Self-focusing transmittances

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In this Letter, we describe the optical field associated with transmittances characterized by a slit-shaped curve. The influence of the curvature is that the diffraction field generates focusing regions. The focusing geometry corresponds to the geometry of the transmittance curve, except for scaling, rotations or translations. A relevant point is that the changes in the morphology of the diffraction field are bounded by the focusing regions. Our experimental and computational results are in good agreement with the theoretical predictions.

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The optical fields are characterized by an amplitude complex function whose spatial evolve may have regions with different physical properties. In general, these regions are separated by caustic regions that are easily identified because they present high- energy with respect to other regions of the optical field [1–3]. The different physical features appear because in the caustic region, the phase function "collapses" generating a singularity and wave behavior is not presented. Rather, a particle feature becomes dominant and this implies the necessity to synthesize focusing regions with different geometries. In this context, the caustic region can be considered as a separatrix region that connects regions with different physical properties. For that reason, in the caustic regions, "birth or death" of interesting features, such as vortex, bifurcation can be expected [4–6]. Indeed, the particle structure of the caustic regions offer important applications, e.g., it can be implemented to illuminate a metal surface generating surface plasmon fields, and as optical tweezers for particle trapping [7,8]. These findings are the motivation to generate and study focusing regions.

In the present study, we describe the development of caustic regions whose geometry corresponds to the geometry of the boundary condition that is characterized by a slit-shaped curve. The model is implemented experimentally and the experiments reveal how bifurcation effects are organized around the caustic regions that are the manifestation of the curvature center of the transmittance function [3]. A property of cycloids, hypocycloids, and epicycloids is that its evolute curve is also a cycloid, hypocycloid, or epicycloid in a scaled version, rotated, or translated with respect to the original curve [9,10]. This property will be used for the design of the self-focusing transmittances. The study is supported by two remarkable mathematical results: a) The evolute of the curve corresponds to the envelope region of the curvature centers; it can also be obtained by the envelope curve of the normal linear to the curve [3,9]; b) The singularity propagates on a plane along the projection of the characteristic curves [11,12]. These two results make it possible to analyze the topological structure of the optical field on the transmittance plane identifying the singularities and then projecting to parallel planes. By using the geometrical theory of diffraction [13], the optical field emerging from a slit-shaped curve can be interpreted as a set of rays, where each ray satisfies the extremal principle, and consequently must satisfy the transversal condition [14]. Subsequently, the diffraction rays must emerge perpendicular to the slit curve. The consequence of this fact is that the optical field can be ordered, signifying that we can identify regions in which only one ray passes, regions generated by an envelope of rays, and regions where two or more rays pass. In this way, the envelope of the ray set, when propagating, generates a cylinder having the evolute as the basis. The focusing regions are the walls of the cylinder, and the physical features attached can be studied by means of the interference between the diffracting field with a plane wave. In addition, the focusing region presents morphological invariance for long distances of propagation and can be considered as a version of nondiffracting optical fields. Each ray in a homogeneous media propagates following a linear path. However, the ray set may generate an envelope whose geometry is not linear.

The study of the diffraction field is obtained from the formal solution for the Helmholtz equation in the two-dimensional version given by

$$\varphi(P) = \frac{1}{2\pi}\int\int_S \frac{e^{ikr}}{r}\left\{\left(ik - \frac{1}{r}\right)Ar - \nabla A\right\}\cdot n\,dl. \qquad (1)$$

In the calculus of the integral, we can identify three regions that are sketched in Fig. 1. When describing the diffraction field of transmittances of slit-shaped curves, it is natural to invoke some results of differential geometry. One of them is the order concept that characterizes the number of trajectories that pass in the neighborhood of a point P (contact point). This point is common to two curves or surfaces. For a tangent surface, the number of trajectories is at least two. The order concept is sketched in Fig. 1; more details concerning this concept can be found in [9]. With this concept, the amplitude for optical field can be approximated as a sum of trajectories. For regions where only a single trajectory passes, the amplitude function can be given as

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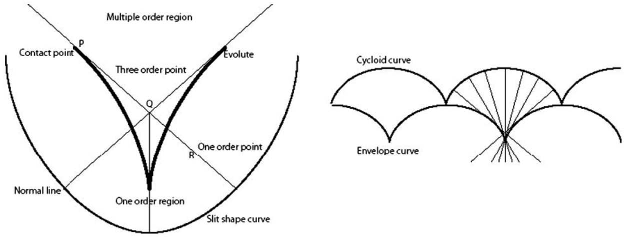

Fig. 1. (Left) Transmittance plane containing a slit curve; the curvature centers generate a focusing region that is the separatrix for regions with different orders. (Right) Transmittance containing a cycloid shape curve and its evolute. For this case, the higher order points are in the neighborhood of the cusped regions.

$$\varphi_B(P) = \frac{ik}{2\pi}\int\int_S \frac{e^{ikr}}{r} Ar\cdot n\mathrm{d}l \approx \frac{ik}{2\pi}\frac{e^{ikR}}{R}B(\theta), \tag{2}$$

where R is the normal distance from a point on the slit curve, k is the wave number, A is the boundary condition associated to a slit shape curve, and $B(\theta)$ is an obliquity factor, which is not important for the study of generic features. For regions where many trajectories pass, the integral can be approximated as

$$\varphi_B(P) = \frac{ik}{2\pi}\int\int_S \frac{e^{ikr}}{r} Ar\cdot n\mathrm{d}l \approx \sum_i^N \frac{ik}{2\pi}\frac{e^{ikR_i}}{R_i}B_i(\theta), \tag{3}$$

where N is the order of the curve. For slit curves we have that the singularity corresponds to the envelope of the trajectories emerging perpendicular to the curve, and the geometry is given by the evolute of the curve. For this curve, the singular regions correspond to the envelope of the curvature centers.

The previous concepts have deep implications in the study of the scattering of tridimensional curves, i.e., curves with curvature and torsion. The concept of order will be applied in brief to explain the experimental results. As mentioned earlier, we are interested in transmittances whose envelopes correspond to the same geometry transmittance except scaling, rotations or translations. These curves are cycloids, hypocycloids, and epicycloids. The representation for curvature centers is

$$\alpha = x - \frac{y'(x'^2 + y'^2)}{x'y'' - x''y'}; \qquad \beta = y + \frac{x'(x'^2 + y'^2)}{x'y'' - x''y'}, \tag{4}$$

when the transmittance is a cycloid slit, it is given by

$$x = a(t - \sin t); \qquad y = a(1 - \cos t). \tag{5}$$

By direct substitution in Eq. (4) it is easy to obtain curvature centers given as

$$\alpha = a(t - \sin t); \qquad \beta = a(1 - \cos t) - 2a. \tag{6}$$

The geometry is the other cycloid curve shifted with respect to the original. In Fig. 2, we show the diffraction field generated with a cycloid curve. It must be noted that all the morphological changes in the diffraction field are bounded by the evolute of the curve.

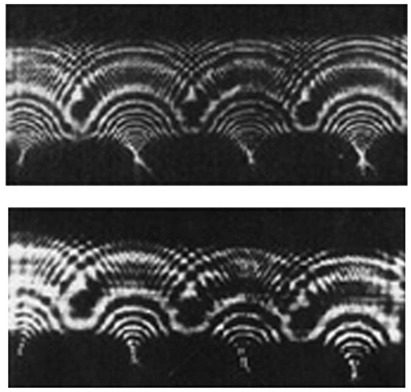

Fig. 2. Experimental results for the diffraction field on two planes separated 15 cm. The transmittance consists in a cycloid slit. It was recorded on high resolution plate and it is contained in a square of 0.5 cm per side. The width of the slit is approximately 0.2 mm. The wavelength used is 623.8 nm.

Another interesting case occurs when the transmittance is a hypocycloid slit whose parametric representation is given by

$$x = (R - R_0)\cos t + R_0 \cos\left(\frac{R - R_0}{R_0}t\right),$$
$$y = (R - R_0)\sin t + R_0 \sin\left(\frac{R - R_0}{R_0}t\right), \tag{7}$$

where R_0 is the radius of the inner circle and $R = nR_0$ is the radius of the outer circle where n is an integer. From these representations, it is easy to show that the evolutes of hypocycloid curves are a scaled and rotated version of the same hypocycloid:

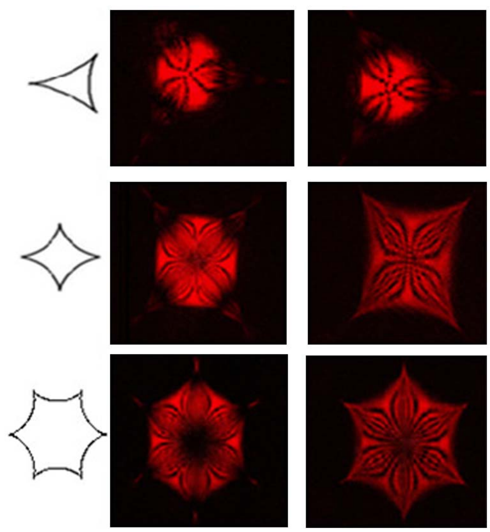

Fig. 3. (Color online) Experimental results on two propagation planes for the diffraction of screens containing an epicycloid slit with $n = 3$, 4, 6, respectively. The curves were recorded on high resolution plate of 5 mm per side.

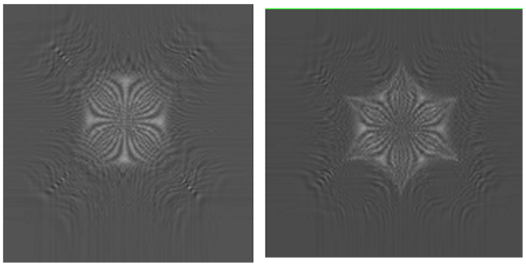

Fig. 4. Computer simulation for the interference between a plane wave with a diffraction field that presents a focusing region with four and six cusps. Bifurcation of the phase function occurs in the neighborhood of the cusped regions where kind speckle pattern is observed.

$$\alpha = \frac{(R - 2R_0)}{R}\left[(R - R_0)\cos t - R_0 \cos\left(\frac{R - R_0}{R_0}t\right)\right]$$
$$\beta = \frac{1}{R}\left[(R - R_0)(R + 2R_0)\sin t + (3R - 2R_0)R_0 \sin\left(\frac{R - R_0}{R_0}t\right)\right]. \tag{8}$$

In Fig. 3, we have shown the experimental results; similar to cycloids, the morphological changes are bounded by the evolute. The next point consists of describing the local topological structure of the phase function. This can be done by interfering the diffracted field with a plane wave. In Fig. 4, we show the computer simulations.

Numerical simulation shows that the interference occurs only in the region of order one and the evolute and the inner region remain practically unchanged; this behavior was also observed experimentally. These results show that the focusing region is structurally stable. The diffraction field, at a global scale, must be organized around the focusing regions, and the local bifurcation effects of the phase function appear mainly around the cusped regions. In conclusion, we report the existence of transmittances slit, whose diffraction fields have associated focusing regions and whose geometry corresponds to the evolute of the curve. The transmittance geometry corresponds to cycloids and epicycloids. An important feature is that the optical field is bounded by the singular region and the phase function presents bifurcation effects in the neighborhood of cusped regions; the experimental results reveal these effects. In a forthcoming paper, a study will be presented to describe the scattered field generated by illuminating a three-dimensional slit- shaped curve that presents curvature and torsion. The concept of order and contact point allows classifying this kind and co-dimension of the local bifurcation effects. This will be used to describe the cinematic focusing region as it propagates through inhomogeneous media and explains the generation of vortex and bifurcation effects.

References

1. M. V. Berry and C. Upstill, Vol. 42 of Elsevier Series on *Progress in Optics*, E. Wolf, ed. (North Holland, 1980), 257–346.
2. M. S. Sosskin and M. V. Vasnetsov, Vol. 42 of Elsevier Series on *Progress in Optics*, E. Wolf, ed. (North Holland, 2001), 219–276.
3. G. Martinez-Niconoff, J. Carranza, and A. C. Rodriguez, in *Elsevier Series on Optics Communications* (1995), Vol. 114, pp. 194–198.
4. Y. S. Kivshar and E. A. Ostrovskaya, Opt. Photon. News **12** (4), 24 (2001).
5. D. P. Rhodes, G. P. T. Lancaster, J. Livesey, D. Mcgloin, J. Arlt, and K. Dholakia, Opt. Commun. **214**, 247 (2002).
6. W. Liu, D. N. Neshev, I. V. Shadrivov, A. E. Miroshnichenko, and Y. S. Kivshar, Opt. Lett. **36**, 1164 (2011).
7. K. Dholakia and T. Cizmar, Nat. Photonics **5**, 335 (2011).
8. A. Mourka, J. Baumgartl, C. Shanor, K. Dholakia, and E. M. Wright, Opt. Express **19**, 5760 (2011).
9. W. C. Graustein, *Differential Geometry* (Dover, 2006).
10. D. J. Struik, *Lectures on Classical Differential Geometry* (Dover, 1988).
11. V. I. Arnold, *Singularities of Caustics and Wave Fronts* (Springer, 1990).
12. P. R. Garabedian, *Partial Differential Equations* (Wiley, 1967).
13. J. B. Keller, J. Opt. Soc. Am. **52**, 116 (1962).
14. L. Elsgoltz, *Differential Equations and Variational Calculus* (MIR, 1977).