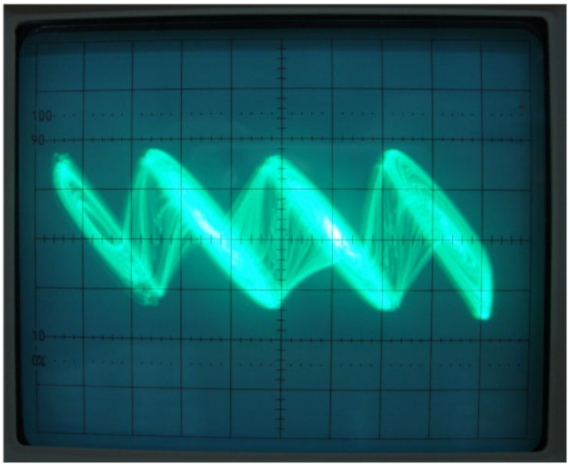

Fig. 9 Experimental verification of 4-scroll attractor; horizontal-axes: 1 V/div and vertical-axes: 0.1 V/div

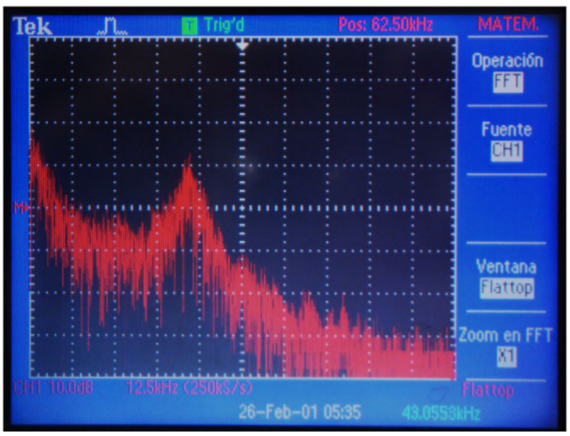

Fig. 10 Experimental measurement of the chaotic spectrum of the 4-scroll attractor

numerical simulations are carried out to generate the attractors and later, the SNFS are scaled to show that the chaotic attractors are within the DRs of real active devices [17, 19–22]. Also, since Opamps have limitations in slew-rate and phase, the chaotic attractors still operate in low frequency, as shown in [21]. On the other hand, our proposed synthesis methodology offers several advantages. First, n-scroll parameters are approximated by using nonlinear system parameters along with the DR associated with real physical active devices. Therefore, it is not necessary to carry out a scaling of the DRs associated with SNFS. Also, recursive formulas to approximate Ep_j and Bp_j have been deduced and they are dependent on the DR and on the number of scroll. Second, the nonlinear system has been synthesized with CCII+s, taking into account their parasitic elements associated with each terminal. Additionally, the proposed topology is less complex compared with the topologies introduced in [17, 19–22]. Third, because a CCII± is a cell very suitable for its performance in high speed and high slew-rate, it is used to design the SNFS. As a consequence, CCII+s-based SNFS have better performance in frequency than Opamp-based SNFS and therefore, high-frequency n-scroll chaotic attractors can be achieved. However, the parasitic capacitances in the Y and Z terminals of the AD844 impose a low-limit on the value of C_z, C_y and C_x [1]. Therefore, the discrete capacitors should be large to minimize the effects of the parasitic capacitors. In order to exploit the capabilities offered by CCII± in the generation of n-scroll chaotic attractors and to improve the frequency response, CMOS technologies should be used [15]. A limitation of the proposed synthesis method is that SNFS are ideally modeled as staircase functions. Therefore, errors are obtained when Ep_j and Bp_j are computed and compared with measured date, as shown in Table 4. This limitation can be solved if the SNFS are modeled as in [19, 21], but taking into account the real parameters of the circuit used to design the SNFS. Thus, a maximum number of scrolls can be obtained to any given DR.

5 Conclusions

The nonlinear system parameters along with real physical active device parameters have been used to approximate n-scroll attractor parameters. Moreover, a novel circuit based on CCII+s has been designed to synthesize the nonlinear chaotic system, where the SNFS have also been designed with CCII+s in configuration of high-voltage amplifier. Experimental results for 3- and 4-scroll attractors have been provided to verify the good agreement with numerical approximations.

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