ELectronics EXpress

Homotopy method with a formal stop criterion applied to circuit simulation

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Abstract: The continuous scaling for fabrication technologies of electronic circuits demands the design of new and improved simulation techniques for integrated circuits. Therefore, this work shows a new double bounded polynomial homotopy based on a polynomial formulation with four solution lines separated by a fixed distance. The new homotopy scheme presents a bounding between the two internal solution lines and the symmetry axis, which allows to establish a stop criterion for the simulation in DC. Besides, the initial and final points on this new double bounded homotopy can be set arbitrarily. Finally, mathematical properties for the new homotopy are introduced and exemplified using a benchmark circuit.

Keywords: homotopy continuation methods, multistable circuits
Classification: Integrated circuits

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DOI: 10.1587/elex.8.1808
Received September 14, 2011
Accepted October 05, 2011
Published November 10, 2011

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1 Introduction

The task of finding DC operating points is important because this analysis is the starting point for the rest of common tests regularly done through the circuit design process (for instance, small-signal analysis). This analysis consists in finding the solutions for a non-linear algebraic equation system (NAEs) (equilibrium equation) from the ICs [1]. These NAEs becomes complex due to the accelerated increase on the density of transistors inside the IC and by the use of complex models (as result of reducing dimensions of the components) causing two phenomena: existence of multiple unexpected operating points and convergence failures for the Newton-Raphson (NR) method. The NR method is employed by the majority of integrated circuit simulators. The reason for the widespread use of the NR method is its quadratic convergence rate which reduces computing time to complete the simulation. Nevertheless, the NR method [2] suffers some convergence issues like: oscillation and divergence.

Circuit designers face convergence failures for DC analysis, commonly, using the NR method and back-up methods, thus, as last resort, the modification of some parameters for the numerical engine are enforced expecting to reach convergence. This situation increases design times, thus making the entire design cycle slow and expensive. This situation, by itself, justifies the use of alternative methods to NR, like homotopy, to locate the operating point. Nevertheless, there are more reasons to use homotopy methods like the existence of multiple operating points [3]. This is because, unlike NR, homotopy is capable to locate multiple operating points. This is important because there is a chance the designer implements a circuit under the assumption that certain operating point exist (calculated by the NR method), which, in fact, is not always physically stablished due to the existence, unnoticed, of multiple operating points; that is, the DC operating point physically present

DOI: 10.1587/elex.8.1808
Received September 14, 2011
Accepted October 05, 2011
Published November 10, 2011

IEICE Electronics Express, Vol.8, No.21, 1808–1815

is different. This is translated into malfunctioning of the circuit which could, in the end; represent high loses for the company in financial terms.

The homotopy method reported by [4] is a highly efficient multiparameter method [5] that locates the operating point for large circuits containing just MOS transistors. In [6], an excellent revision of globally convergent probability-one homotopy methods applied to circuit simulation with bipolar transistors is presented. All the homotopy methods [4, 5, 6] described above have been proved useful to locate one or more operating points that converge to solutions where the NR method is unable to calculate. Nonetheless, such methods lack a formal stop criterion [7]; the stop criterion allows to complete the simulation with the mathematical certainty that no more solutions are left to be found along the traced homotopy path. On one hand, in [8], a homotopy method containing a stop criterion is reported, it consists on creating a boundary for the search space. Nevertheless, such method is only useful to analyse circuits with bipolar transistors; besides, to program and use this algorithm, requires deep understanding on the behaviour of multistable circuits [3]. On the other hand, [7, 9] proposed a homotopy with stop criterion named double bounded homotopy (DBH), which is based on the manipulation of the homotopy path until it is converted in a closed path; allowing to establish a formal method to conclude the homotopy simulation. Besides, DBH homotopy can be applied to a wide spectrum of non-linear circuits that include bipolar transistors, tunnel diodes, MOS transistors, among others [9].

This work presents a homotopy function based on the qualitative properties of the homotopy proposed in [7], it is called double bounded polynomial homotopy (DBPH). This homotopy is capable to find multiple operating points in a closed path. Like [7], DBPH homotopy includes symmetry axis and a formal stop criterion; features that will be described bellow.

2 Double bounded polynomial homotopy with four solution lines

The double bounded polynomial homotopy with 4 solution lines is defined by this equation

$$H(f(x),\lambda) = \lambda(\lambda+a)(\lambda-a)(\lambda-2a)(x-x_i)(x-x_f) + C(\lambda-a/2)^2 f(x)^2, \quad (1)$$

where λ is the homotopy parameter, $f(x)$ the equilibrium equation [10] of the circuit, a is a constant that represents separation between solution lines, x_i is the initial point, x_f the final point, and C an arbitrary constant.

Based on the previous, homotopy can be expressed in general way as

$$H(f(x),\lambda) = \begin{cases} f(x^*) = 0 & \text{for } \lambda = 0 \text{ and } x = x^* \\ (x-x_i)(x-x_f) = 0 & \text{for } \lambda = a/2 \\ f(x^*) = 0 & \text{for } \lambda = a \text{ and } x = x^* \end{cases}$$

where x^* is any solution for $f(x)$, x_i and x_f are homotopy's initial and final points, respectively.

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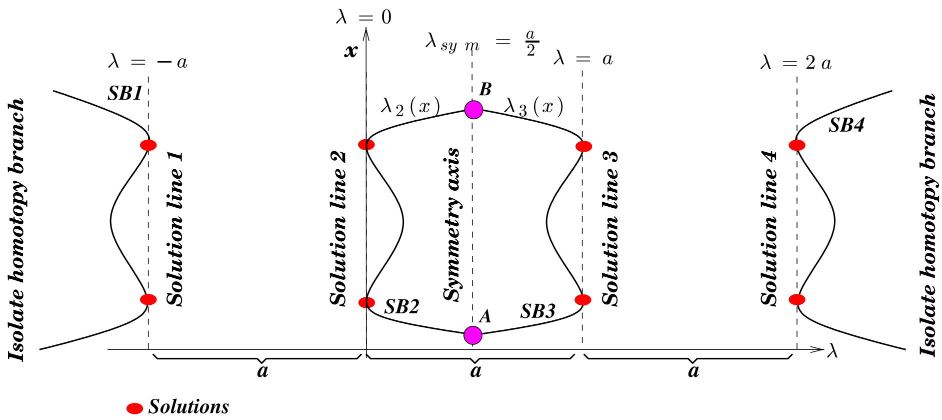

Fig. 1. Double bounded homotopy with four solution lines.

This homotopy contains four solution lines ($\lambda = -a$, $\lambda = 0$, $\lambda = a$, and $\lambda = 2a$)(see Fig. 1). Nevertheless, the two solution lines for both ends are unconnected branches ($SB1$ and $SB4$) not used for tracing purposes. Squaring the function $f(x)$ has the finality to establish an even number of solutions (or operating points) which precisely produces the bounding and closes the homotopy path inside the middle region.

Fig. 1 shows how homotopy path starts at $A = (x_i, a/2)$ on the symmetry axis, finds two roots (in region $SB3$) and finishes when a new crossing through the symmetry axis at $B = (x_f, a/2)$ is detected, which means that tracing for a symmetrical branch has been completed and fulfilling the stop criterion [9]. The properties for this new homotopy are presented in the following subsections:

2.1 Symmetrical branches

To obtain the branches for the homotopy path, first the equation (1) is reformulated as follows

$$H(f(x), \lambda) = \lambda(\lambda + a)(\lambda - a)(\lambda - 2a) + (\lambda - a/2)^2 J(x) = 0, \qquad (2)$$

where

$$J(x) = \frac{Cf(x)^2}{(x-x_i)(x-x_f)}.$$

In order to trace the homotopy path [9], the unconnected symmetrical paths *SB1* and *SB4* will be ignored because these open branches would make not possible to apply the stop criterion. Symmetrical branches $SB2$ ($\lambda_2(x)$) and $SB3$ ($\lambda_3(x)$) shown in Fig. 1 can be derived solving λ from equation (2). Given the fact that $SB2$ and $SB3$ are connected and symmetrical, only one should be traced to obtain the full path and finalize the simulation. $SB3$ is chosen as the tracing path, which tangentially touches the solution line $\lambda = a$. Therefore, the symmetrical branch $SB2$ is

$$\lambda_2(x) = \frac{a - \sqrt{5a^2 - 2\sqrt{(J(x) + 4a^2)(J(x) + a^2)} + 2J(x)}}{2}. \qquad (3)$$

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Accepted October 05, 2011
Published November 10, 2011

The symmetrical branch $SB3$ is

$$\lambda_3(x) = \frac{a+\sqrt{5\,a^2-2\,\sqrt{(J(x)+4\,a^2)(J(x)+a^2)}+2\,J(x)}}{2}. \tag{4}$$

To demonstrate that $\lambda_2(x)$ is linked to the solution line $\lambda = 0$, the following limit is calculated

$$\lim_{f(x)\to 0} \lambda_2(x) = 0, \tag{5}$$

where the equilibrium equation $f(x)$ tends to zero when x tends to solution x^* as shown in the following limit calculation

$$\lim_{x\to x^*} f(x) = 0. \tag{6}$$

Now, to demonstrate that $\lambda_3(x)$ is linked to the solution line $\lambda = a$, the following limit is calculated as

$$\lim_{f(x)\to 0} \lambda_3(x) = a. \tag{7}$$

This shows that solutions x^* are placed at $\lambda = a$.

2.2 Symmetry axis

The symmetry axis is an important property for the double bounded homotopy [7]. In the particular case of the double bounded polynomial homotopy the symmetry axis is

$$\lambda_{sym} = \frac{a}{2}. \tag{8}$$

This symmetry axis belong to the symmetry relationship between $SB2$ and $SB3$ branches.

As shown in Fig. 1, this relationship must be fulfilled as follows

$$\lambda_3(x) - \lambda_{sym} = \lambda_{sym} - \lambda_2(x).$$

Replacing the value for λ_{sym}, we obtain

$$\lambda_3(x) - 0.5a = 0.5a - \lambda_2(x).$$

Then, replacing $\lambda_2(x)$ and $\lambda_3(x)$ for their respective functions the next relationship is found

$$0.5a + 0.5\sqrt{G(x)} - 0.5a = 0.5a - 0.5a + 0.5\sqrt{G(x)},$$

where $G(x) = \sqrt{5\,a^2 - 2\,\sqrt{(J\,(x) + 4\,a^2)\,(J\,(x) + a^2)}}.$

Reducing terms

$$0.5\sqrt{G(x)} = 0.5\sqrt{G(x)},$$
$$0 = 0.$$

The proof for this equality shows that the homotopy path is symmetrical around the symmetry axis.

DOI: 10.1587/elex.8.1808
Received September 14, 2011
Accepted October 05, 2011
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IEICE Electronics Express, Vol.8, No.21, 1808–1815

3 Study case: circuit with bipolar transistors and a diode

In [11], a circuit was resolved using fixed point homotopy. This circuit has 3 operating points. The Ebers-Moll model is used for all the transistors. The equation for the model is given as

$$\begin{bmatrix} i_{D_E} \\ i_{D_C} \end{bmatrix} = \begin{bmatrix} 1 & \text{-0.01} \\ \text{-0.99} & 1 \end{bmatrix} \begin{bmatrix} 10^{-9}(e^{(40v_{be})}-1) \\ 10^{-9}(e^{(40v_{bc})}-1) \end{bmatrix}.$$

As for the diode, the model is

$$i_d = 10^{-9}(e^{40u}-1).$$

First, the equilibrium equation is formulated using the modified nodal analysis [10] with the result of a system having 14 equations $(f_1, f_2, \ldots, f_{14})$ and 14 variables $(v_1, v_2, \ldots, v_{13}, I_E)$. The circuit is shown in Fig. 2 (a).

Now, applying the DBPH homotopy $(a = 1, C = 1)$, the formulation is expressed as follows

$$\begin{aligned} H_1(f_1, \lambda) &= \lambda(\lambda+1)(\lambda-1)(\lambda-2)(v_1+13)(v_1-13) + (\lambda-0.5)^2 f_1^2 = 0, \\ H_2(f_2, \lambda) &= \lambda(\lambda+1)(\lambda-1)(\lambda-2)(v_2+13)(v_2-13) + (\lambda-0.5)^2 f_2^2 = 0, \\ &\vdots \\ H_{14}(f_{14}, \lambda) &= \lambda(\lambda+1)(\lambda-1)(\lambda-2)(I_E+13)(I_E-13) + (\lambda-0.5)^2 f_{14}^2 = 0. \end{aligned}$$

The initial point for every electrical variable may take the value +13 or −13. Therefore, there are n^2 possible combinations for each initial point (n is the number of electrical variables). For this simulation the selected initial point (x_{i_1}) is shown in Table I. The supply voltage for the circuit (E) provides 12V, this restricts the value of the nodal voltages at 12V maximum. Besides, for practical reasons, the test circuit will not handle currents beyond 13A, therefore, it is valid to assume that the operating point is within the range of ± 13. Hence, choosing the value of ± 13 for the initial point x_{i_1} and final point x_{f_1} is a way to guarantee the chance that the homotopy path contains all the operating points of the circuit.

Table I. Relevant points.

R.P	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
x_{i_1}	+13	-13	+13	-13	-13	-13	-13	-13
x_{i_2}	11.99	-15.41	-1.42	-15.04	-127.15	40.22	-1.40	-420.75
S_1	12	0.405	0.366	0.685	0.349	6.796	0.070	7.038
S_2	12	0.883	0.278	0.590	0.631	0.812	0.315	1.074
S_3	12	5.995	0.085	0.368	0.712	0.436	0.390	0.699
x_{f_1}	+13	+13	+13	+13	-13	+13	-13	+13
x_{f_2}	11.99	0.84	0.83	1.20	-110.90	45.64	-1.40	-420.73

R.P cont.	v_9	v_{10}	v_{11}	v_{12}	v_{13}	I_E	λ
x_{i_1}	-13	-13	-13	-13	-13	-13	0.5
x_{i_2}	-1.71	-1.41	-1.35	-0.34	0.32	-0.03	0.5
S_1	11.839	0.4e-5	0.039	0.039	0.321	-0.0085	1
S_2	11.647	0.4e-5	0.039	0.039	0.321	-0.0100	1
S_3	11.635	0.4e-5	0.039	0.039	0.321	-0.0089	1
x_{f_1}	+13	+13	+13	+13	+13	+13	0.5
x_{f_2}	-1.71	1.38	-1.35	-0.034	0.32	-0.03	0.5

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IEICE Electronics Express, Vol.8, No.21, 1808–1815

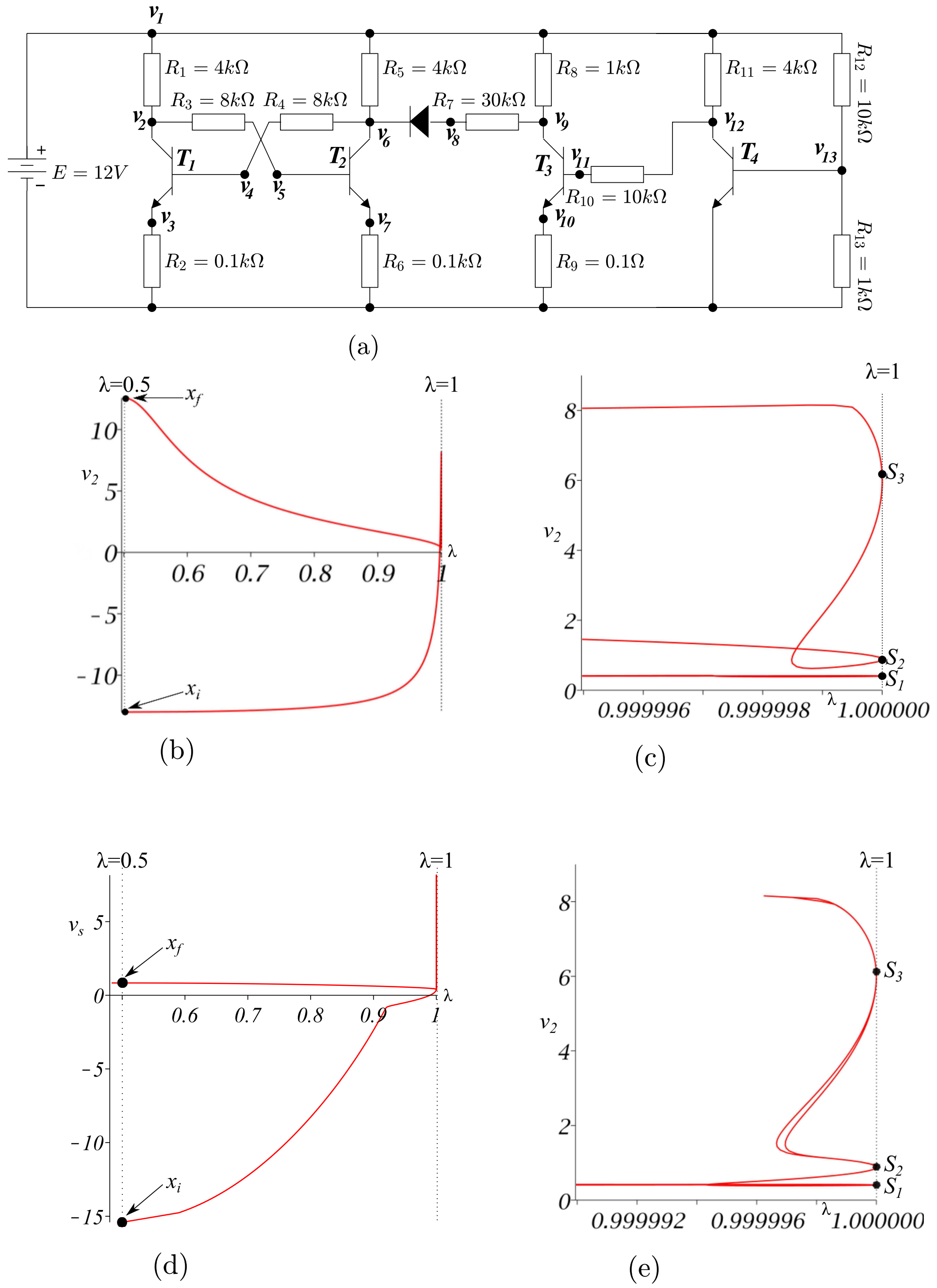

Fig. 2. (a) Benchmark circuit. (b) Homotopy path $\lambda - v_2$ for homotopy DBPH. (c) Zoom to the solutions of (b). (d) Homotopy path $\lambda - v_2$ for DBH. (e) Zoom to the solutions of (d).

The operating points for the benchmark circuit were located using double bounded polynomial homotopy and double bounded homotopy [7] (using $a = 0$, $b = 1$, $C = 1$ and $D = 1940$); resulting that both methods located all three operating points of the circuit (S_1, S_2, and S_3), shown in Table I. Initial point x_{i_1} for the double bounded polynomial homotopy was proposed arbitrarily; as for the double bounde homotopy, it was obtained by numerical solution of the homotopy function at $\lambda = 0.5$. Numerical path following [9] for both homotopies started and ended at $\lambda = 0.5$ (initial step length $h = 0.09$), nevertheless, double bounded homotopy needed a total of 10348 iterations to reach the final point at x_{f_2} while the double

DOI: 10.1587/elex.8.1808
Received September 14, 2011
Accepted October 05, 2011
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bounded polynomial homotopy only needed 2229 iterations to reach final point at x_{f_1}. Both final points (x_{f_1} and x_{f_2}) can be seen in Table I.

On one hand, Fig. 2 (b) and Fig. 2 (c) shows the homotopy path for variable v_2 using the double bounded polynomial homotopy. On the other hand, Fig. 2 (d) and Fig. 2 (e) shows the homotopy path for variable v_2 using the double bounded homotopy. It can be seen that both methods locate all three operating points, having the same order of appearance but different homotopy path. These solutions are shown in Table I, where S_1, and S_3 are stable and S_2 is unstable.

In Table I it is possible to see that initial point x_{i_1} has arbitrary values of $+13$ and -13; this obeys that using other sign combinations resulted in convergence of just one or two solutions. Therefore, the appropriate selection of the initial point plays an important role on the number of solutions to be found, hereafter a study about an optimal initial point selection should be performed in a near future.

The homotopy DBPH does not directly interfere with models of electronic devices, so it is not restricted to simulate circuits that may contain bipolar devices, diodes, tunnel diodes or MOS transistors [9]. In general terms, it can be applied to circuits containing devices that possess analytic models.

4 Conclusion

The stop criterion for homotopy methods consist, mainly, in two heuristic criteria: the number of iterations is fixed to a maximum number (arbitrarily number) of integration steps or the algorithm stops when the homotopy path cross the solution line. Nevertheless, this kind of stop criterion may end without finding all roots on the homotopy path. Therefore, a homotopy method with formal stop criterion was proposed, it is named as double bounded polynomial homotopy. The proposed homotopy shows interesting properties like: it allows the homotopy path to be bounded between two limits known as solution lines and possess a symmetry axis which allows to implement a stop criterion. The operating points of a benchmark circuit containing bipolar devices were found using the double bounded polynomial homotopy and the double bounded homotopy, comparing results it can be concluded that the DBPH homotopy has advantages over DBH homotopy like arbitrary initial and final points, and requires less iterations.

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