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High NOON states in trapped ions

D Rodríguez-Méndez and H Moya-Cessa

INAOE, Apdo. Postal 51 y 216, 72000 Puebla, Pue., Mexico

E-mail: hmmc@inaoep.mx

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Abstract
We show how NOON states may be generated in ion traps. We use the individual interactions of light with each of the two vibrational modes of the ion to entangle them. This allows us to generate NOON states with $N = 8$.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Nonclassical states for mono-mode fields have attracted a great deal of attention over the years because of their fundamental and technological value; among them we have (i) macroscopic quantum superpositions of quasiclassical coherent states with different mean phases or amplitudes [1, 2], (ii) squeezed states [3] and (iii) the particularly important limit of extreme squeezing, i.e. the Fock or number states [4]. More recently, some bimodal fields have also attracted attention because they may be used for high-precision phase measurements: NOON states, i.e. nonclassical states of combined photon pairs [5, 6]. Many applications in quantum imaging, quantum information and quantum metrology [7] depend on the availability of entangled photon pairs because entanglement is a distinctive feature of quantum mechanics that lies at the core of many new applications. These maximally path-entangled multiphoton states may be written in the form

$$|N00N\rangle_{a,b} = \frac{1}{\sqrt{2}}\left(|N\rangle_a |0\rangle_b + |0\rangle_a |N\rangle_b\right). \tag{1}$$

It has been pointed out that NOON states manifest unique coherence properties by showing that they exhibit a periodic transition between spatially bunched and antibunched states when they undergo Bloch oscillations. The period of the bunching/antibunching oscillation is N times faster than the period of the oscillation of the photon density [8].

The greatest N for which NOON states have been produced is $N = 5$ [5]. Most schemes to generate this class of states are either for optical [5, 6] or for microwave [9] fields. In this paper, we would like to analyze the possibility of generating them in ions [10–17], i.e. NOON states of their vibrational motion. We will show that they may be generated with $N = 8$.

2. Ion–laser interaction

The Hamiltonian for an ion in a two-dimensional (2D) Paul trap has the form

$$\hat{H} = \nu_x a_x^\dagger a_x + \nu_y a_y^\dagger a_y + \frac{\omega_{ee}}{2}\hat{A}_{ee} + (\hat{A}_{ge}\lambda E^{(-)}(\hat{x}, \hat{y}, t) + \text{h.c.}), \tag{2}$$

with λ being the electronic coupling matrix element and $E^{(-)}(\hat{x}, \hat{y}, t)$ the negative part of the classical electric field of the driving field. The operators $\hat{A}_{jk}$ take into account the transitions between the states $|j\rangle$ and $|k\rangle$ (g for ground and e for excited). We assume the ion is driven by a plane wave

$$E^{(-)}(\hat{x}, \hat{y}, t) = E_0 \mathrm{e}^{-\mathrm{i}(k_x\hat{x} + k_y\hat{y} + \omega t)} \tag{3}$$

with k_j, $j = x, y$ being the wavevectors of the driving field, and define the Lamb–Dicke parameters

$$\eta_x = 2\pi\frac{\sqrt{{}_x\langle 0|\Delta\hat{x}^2|0\rangle_x}}{\lambda_x}, \quad \eta_y = 2\pi\frac{\sqrt{{}_y\langle 0|\Delta\hat{y}^2|0\rangle_y}}{\lambda_y}, \tag{4}$$

such that we redefine

$$k_x\hat{x} = \eta_x(a_x + a_x^\dagger), \quad k_y\hat{y} = \eta_y(a_y + a_y^\dagger). \tag{5}$$

In the resolved sideband limit, the vibrational frequencies ν_x and ν_y are much larger than other characteristic frequencies and the interaction of the ion with the two lasers can be treated separately, using a nonlinear Hamiltonian [18, 19]. The Hamiltonian (2) in the interaction picture can then be

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written as

$$H_{\rm I} = \begin{cases} \Omega_x^{(k)} {\rm e}^{-\eta_x^2/2} \hat{A}_{eg} \frac{\hat{n}_x!}{(\hat{n}_x+k)!} L_{\hat{n}_x}^{(k)}(\eta_x^2) a_x^k + {\rm h.c.}, \\ \quad \eta_y = 0, \quad \delta = k\nu_x, \\ \Omega_y^{(k)} {\rm e}^{-\eta_y^2/2} \hat{A}_{eg} \frac{\hat{n}_y!}{(\hat{n}_y+k)!} L_{\hat{n}_y}^{(k)}(\eta_y^2) a_y^k + {\rm h.c.}, \\ \quad \eta_x = 0, \quad \delta = k\nu_y, \end{cases} \tag{6}$$

where $L_{\hat{n}_j}^{(k)}$, $j = x, y$ are the operator-valued associated Laguerre polynomials, the Ωs are the Rabi frequencies and $\hat{n}_j = a_j^\dagger a_j$, $j = x, y$. If we consider $\eta_y = 0$ and $\eta_x \ll 1$ and take $\delta = 4\nu_x$, we obtain

$$H_{\rm I}^{(x)} \approx g A_{21} a_x^4 + {\rm h.c.} \tag{7}$$

with $g = \Omega_x \frac{\eta_x^4}{4!}$. The evolution operator for the interaction Hamiltonian is then (in the atomic basis, see [20])

$$U_{\rm I}^{(x)}(t) = {\rm e}^{-{\rm i}H_{\rm I}^{(x)}t} = \begin{pmatrix} C_{\hat{n}}^x & -{\rm i}\, S_{\hat{n}}^{\dagger x} V_x^4 \\ -{\rm i}\, V_x^{\dagger 4} S_{\hat{n}}^x & C_{\hat{n}-4}^x \end{pmatrix}, \tag{8}$$

where

$$\begin{aligned} C_{\hat{n}}^x &= \cos(\sqrt{(\hat{n}_x+4)(\hat{n}_x+3)(\hat{n}_x+2)(\hat{n}_x+1)}gt), \\ S_{\hat{n}}^x &= \sin(\sqrt{(\hat{n}_x+4)(\hat{n}_x+3)(\hat{n}_x+2)(\hat{n}_x+1)}gt), \end{aligned} \tag{9}$$

and the operator

$$V_x = \frac{1}{\sqrt{\hat{n}_x+1}} a_x \tag{10}$$

is the Susskind–Glogower (phase) operator [21].

3. Generation of NOON states

In figure 1, we show that (for simplicity we do the treatment in one of the dimensions x or y), for a particular time, $\tau_{\rm p}$, if the ion was initially in its excited state, the probability of finding it in the excited state again is zero. This also applies to an initial ground state.

By starting with the ion in the excited state and the initial vibrational state in the vacuum state, i.e. $|0\rangle_x|0\rangle_y$, if we set $\eta_y = 0$, after a convenient time, i.e. the time when the probability of finding the ion in its excited state is zero (meaning that the ion, by passing from its excited to its ground state, gives four phonons to the vibrational motion), we can generate the state $|4\rangle_x|0\rangle_y$. Repeating this procedure (with the ion reset again to the excited state, via a rotation), but now with $\eta_x = 0$, four phonons are added to the y-vibrational motion, generating the 2D state $|4\rangle_x|4\rangle_y$.

Therefore, if we consider the ion initially in a superposition of ground and excited states, and the $|4\rangle_x|4\rangle_y$ vibrational state

$$|\psi_{\rm init}\rangle = \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)|4\rangle_x|4\rangle_y, \tag{11}$$

for $\eta_y = 0$ and $\tau_{\rm p}$, the state generated is

$$|\psi_{\eta_y=0}\rangle = \frac{\rm i}{\sqrt{2}}(|e\rangle|0\rangle_x + |g\rangle|8\rangle_x)|4\rangle_y. \tag{12}$$

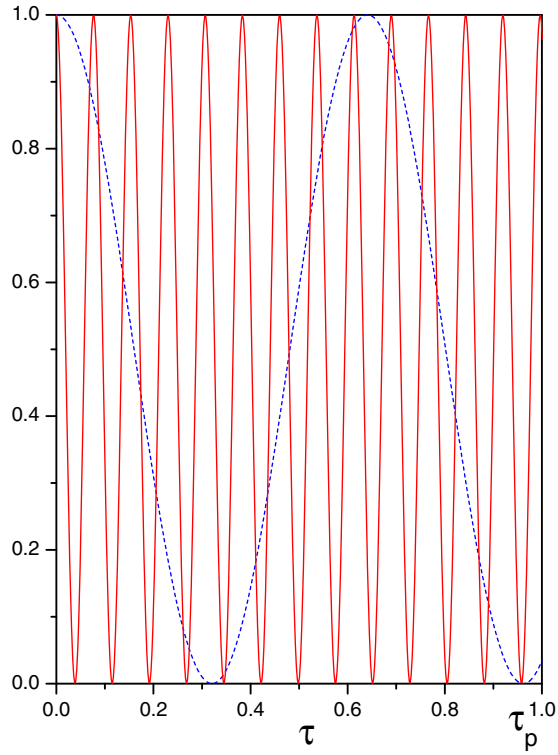

Figure 1. Probability of finding the ion in its excited state (solid line) when the initial state is excited and the probability of finding the ion in its ground state when the initial state of the ion is in the ground state (dashed line). The initial vibrational state is the number state $|n = 4\rangle$. It may be seen that for $\tau_{\rm p}$ both probabilities approach zero.

Now we consider this state as the initial state for the next interaction with $\eta_x = 0$ and the same interaction time $\tau_{\rm p}$, to produce

$$|\psi_{\eta_x=0}\rangle = -\frac{1}{\sqrt{2}}(|e\rangle|0\rangle_x|8\rangle_y + |g\rangle|8\rangle_x|0\rangle_y). \tag{13}$$

Next, the ion is rotated via a classical field (an on-resonance interaction) such that the state

$$|\psi_{\rm R}\rangle = -\frac{1}{2}\left[|e\rangle(|0\rangle_x|8\rangle_y - |8\rangle_x|0\rangle_y) + |g\rangle(|0\rangle_x|8\rangle_y + |8\rangle_x|0\rangle_y\right] \tag{14}$$

is obtained. Finally, by measuring the ion in its excited state, we produce the NOON state

$$|{\rm NOON}\rangle_e = \frac{1}{\sqrt{2}}(|0\rangle_x|8\rangle_y - |8\rangle_x|0\rangle_y), \tag{15}$$

and if the ion is measured in the ground state, also a NOON state is produced:

$$|{\rm NOON}\rangle_g = \frac{1}{\sqrt{2}}(|0\rangle_x|8\rangle_y + |8\rangle_x|0\rangle_y). \tag{16}$$

4. Conclusions

In this paper, we have shown a way of producing high NOON states ($N = 8$) by entangling the vibrational motion of an ion trapped in 2D. The entanglement is produced by a set of interactions of the trapped ion with laser fields

conveniently tuned to produce four-phonon transitions. It should be noted that, because the Lamb–Dicke regime is assumed, the four-phonon transitions we are proposing are difficult to achieve. Decoherence processes are usually not taken into account in ion–laser interactions because these interactions are, in general, not affected by the environment. However, due to the interaction times needed for generating the states presented here, they may be considered in this case. Here, however, we have only treated the ideal case as a first approach. We will study the consequences of the environment elsewhere.

References

[1] Schrödinger E 1935 *Naturwissenschaften* **23** 807
Schrödinger E 1935 *Naturwissenschaften* **23** 823
Schrödinger E 1935 *Naturwissenschaften* **23** 844

[2] Yurke B and Stoler D 1986 *Phys. Rev. Lett.* **57** 13

[3] Loudon R and Knight P L 1987 *J. Mod. Opt.* **34** 709
Moya-Cessa H and Fernández-Guasti M 2003 *Phys. Lett.* A **311** 1

[4] Krause J, Scully M O, Walther T and Walther H 1989 *Phys. Rev.* A **39** 1915

[5] Afek I, Ambar O and Silberberg Y 2010 *Science* **328** 879

[6] Dowling J P 2008 *Contemp. Phys.* **49** 125

[7] Wolfgramm F, Cere A and Mitchell M W 2010 *J. Opt. Soc. Am.* B **27** A25

[8] Bromberg Y, Lahini Y and Silberberg Y 2011 *Phys. Rev. Lett.* **105** 263604

[9] Rodríguez-Méndez D and Moya-Cessa H 2011 *Opt. Commun.* **284** 3345

[10] Wineland D J, Bollinger J J, Itano W M, Moore F L and Heinzen D J 1992 *Phys. Rev.* A **46** R6797

[11] Leibfried D, Blatt R, Monroe C and Wineland D 2003 *Rev. Mod. Phys.* **75** 281

[12] Wineland J *et al* 1998 *J. Res. Natl Inst. Stand. Technol.* **103** 259

[13] Leibfried D, Meekhof D M, King B E, Monroe C, Itano W M and Wineland D J 1996 *Phys. Rev. Lett.* **77** 4281

[14] Wallentowitz S and Vogel W 1999 *Phys. Rev.* A **59** 531

[15] Sasura M and Buzek V 2002 *J. Mod. Opt.* **49** 1593

[16] Moya-Cessa H and Tombesi P 2000 *Phys. Rev.* A **61** 025401

[17] Moya-Cessa H, Jonathan D and Knight P L 2003 *J. Mod. Opt.* **50** 265

[18] de Matos Filho R L and Vogel W 1996 *Phys. Rev. Lett.* **76** 608

[19] de Matos Filho R L and Vogel W 1996 *Phys. Rev.* A **54** 4560

[20] Phoenix S J D and Knight P L 1990 *J. Opt. Soc. Am.* B **7** 116

[21] Susskind L and Glogower J 1964 *Physics* **1** 49