

Maximally Flat CIC Compensation Filter: Design and Multiplierless Implementation

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Abstract—This brief introduces a design and implementation of maximally flat cascaded integrator comb compensation filters. In particular, we consider second- and fourth-order linear phase filters for narrow-band and wideband compensation. Closed-form equations for the computation of the filter coefficients are given. The multiplierless implementation is also considered. The number of adders is a function of the decimation factor D and the number of stages N . The implementation complexity is discussed, and comparisons with some methods reported in the literature are provided.

Index Terms—Cascaded integrator comb filters, decimation, finite impulse response filters, linear phase.

I. INTRODUCTION

THE cascaded integrator comb (CIC) filters are an economical class of linear phase finite-impulse response (FIR) filters [1]; they are used for decimation and interpolation because of their simple multiplierless structure. The transfer function of the CIC filter is defined by

$$H(z) = \left(\frac{1}{D} \frac{1 - z^{-D}}{1 - z^{-1}} \right)^N \quad (1)$$

where D and N are the decimation factor and the number of stages, respectively.

The resulting CIC magnitude response characteristic has a low attenuation in the alias bands and a droop in the passband region, which consequently introduce distortion in the signal after decimation. The simplest method to improve alias rejection is to increase the number of the CIC stages. It is common in many applications to use $N = 5$ stages. Increasing the number of the stages causes an increase in the passband droop. Alternative methods improve the stopband alias rejection, e.g., [2]. Similarly to [3]–[8], in this brief, we only consider the problem of the passband droop compensation.

A second-order FIR compensation filter is proposed in [4]. The method is based on the minimization of the error function in the least-square sense. In a similar way, a generalized approach that includes least-square, minimax, and maximally flat designs is given in [6]. An extension to include the new L_1

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optimization technique is described in [7]. This design results in less peak error ripple in the region of interest compared with the least-square and minimax optimizations. Additionally, the resulting implementation structures of the methods in [4], [6], and [7] require at least one multiplier.

The method proposed in [3] describes the multiplierless design of a second-order compensation filter. The filter coefficients expressed as a sum of power of two (SOPOT) are computed using a random search algorithm [3]. Similarly, in [5] and [8], the authors proposed simple second-order compensation filters for the narrow-band and wideband compensation, respectively, with low computational complexity.

The main goal of this brief is to extend the results given in [6] to include the wideband compensation as well as to efficiently implement the compensator filters. The novelties of this brief are new maximally flat compensators for the narrow-band and wideband, where the closed-form equations are proposed for computing the filter coefficients. Additionally, the multiplierless implementation is proposed.

The rest of this brief is organized as follows. In Section II, we propose a set of linear equations to design a linear phase maximally flat compensation filter of even order. In particular, narrow-band design uses the second-order filter, whereas the fourth-order filter is used for wideband design. Closed-form equations to compute the filter coefficients for narrow-band and wideband designs are also given. Section III deals with the multiplierless implementation of the proposed maximally flat compensators. The implementation complexity in terms of the number of adders is also provided. Finally, discussions of results are given in Section IV.

II. MAXIMALLY FLAT COMPENSATORS

This section deals with the design of a linear phase FIR compensation filter, with the maximally flat magnitude response. The proposed compensated CIC filter is expressed as

$$G(z) = H(z)P(z^D) \quad (2)$$

where $H(z)$ is the CIC filter defined in (1), and $P(z)$ is a linear phase filter. In general, there are four cases of linear phase filters that should be considered, i.e., Type I–IV FIR filters. However, the Type II–IV FIR filters imply at least one zero on unit circle [9], which is inconvenient for the expanded filter design $P(z^D)$. Accordingly, we consider the Type I linear phase FIR filter.

Usually, the CIC decimator filter is followed by a second decimator stage. The decimator factor ν of the second stage determines the passband edge frequency ω_p , where the worst passband droop occurs, $\omega_p = \pi/(D\nu)$ [2]. Depending on the value ν , we have narrow-band and wideband designs, that is, narrow-band results if $\nu \geq 4$; otherwise, wideband is considered.

From (1), the frequency response of the CIC filter is given by

$$H(e^{j\omega}) = e^{-j(D-1)N\omega/2} H_R(\omega) \quad (3)$$

where $H_R(\omega)$ is a real-valued function given by

$$H_R(\omega) = \left(\frac{1}{D} \frac{\sin(D\omega/2)}{\sin(\omega/2)} \right)^N. \quad (4)$$

We define the Type I linear phase FIR compensation filter $P(z)$ as

$$P(z) = \sum_{n=0}^L a_n z^{-n} \quad (5)$$

where L is an even integer and is the order of $P(z)$, a_n , $n = 0, \dots, L$, are the filter coefficients, and a_n satisfies $a_n = a_{L-n}$. The corresponding frequency response is [9]

$$P(e^{j\omega}) = e^{-jL\omega/2} P_R(\omega) \quad (6)$$

where the real-valued function $P_R(\omega)$ is expressed as [9]

$$P_R(\omega) = a_{L/2} + 2 \sum_{n=0}^{L/2-1} a_n \cos(\omega(L/2 - n)). \quad (7)$$

Consequently, the corresponding frequency response of $G(z)$ can be written as

$$G(e^{j\omega}) = e^{-j\omega((D-1)N+LD)/2} H_R(\omega) P_R(D\omega). \quad (8)$$

From (8), it is clear that the overall filter $G(e^{j\omega})$ has a linear phase, which avoids phase distortion of the input signal in the passband. We now impose the maximally flat condition onto the magnitude response.

In order to design the compensation filter $P(z)$, we define the error function as [6]

$$E(\omega) = P_R(D\omega) H_R(\omega) - 1. \quad (9)$$

The condition that the error function $E(\omega)$ is maximally flat at $\omega = 0$ is that it has as many derivatives as possible that are vanishing at $\omega = 0$ [10]. Since the error function is an even function of ω , its odd indexed derivatives evaluated at $\omega = 0$ are automatically zero [10]. Therefore, the maximally flat conditions are

$$E(0) = 0 \quad (10a)$$

$$\left. \frac{d^p E(\omega)}{d\omega^p} \right|_{\omega=0} = 0 \quad (10b)$$

where p is even and positive integer, i.e., $p = 2q$ for $q = 1, \dots, L/2$.

Using (9), (10a) implies

$$P_R(0) = 1. \quad (11)$$

Now, substituting (9) into (10b) and using the general Leibniz rule for the p th derivative of a product, we arrive at

$$\frac{d^p H_R(\omega)}{d\omega^p} + \sum_{\ell=1}^p \binom{p}{\ell} \left[\frac{d^\ell P_R(D\omega)}{d\omega^\ell} \frac{d^{p-\ell} H_R(\omega)}{d\omega^{p-\ell}} \right]_{\omega=0} = 0 \quad (12)$$

where the binomial coefficient is given by

$$\binom{p}{\ell} = \frac{p!}{\ell!(p-\ell)!}. \quad (13)$$

The odd indexed derivatives of $P_R(D\omega)$ evaluated at $\omega = 0$ are zero. Therefore, from (7), we obtain

$$\left. \frac{d^\ell P_R(D\omega)}{d\omega^\ell} \right|_{\omega=0} = \begin{cases} 2(-1)^{\ell/2} D^\ell \sum_{n=0}^{L/2-1} (L/2 - n)^\ell a_n, & \ell \text{ even} \\ 0, & \ell \text{ odd.} \end{cases} \quad (14)$$

Substituting (14) into (12), (10b) can be rewritten as

$$2 \sum_{\ell=1}^q \binom{2q}{2\ell} (-1)^\ell D^{2\ell} \times \sum_{n=0}^{L/2-1} (L/2 - n)^{2\ell} a_n \left[\frac{d^{2(q-\ell)} H_R(\omega)}{d\omega^{2(q-\ell)}} \right]_{\omega=0} = - \frac{d^{2q} H_R(\omega)}{d\omega^{2q}} \quad (15)$$

for $q = 1, \dots, L/2$. The coefficients of the linear phase maximally flat compensation filter $P(z)$ of the order L are obtained by solving the set of linear equations (15). In particular, we consider the second-order filter for narrow-band design and the fourth-order filter for wideband design as they are described in the following subsections.

A. Narrow-Band Compensator

This section focuses on the design of the narrow-band compensator filter based on the second-order linear phase filter, which is equivalent to that of [6]. The resulting magnitude characteristics of the compensated CIC filter exhibits a maximally flat characteristic at $\omega = 0$. The multiplierless implementation of the narrow-band compensator is discussed in Section III.

Using (5) with $L=2$, the transfer function of $P(z)$ is given by

$$P(z) = a_0 + a_1 z^{-1} + a_0 z^{-2}. \quad (16)$$

Accordingly, the maximally flat conditions given by (11) and (15) are equivalent to

$$a_1 + 2a_0 = 1 \quad (17a)$$

$$a_0 = \frac{1}{2D^2} \left. \frac{d^2 H_R(\omega)}{d\omega^2} \right|_{\omega=0} \quad (17b)$$

where

$$\left. \frac{d^2 H_R(\omega)}{d\omega^2} \right|_{\omega=0} = - \frac{N(D^2 - 1)}{12}. \quad (18)$$

Solving the set of linear equations (17), we arrive at

$$a_0 = - \frac{N}{32} \frac{1 - D^{-2}}{1 - 2^{-2}}, \quad a_1 = 1 - 2a_0. \quad (19)$$

We define the following constant:

$$A = \frac{1 - D^{-2}}{1 - 2^{-2}}. \quad (20)$$

Consequently, the filter coefficient a_0 can be rewritten as

$$a_0 = -2^{-5} N \cdot A. \quad (21)$$

The proposed filter structure for the second-order compensator is given in Fig. 1. Observe that the structure has one multiplier since the multiplier of value N can be expressed using canonic signed digit (CSD) representation [11].

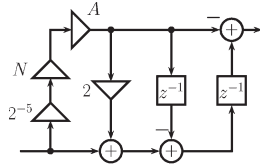


Fig. 1. Second-order compensation filter structure.

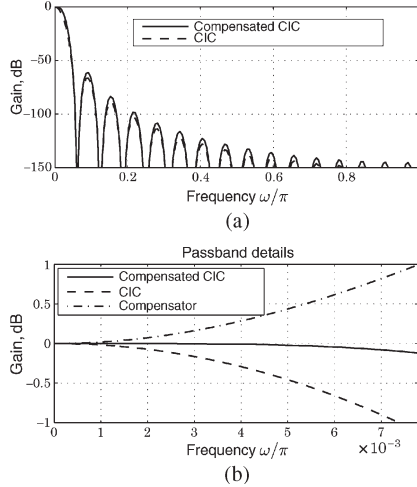


Fig. 2. Magnitude responses of the CIC, compensated CIC, and compensation filters in Example 1.

Example 1: Consider the following design parameters: the decimator factor D is equal to 32, the number of stages N is 5, and ν is 4.

Using the values D , N , and ν , the resulting passband frequency ω_p is 0.0078π rad. Fig. 2(a) illustrates the magnitude responses of both CIC and compensated CIC filters. Passband details of CIC, compensated CIC, and compensation filters are shown in Fig. 2(b). Note that the gains of CIC and compensated CIC filters at ω_p are -1.12 and -0.12 dB, respectively. Furthermore, the compensated CIC filter has the same attenuation in the alias bands as the CIC filter. Therefore, for practical purposes, the compensation does not deteriorate the attenuation in the alias bands of the CIC filter.

B. Wideband Compensator

Now, we turn our attention to the design of the wideband compensator using the fourth-order linear phase FIR filter. For $L = 4$, the transfer function $P(z)$ is

$$P(z) = a_0 + a_1 z^{-1} + a_2 z^{-2} + a_1 z^{-3} + a_0 z^{-4}. \quad (22)$$

Consequently, the maximally flat conditions (11) and (15) are rewritten as

$$2a_0 + 2a_1 + a_2 = 1 \quad (23a)$$

$$4a_0 + a_1 = \frac{1}{2D^2} \left. \frac{d^2 H_R(\omega)}{d\omega^2} \right|_{\omega=0} \quad (23b)$$

$$16a_0 + a_1 = \frac{1}{2D^4} \left(6 \left[\left. \frac{d^2 H_R(\omega)}{d\omega^2} \right]_{\omega=0}^2 - \left. \frac{d^4 H_R(\omega)}{d\omega^4} \right|_{\omega=0} \right) \quad (23c)$$

where

$$\left. \frac{d^4 H_R(\omega)}{d\omega^4} \right|_{\omega=0} = \frac{N(D^2 - 1)(D^2(5N - 2) - 5N - 2)}{240}. \quad (24)$$

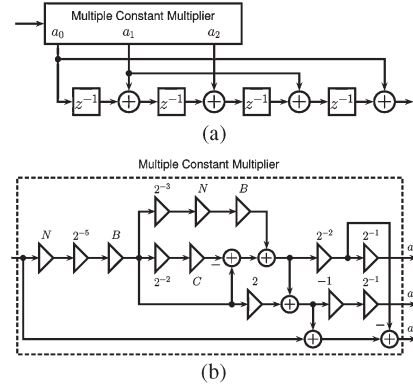


Fig. 3. Fourth-order compensation filter structure.

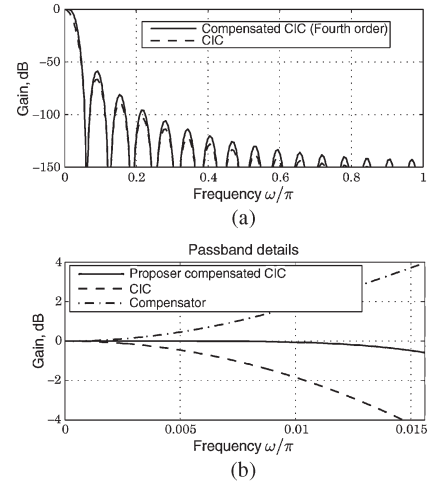


Fig. 4. Magnitude responses of the CIC and compensated CIC filters in Example 2.

Solving the set of linear equations (23), we obtain

$$a_0 = 2^{-8} N \cdot B (2^{-3} N \cdot B + 1 - 2^{-2} C) \quad (25)$$

$$a_1 = -2^{-6} N \cdot B (2^{-3} N \cdot B + 3 - 2^{-2} C) \quad (26)$$

$$a_2 = 1 - 2a_0 - 2a_1 \quad (27)$$

where

$$B = \frac{1 - D^{-2}}{1 - 2^{-2}} \quad C = \frac{1 - (2D)^{-2}}{1 - 2^{-4}}. \quad (28)$$

Observe that the filters coefficients a_0 and a_1 are related as

$$a_1 = -2^2 a_0 - 2^{-5} N \cdot B. \quad (29)$$

Consequently, the compensator can be implemented as shown in Fig. 3. Fig. 3(a) shows the implementation using a multiple constant multiplier block, which is described in Fig. 3(b). The resulting filter structure involves three multipliers, that is, two multipliers B and one multiplier C . The following example illustrates the design of the wideband compensator filter.

Example 2: Consider the following design parameters: $D = 32$, $N = 5$, and $\nu = 2$.

The passband edge frequency ω_p is 0.0156π . Fig. 4(a) shows the magnitude responses of compensated CIC and CIC filters. The gains of CIC and compensated CIC filters at ω_p are -4.54 and -0.58 dB as is shown in Fig. 4(b). Similarly, as in Example 1, the decrease in attenuation occurs only in the don't care bands and does not affect the alias bands, as shown in Fig. 4(a).

The closed-form equations for the computation of the filter coefficients are valid for any value of the decimation factor D

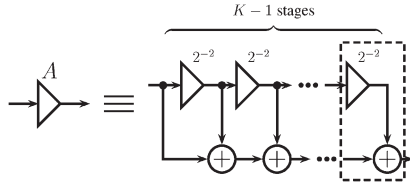


Fig. 5. Multiplierless implementation of coefficient A . The structure exhibits the hierarchical property.

and the number of stages N . In general, the implementation of the compensation filter requires multipliers. However, for some values of D , multiplierless implementation is possible.

The next section discusses the multiplierless implementation of the second- and fourth-order compensation filters, for the case in which the decimation factor is a power of two.

III. MULTIPLIERLESS IMPLEMENTATION

A. Narrow-Band Compensator

This subsection describes how coefficient A can be implemented without multipliers for the values of D expressed as a power of two. Accordingly, using $D = 2^K$, for $K > 0$, the value A becomes

$$A = \frac{1 - 2^{-2K}}{1 - 2^{-2}}. \quad (30)$$

We can recognize that A can be rewritten using a geometric series

$$A = \sum_{\ell=0}^{K-1} 2^{-2\ell}. \quad (31)$$

The multiplierless implementation of coefficient A is obtained by using the following identity:

$$\sum_{\ell=0}^{K-1} 2^{-2\ell} = 1 + 2^{-2} \sum_{\ell=0}^{K-2} 2^{-2\ell}. \quad (32)$$

Applying (32), recursively, we get the structure shown in Fig. 5, which exhibits an important property, i.e., the *hierarchical property*. This means that the resulting structure has a basic building block (see Fig. 5), which is highlighted using dashed lines. The structure has $K - 1$ stages for the decimation factor 2^K . Increasing the decimation factor D to 2^{K+1} , the new value of A is obtained just by adding a new stage to the previous structure. Additionally, all coefficients have value 2^{-2} . This favorable hierarchical property is important, for example, in a direct silicon compilation.

B. Wideband Compensator

This subsection introduces the multiplierless implementation of the fourth-order compensator. Similar to the case of the second-order compensator, we consider the case when the decimation factor D is a power of two, particularly the case $D = 2^{2M-1}$ where M is a positive integer. Consequently, from (28), coefficients B and C are rewritten as

$$B = \sum_{m=0}^{2M-2} 2^{-2m} \quad C = \sum_{m=0}^{M-1} 2^{-4m}. \quad (33)$$

From (33), it is straightforward to develop multiplierless structures with hierarchical property.

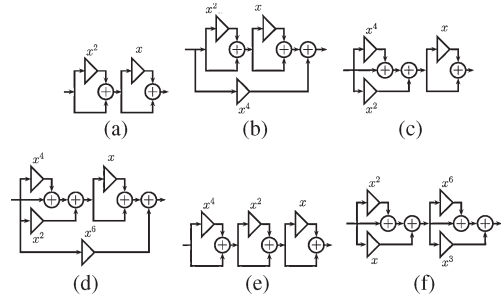


Fig. 6. Optimal structures based on the optimal approach [12]. (a) $S = 4$. (b) $S = 5$. (c) $S = 6$. (d) $S = 7$. (e) $S = 8$. (f) $S = 9$.

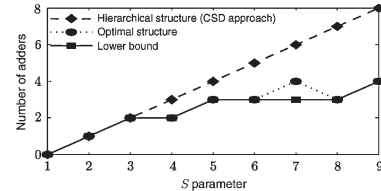


Fig. 7. Number of adders for the hierarchical and the optimal structure.

C. Optimal Structures

Observe that the SOPOT for A , B , and C given in (31) and (33) are in CSD representation, i.e., these satisfy the condition that the product of the coefficient for $2^{-\ell}$, $\ell \geq 0$, and of the coefficient for $2^{-\ell-1}$ is zero [11]. The CSD representation is unique and contains the minimum possible number of nonzero digits. Consequently, structures based on CSD representation result in a minimum number of adders. It is possible to restate all these coefficients using subexpressions to get fewer adders and optimal structures [12]. Therefore, using the approach from [12], this subsection provides optimal structures for coefficients A , B , and C .

First note that the relationships (31) and (33) have the following general form:

$$k = \sum_{\ell=0}^{S-1} x^\ell \quad (34)$$

where the values (x, S) are $(2^{-2}, K)$, $(2^{-2}, 2M - 1)$, and $(2^{-4}, M)$ for A , B , and C , respectively. The main goal is to implement coefficient k given in (34) using a minimum number of adders. Based on the optimal approach proposed in [12], we develop optimal structures shown in Fig. 6 for $S = 4, \dots, 9$. Fig. 7 compares the number of adders of the hierarchical and the optimal structure with the lower bound proposed in [13]. Notice that the hierarchical structure is optimal for $S = 1, 2, 3$. Moreover, the optimal structure achieves the lower bound except for $S = 7$.

D. Compensation Filter Complexity

The optimal implementation for the second-order compensator requires N_a adders, that is

$$N_a = 3 + N_a^A + N_a^N \quad (35)$$

where N_a^A and N_a^N denote the number of adders of A and N , respectively. Similarly, the optimal fourth-order compensator involves

$$N_a = 5 + 2N_a^B + N_a^C + 2N_a^N \quad (36)$$

where N_a^B and N_a^C stand for the number of adders of B and C , respectively. To estimate N_a^A , N_a^B , and N_a^C , we first

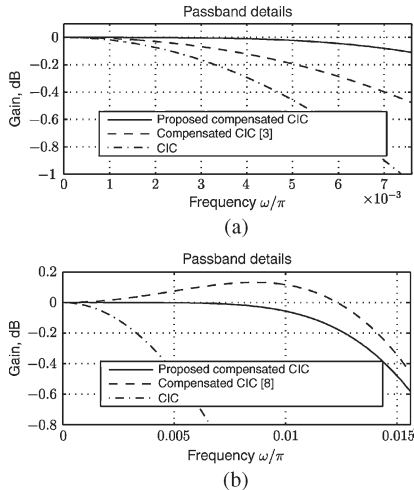


Fig. 8. Compensated CIC filters.

decompose the number S in terms of prime factors $f_i^{(S)}$, i.e., $S = \prod_{i=1}^{\Omega_S} f_i^{(S)}$, where Ω_S represents the total number of prime factors.¹ Consequently, combining the prime factorization and results [13], we have

$$N_a^A = \sum_{i=1}^{\Omega_K} \left\lceil \log_2 \left(f_i^{(K)} \right) \right\rceil$$

$$N_a^B = \sum_{i=1}^{\Omega_{(2M-1)}} \left\lceil \log_2 \left(f_i^{(2M-1)} \right) \right\rceil \quad (37)$$

$$N_a^C = \sum_{i=1}^{\Omega_M} \left\lceil \log_2 \left(f_i^{(M)} \right) \right\rceil \quad (38)$$

where $\lceil \cdot \rceil$ stands for the ceiling function. Additionally, the complexity of N is based on the CSD representation.

Example 3: We compute the implementation complexity for Examples 1 and 2. Considering that N needs one adder, and using $K = 5$, the optimal second-order compensation filter requires seven adders. Similarly, with $M = 3$, the optimal fourth-order compensator needs 15 adders.

IV. DISCUSSIONS

The proposed maximally flat compensated CIC filter is compared with some methods recently proposed in the literature. We first compare the performance of the proposed compensation filter with the method [3] for narrow-band compensation. That method presented a second-order compensation filter where the filter coefficients expressed as a SOPOT are computed using a random search algorithm [3]. Fig. 8(a) compares the proposed approach and the method [3] using the design parameters given in Example 1. Observe that the proposed compensator provides better compensation in the passband region, i.e., $0 \leq \omega \leq \omega_p$. Additionally, our compensator offers improved frequency response and implementation complexity. The implementation of the compensator [3] involves three adders and two delays. Unfortunately, the compensator [3] does not provide optimal magnitude response.

For the wideband compensator, we compare the performance of the compensation filter with the method in [8]. In [8], the author proposed a multiplierless compensator filter as a cascade of second-order linear phase FIR filters. The filter coefficients of

the second-order filter are computed by the minimization of the squared error in the passband. Fig. 8(b) illustrates the magnitude responses of the proposed compensator filter and the compensator [8] using the design parameters given in Example 2. Notice that both methods approximately offer the same compensation in the passband region. However, our design provides flat magnitude response in the first half of the passband and optimal implementation complexity. The complexity of the compensator [8] is 12 adders and 12 delays.

V. CONCLUDING REMARKS

This brief introduces a design of maximally flat CIC passband compensator filters. The filter coefficients are obtained by solving a set of linear equations. We described in detail two special cases, i.e., second- and fourth-order filters for the narrow-band and wideband compensations, respectively. The corresponding filter coefficients have closed-form equations. The multiplierless implementation complexity of the proposed filters depends on the decimation factor D and the number of stages N . However, there is a restriction on the values of the decimation factor: for the narrow-band design, decimation factor D must be a power of two, whereas for the wideband design, it must be in the form 2^{2M-1} , for $M > 0$. In the stopband region, the compensation filter does not deteriorate the attenuation in the alias bands of the CIC filter.

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