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Optical realization of the atom–field interaction in waveguide lattices

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Abstract
A classical realization of the atom–field interaction Hamiltonian, based on the transport of light in engineered optical waveguide lattices, is theoretically proposed. The optical lattice enables direct visualization of atom–field dynamics in Fock space.

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(Some figures may appear in colour only in the online journal)

1. Introduction

In this paper, we will show how to model the atom–field interaction by using *diatomic* waveguide arrays. We have recently shown how evanescently coupled waveguides may be used to emulate coherent, displaced number states [1] and nonlinear coherent states [2]. Photonic lattices have also been used for the optical realization of the two-site Bose–Hubbard model [3] and the classical realization of two-site Fermi–Hubbard systems [4]. Here, via a transformation of the atom–field Hamiltonian we will show that by choosing adequately the initial state of the atom–field wavefunction, we can arrive at systems of differential equations that arise in the study of diatomic waveguide arrays, such that we model in these systems the quantum interaction between a two-level atom and a quantized field.

2. The quantum interaction

The Hamiltonian for the atom–field interaction is given by (we set $\hbar = 1$) [5, 6]

$$H = \omega\hat{n} + \frac{\omega_0}{2}\sigma_z + g(a + a^\dagger)(\sigma_+ + \sigma_-), \tag{1}$$

where ω is the field frequency, ω_0 is the atomic transition frequency and g is the atom–field interaction constant. The operators a and $a^\dagger$ are the annihilation and creation operators, respectively, with $\hat{n} = a^\dagger a$ the atomic operators being the usual Pauli spin matrices, with $[\sigma_+, \sigma_-] = \sigma_z$ and $[\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm$. In matrix notation, we can rewrite the Hamiltonian (1) as

$$H = \begin{pmatrix} \omega\hat{n} + \frac{\omega_0}{2} & g(a + a^\dagger) \\ g(a + a^\dagger) & \omega\hat{n} - \frac{\omega_0}{2} \end{pmatrix}. \tag{2}$$

Because this Hamiltonian is equivalent to the on-resonance ion–laser interaction [7], what we will show for the atom–field interaction (2) will also be valid for the ion–laser interaction.

Given a Hamiltonian one needs to solve the Schrödinger equation

$$\mathrm{i}\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle. \tag{3}$$

In order to simplify the Hamiltonian (2), we transform the wavefunction via $|\psi(t)\rangle = T^\dagger|\psi_T(t)\rangle$, with the unitary operator

$$T = \frac{1}{2}\begin{pmatrix} (-1)^{\hat{n}} - 1 & (-1)^{\hat{n}} + 1 \\ -(-1)^{\hat{n}} - 1 & 1 - (-1)^{\hat{n}} \end{pmatrix}, \tag{4}$$

such that we obtain the transformed Hamiltonian diagonal in the atomic basis

$$H_T = THT^\dagger = \begin{pmatrix} \omega\hat{n} + \frac{\omega_0}{2}(-1)^{\hat{n}} - g(a + a^\dagger) & 0 \\ 0 & \omega\hat{n} - \frac{\omega_0}{2}(-1)^{\hat{n}} - g(a + a^\dagger) \end{pmatrix}, \tag{5}$$

and the Schrödinger equation for the transformed wave function reads

$$\mathrm{i}\frac{\partial|\psi_T(t)\rangle}{\partial t} = H|\psi_T(t)\rangle. \tag{6}$$

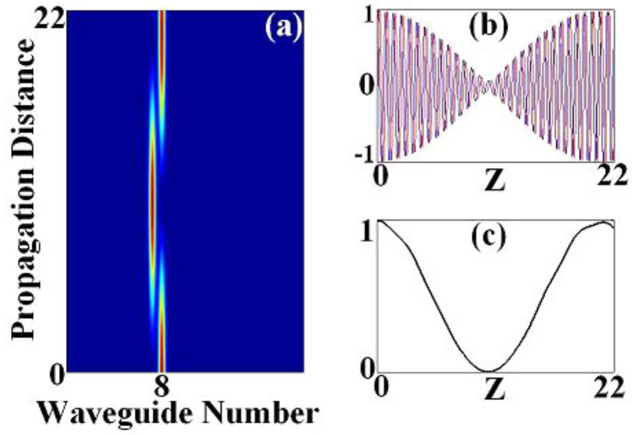

Figure 1. (a) Numerical simulation for a single input excitation for the lattice parameters $g = 0.05$, $\omega = 1$ and $\omega_0 = 1$, (b) the evolution of the field in the excitation channel (the blue/red curve is the real/imaginary part of the field) and (c) the evolution of intensity in the excitation channel.

Because H and H_T are connected by the unitary transformation (4) they are equivalent; solving one of them gives the solution for the other. Of course, one also needs to transform the initial condition

$$|\psi_T(0)\rangle = T|\psi(0)\rangle. \tag{7}$$

We have the following transformations for different initial conditions, for even number states and atoms in the excited or ground states:

$$\begin{aligned} |\psi_T^{(\mathrm{e,e})}(0)\rangle &= T\begin{pmatrix}|2n\rangle\\0\end{pmatrix} = -\begin{pmatrix}0\\|2n\rangle\end{pmatrix},\\ |\psi_T^{(\mathrm{e,g})}(0)\rangle &= T\begin{pmatrix}0\\|2n\rangle\end{pmatrix} = \begin{pmatrix}|2n\rangle\\0\end{pmatrix}, \end{aligned} \tag{8}$$

and for odd number states and excited or ground states:

$$|\psi_T^{(\mathrm{o,e})}(0)\rangle = T\begin{pmatrix}|2n+1\rangle\\0\end{pmatrix} = -\begin{pmatrix}|2n+1\rangle\\0\end{pmatrix}, \tag{9}$$

$$|\psi_T^{(\mathrm{o,g})}(0)\rangle = T\begin{pmatrix}0\\|2n+1\rangle\end{pmatrix} = \begin{pmatrix}0\\|2n+1\rangle\end{pmatrix}. \tag{10}$$

The above means that if we start with an atomic ground state and an even number state we will have as the transformed initial condition the second equation in (8), $|\psi_T^{(\mathrm{e,g})}(0)\rangle = |e\rangle|2n\rangle$, and therefore we will need to solve for the Hamiltonian (see (5))

$$H_{\mathrm{e}} = \omega\hat{n} + \frac{\omega_0}{2}(-1)^{\hat{n}} - g(a + a^{\dagger}). \tag{11}$$

By writing $|\psi_T(t)\rangle = \sum_{n=0}^{\infty} E_n(t)|n\rangle$ and using (10) and (6), we obtain the system of differential equations

$$\mathrm{i}\frac{\mathrm{d}E_0}{\mathrm{d}t} = \frac{\omega_0}{2}E_0 - gE_1, \tag{12}$$

$$\mathrm{i}\frac{\mathrm{d}E_n}{\mathrm{d}t} = \left(\omega n + \frac{\omega_0}{2}(-1)^n\right)E_n - g(\sqrt{n}E_{n-1} + \sqrt{n+1}E_{n+1}),\quad n > 1. \tag{13}$$

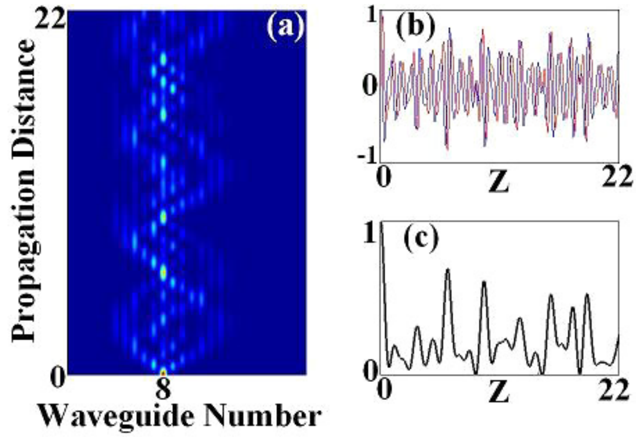

Figure 2. (a) Numerical simulation for a single input excitation for the lattice parameters $g = 0.5$, $\omega = 1$ and $\omega_0 = 1$, (b) the evolution of the field in the excitation channel (the blue/red curve is the real/imaginary part of the field) and (c) the evolution of intensity in the excitation channel.

3. Waveguide arrays to model the quantum interaction

Evanescently coupled waveguides [8] have emerged recently as a promising candidate for the realization of an ideal, one-dimensional lattice with tunable hopping [1]. We now show how the system of differential equations given in (11) and (12) may be produced in waveguide arrays, therefore modelling the ion laser Hamiltonian. By taking $t \to -Z$ we arrive at the following equation, a differential equation for propagation in a waveguide array:

$$\mathrm{i}\frac{\mathrm{d}E_0}{\mathrm{d}Z} + \frac{\omega_0}{2}E_0 - gE_1 = 0, \tag{14}$$

$$\mathrm{i}\frac{\mathrm{d}E_n}{\mathrm{d}Z} + \left(\omega n + \frac{\omega_0}{2}(-1)^n\right)E_n - g(\sqrt{n}E_{n-1} + \sqrt{n+1}E_{n+1}) = 0,\quad n > 1. \tag{15}$$

Because the propagation constant corresponds to the wave number in a potential well, then, if we change the width of the potential well, we are also changing the value of its corresponding wave number. Therefore, the best way to implement the structures described above is by changing the width of each waveguide in a descending/ascending fashion [9]. We solve numerically equations (13) and (14) and plot in figure 1 how light propagates when it is excited the eighth waveguide. The parameters used correspond to a solvable case for equation (1), because when $\omega, \omega_0 \gg g$, the rotating wave approximation (RWA) [10] can be used and we can drop from Hamiltonian (1) the counter-rotating terms $a\sigma_-$ and $a^{\dagger}\sigma_+$. In this case, if we have for instance the initial state

$$|\psi(0)\rangle = \begin{pmatrix}0\\|8\rangle\end{pmatrix} = |g\rangle|8\rangle, \tag{16}$$

i.e. the atom in its ground state and the field in a Fock (number) state $|8\rangle$, the field (and the atom, by looking at the atomic inversion) will undergo Rabi oscillations because the only states involved in the interaction are $|g\rangle|8\rangle$ and $|7\rangle|e\rangle$. This may be observed in figure 1, where it is shown how the field changes from the eighth to the seventh waveguides. In the same figure, the evolution of intensity in the excitation channel plays the role of atomic inversion. In figure 2, we plot the same quantities, but for ω, ω_0 of the same order as g. In this case, the RWA may not be applied anymore and the counter-rotating terms play a role: not only the seventh and eighth waveguides become excited now, but because of the terms $a\sigma_-$ and $a^\dagger\sigma_+$ more waveguides are involved in the interaction. This problem, of course, may not be treated analytically anymore. The oscillations in the evolution of the intensity in the excitation channel (atomic inversion) lose their ordered behavior, and no Rabi oscillations may be observed.

4. Conclusions

We have shown how the interaction of a two-level atom with a quantized field may be modelled in waveguide arrays. The transformation (4) has allowed us to divide the Hilbert space in such a way that the atom is effectively removed from the interaction. This made it possible to write the Schrödinger equation for the field states as a system of differential equations that emulates the one obtained for a semi-infinite diatomic photonic lattice.

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