Experimental Synchronization of Multiscroll Chaotic Attractors using Current-Feedback Operational Amplifiers

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Abstract

We present the experimental synchronization of two multiscroll chaotic attractors realized with current-feedback operational amplifiers. The synchronization is performed by applying Hamiltonian forms and observer approach. An application example is provided through the implementation of a master-slave communication system for chaotic encryption.

Keywords: Multiscroll, Hamiltonian forms, synchronization, secure communication, amplifier

1. Introduction

A chaotic oscillator consists of a linear part and a nonlinear part [1]. In some cases, the nonlinear part can be approached to a piecewise-linear (PWL) function [2]-[5], which is relatively easy to implement. In this manner, starting from the description of a continuous-time chaotic oscillator by state variables, its design by using current-feedback operational amplifiers (CFOAs) [6], implies the implementation of linear operators such as addition, subtraction, gain, integration and derivation; as well as the nonlinear or PWL functions [3]. In the following sections we show the realization of multiscroll chaos generators using CFOAs, which are synchronized by applying Hamiltonian forms and observer approach [7], and further a secure communication system is experimentally realized.

2. Multiscroll Chaos Generator

Chaotic oscillators based on saturated nonlinear functions (SNLFs) can be modeled using PWL approximations [1]-[5]. In Eq. (1) is described a PWL approximation called series of a SNLF, where $k \geq 2$ is the slope of the SNLF and multiplier factor to saturated plateus, $plateau = \pm nk$ with $n = integer\ odd$ to generate even-scrolls and $n = integer\ even$ to generate odd-scrolls. $h = saturated\ delay$ of the center of the slopes agreeing with $h_i = \pm mk$, where $i = 1,\ldots,[(scrolls-2)/2]$ and

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CONTENTS

$m = 2,4,\ldots,(scrolls-2)$ to generate even-scrolls; and $i = 1,\ldots,[(scrolls-1)/2]$ and $m = 1,3,\ldots,(scrolls-2)$ to generate odd-scrolls; p and q are positive integers.

To generate multiscrolls attractors (n) a non-linear controller is added as shown in Eq. (2), where $f(x;k,h,p,q)$ is defined by Eq. (3), and a,b,c,d are positive constants and must be $0 < a,b,c,d < 1$ to accomplish chaos conditions [2]. Circuit realizations using PWL functions different from SNLF series have been presented in [3], [4], [8].

The circuit realization of the SNLF based multiscroll chaos using CFOAs, is shown in the blocks labeled master and slave in Figure 1, where the SNLF is designed by the parallel connection of CFOA-based cells [1], working in the saturation region with shift bias-levels [3], as shown in Figure 2. The provided saturated voltage from the CFOA-based cell is given by Eq. (4), which includes the shifted-voltage E. To generate the SNLF E takes different values in Eq. (4) to synthesize the required plateaus and slopes. The number of cells is determined by the number of scrolls minus one [1]. The values of the circuit elements in Figure 1 are evaluated by Eq. (5), while the value of the plateaus k, in voltage and current modes, the breakpoints α, the slope and h are evaluated by Eq. (6).

$$f(x;k,h,p,q) = \sum_{i=-p}^{q} f_i(x;h,k) \tag{1}$$

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -ax - by - cz - df(x;k,h,p,q) \end{aligned} \tag{2}$$

$$f(x;k,h,p,q) = \begin{cases} (2q+1)k & \text{if} \quad x > qh+1 \\ k(x-ih)+2ik & \text{if} \quad |x-ih| \le 1 \\ & \qquad -p \le i \le q \\ (2i+1)k & \text{if } ih+1 < x < (i+1)h-1 \\ & \qquad -p \le i \le q-1 \\ -(2p+1)k & \text{if} \quad x < -ph-1 \end{cases} \tag{3}$$

$$V_o = \frac{A_v}{2}\left(\left|V_i + \frac{V_{sat}}{A_v} - E\right| - \left|V_i - \frac{V_{sat}}{A_v} - E\right|\right) \quad V_o = \frac{A_v}{2}\left(\left|V_i + \frac{V_{sat}}{A_v} + E\right| - \left|V_i - \frac{V_{sat}}{A_v} + E\right|\right) \tag{4}$$

$$C = \frac{1}{0.7Rix}, \qquad Rx = Ry = Rz = \frac{1}{0.7C}, \qquad R = \frac{1}{C} \tag{5}$$

$$k = R_{ix} I_{sat}, \qquad I_{sat} = \frac{V_{sat}}{R_C}, \quad \alpha = \frac{R_{iz}|V_{sat}|}{R_{fz}}, \quad s = \frac{h}{\alpha}, \quad h = \frac{E_i}{\left(1 + \frac{R_{iz}}{R_{fz}}\right)} \tag{6}$$

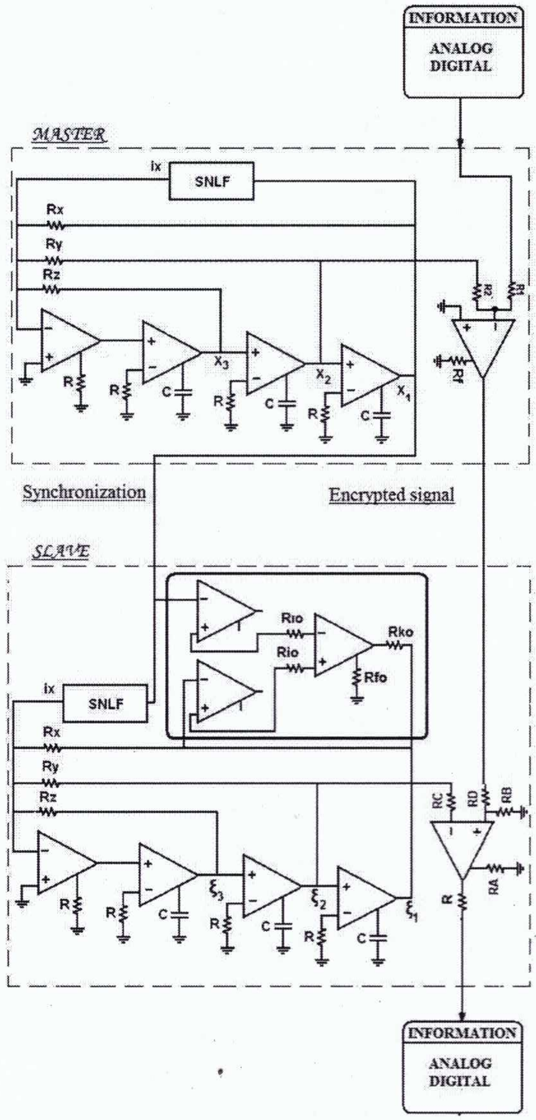


Figure 1. Chaotic transmission system using CFOAs.

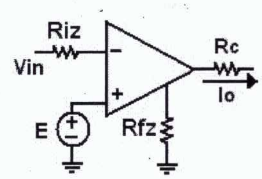


Figure 2. Proposed CFOA-based cell to generate SNLFs.

By using the commercially available CFOA AD844, as already done in [3]-[4], [8], and by setting R_{ix}=10KΩ, C=2.2nf, R=7KΩ, R_x=R_y=R_z=10KΩ, Rf=10KΩ, R_i=10KΩ in Figure 1, and Rix=10KΩ, Rc=64KΩ, Riz=1KΩ, Rfz=1MΩ, E1=±2v with Vsat= +7.24v and -7.28v, we get a chaos generator with N=4-scrolls, F=10Khz and EL=±4v, as shown in Figure 3.

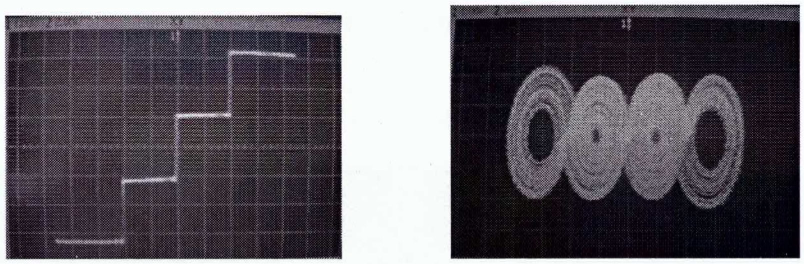


Figure 3. Experimental result of the SNLF and generation of 4-scroll attractors.

3. Synchronization of Two Multiscroll Chaos Generators

The first chaotic synchronization method was proposed by Pecora and Caroll [9], and they implicitly propose to create this stable dynamic error system by observing what they called the transverse Lyapunov exponent [5], of the observer. These exponents were used to indicate the stability of the error system. Recently, other implicitly based method which requires no Lyapunov exponent calculation is a Hamiltonian forms and observer approach [7], that we will use to synchronize the multiscroll chaos generator using commercially available CFOAs.

Lets us consider the dynamic system described by the master circuit in Eq. (7). A slave system is a copy of the master and can be described by Eq. (8). Therefore: Two chaotic systems described by a set of states $x_1, x_2, \ldots x_n$ and $\xi_1, \xi_2, \ldots \xi_n$ will synchronize if Eq. (9) fulfills [7], for any initial conditions $x(0) \neq \xi(0)$. Due to the real limitations of electronic devices [3], a tolerance value is used in practical applications [8], as shown by Eq. (10), where ε is the allowed tolerance value and a time $t_f < \infty$ is assumed. The synchronization error is then defined by Eq. (11).

$$\dot{x} = F(x) \qquad \forall x \in \Re^n \tag{7}$$

$$\dot{\xi} = F(x) \qquad \forall x \in \Re^n \tag{8}$$

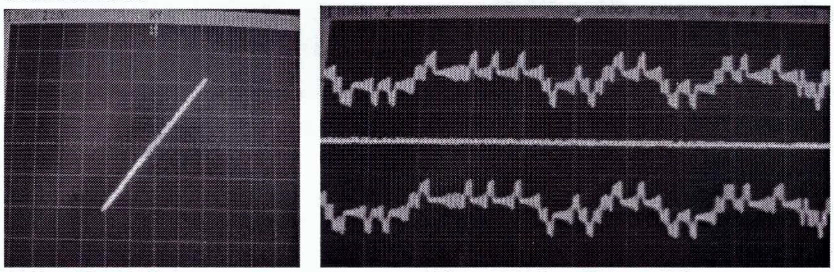
$$\lim_{t \to \infty} |x(t) - \xi(t)| \equiv 0 \tag{9}$$

$$|x(t) - \xi(t)| \leq \varepsilon_t \qquad \forall t \geq t_f \tag{10}$$

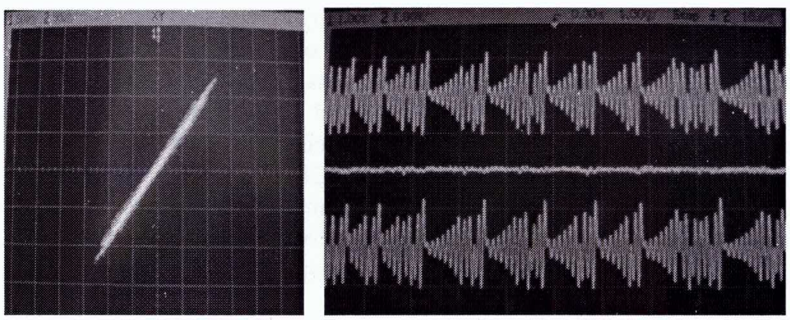
$$e(t) = x(t) - \xi(t) \tag{11}$$

To synchronize two multiscroll chaos generators systems by applying Hamiltonian forms and observer approach, their equations must be placed in the Generalized Hamiltonian Canonical form [7], [8]. Our proposed circuit realization by using CFOAs is the master-slave topology shown in Figure 1. The synchronization circuitry is embedded into the slave. In the right side is shown the communication channel for information encryption.

The experimental synchronization result by selecting Rio = 10kΩ, Rfo = 3.9MΩ and Rko=3Ω is shown in Figure 4. The transmission method is performed on a single channel (for simplicity). The confidential message *m(t)* is encrypted with a chaotic signal $x_2(t)$ by an additive process. Message recovery is performed by a subtraction to the signal received $\bar{y} = x_2(t) + m(t)$. The error in synchrony is given by $e_1(t) = x_1(t) - \hat{x}_1(t) = 0$, thus $\hat{m}(t) = m(t)$ [8]. In our experiment, the message to convey is a sine wave of frequency f = 10Khz and 500mV in amplitude. Figure 6 shows the experimental result of the secure transmission.

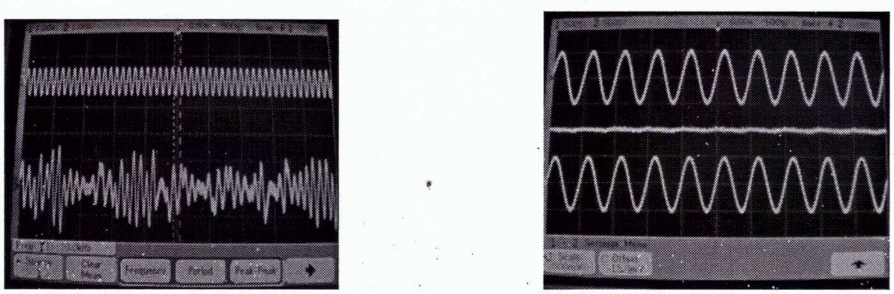


(a) (X: 2V/div; Y:2V/div) (Ch1: 5V/div; Ch2:5V/div)



(b) (X: 500mV/div; Y:500mV/div) (Ch1: 1V/div; Ch2:1V/div)

Figure 4. Experimental synchronization: Diagram phase plane and time signal (a) X_1 vs ξ_1, (b) X_2 vs ξ_2.



(a) Ch1:1V/div; Ch2:1V/div
Ch1 :*m(t)*, Ch2 : $x_2(t) + m(t)$

(b) Ch1:500mV/div; Ch2:500mV/div;
Center:500mV/div

Figure 5. (a) Message and encrypted information, (b) Information retrieval, input and output signals.

Conclusion

It was presented the experimental realization of multiscroll chaos generators consisting of PWL functions as nonlinear elements. The commercially available CFOA AD844 was used to implement the PWL function to generate multiscroll attractors.

Two multiscroll chaos generators were synchronized by Hamiltonian forms and observer approach, and by using CFOAs to implement a chaos-based secure communication system. As a result, the experiments demonstrate the usefulness of the CFOA to realize chaos systems applications.

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