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Simulation Algorithm for Ronchigrams of Spherical and Aspherical Surfaces, with the Lateral Shear Interferometry Formalism

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With the Ronchi test a technician controls the manufacturing process using the following procedure: first, a Ronchigram is simulated which is scale-printed and placed on the surface as a mask and is usually printed with a low spatial resolution. This simulated Ronchigram is compared visually with the experimental pattern observed. This way of comparison leads to systematic errors in the evaluation of the surface, because it depends largely on the experience of the technician. Therefore, the main objectives of this work are to increase the spatial resolution and eliminate the dependence on the technician's experience. Therefore, we compare the simulated Ronchigrams obtained by lateral shear interferometry, whose profiles are cosine, with Ronchigrams obtained experimentally. We present the simulation algorithm for the Ronchigrams of spherical and aspherical surfaces based on the expressions of a lateral shear interferometer. We show the results, of the comparison between simulated Ronchigrams (ray tracing and lateral shear interferometry) and experimental Ronchigrams.

Keywords: Ronchi test, aspherical surfaces, lateral shear interferometry

1. Introduction

In the Ronchi test, one observes at the exit pupil the reflection of a ray of light passing through a ruling with bright and dark parallel bands equally spaced, placed near the radius of curvature, producing an interference pattern, called Ronchigram (Fig. 1). This is one of the most powerful methods used to measure aberrations of optical surfaces and systems; the wavefronts errors can be estimated from the deviation of the observed fringe pattern on the exit pupil in the comparison between simulated and calculated patterns.[1,2)]

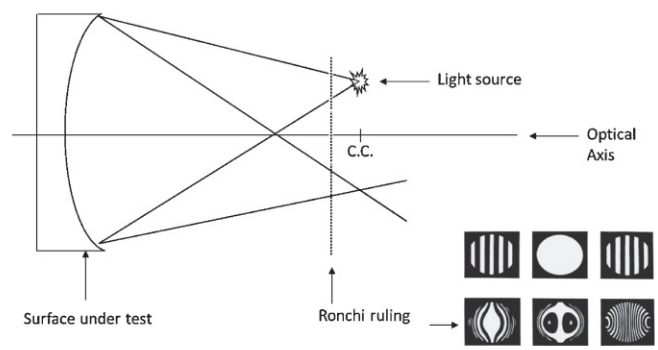


Fig. 1. Basic configuration of Ronchi test.

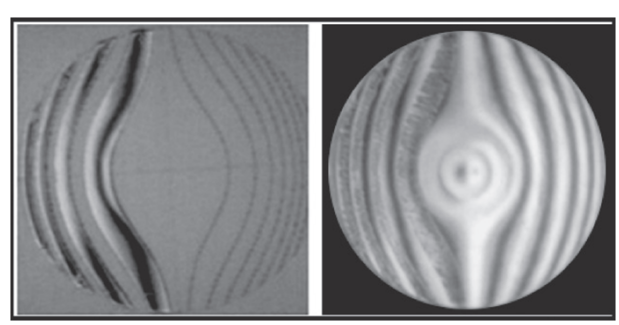


Fig. 2. Ronchigram simulated by ray tracing. (a) Mask placed on the surface being produced. (b) Theoretical fringes marked on the surface.

Generally, in optical fabrication shops, a technician uses the Ronchi test to verify the manufacturing process of an optical surface, by simulating a Ronchigram generated from a ray tracing algorithm,[2–5)] which is scale-printed as a mask. This Ronchigram generally has a low spatial resolution (Fig. 2), i.e., it is composed of three to seven fringes; consequently, it has areas not adequately verified. The Ronchigram is placed on the surface being produced and is compared with the experimental pattern, the manufacturing process of the surface ends when the experimental fringes match the mask. This process leads to systematic errors in the evaluation of the surface, one of which depends on the criterion of the observer, which is qualitative. Another error introduced is the generation of mask fringes, which are generated in the binary form, while the observed irradiance pattern on the exit pupil of the system is in a cosine form, which also limits a direct comparison of images by computational methods.

In this paper we describe a method for increasing the spatial resolution (i.e., have more fringes and also eliminate the criterion of the observer, thus providing a quantitative test) by comparing the experimental Ronchigram with a simulated one, which is obtained from an algorithm based on lateral shear interferometry. With this new proposed algorithm, we can calculate the irradiance pattern with a cosine form that is actually observed experimentally and automatically.

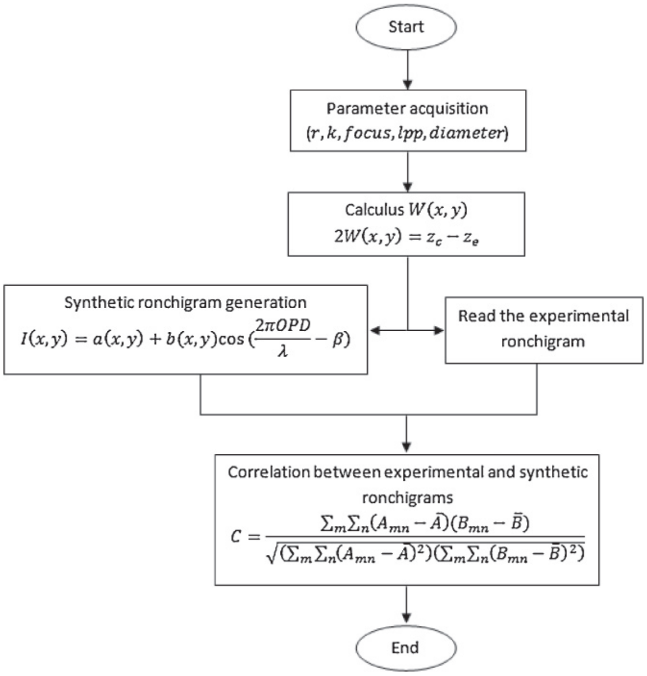


Fig. 3. Flowchart of the algorithm.

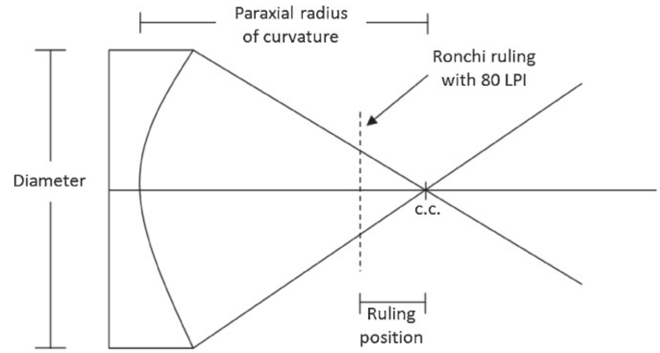


Fig. 4. Parameters used in the Ronchi test.

Table 1. Values of conic constants for conic surfaces.

Type of conic surface	Conic constant value (k)
Hyperboloid	$k < -1$
Paraboloid	$k = -1$
Ellipse rotated about its major axis	$-1 < k < 0$
Sphere	$k = 0$
Ellipse rotated about its minor axis	$k > 0$

In the following section, we describe the algorithm we developed for simulating Ronchigrams with cosine irradiance. In Sect. 3, we present examples of Ronchigrams simulated with our algorithm and compared against experimental and simulated by the conventional ray tracing techniques. Finally, some conclusions are presented about the work done.

2. Algorithm Description

In this section we describe the algorithm we developed, shown in a flowchart in Fig. 3.

Step 1: Parameter acquisition

In this step we introduce the geometric parameters of the surface that we wish to generate, which are the paraxial radius of curvature r, the conic constant k, and the diameter; and also the distance from the exit pupil of the surface to the Ronchi ruling, D, and the lines per millimeter of the ruling, *lpi* (see Fig. 4).

Step 2: Optical path difference calculus (OPD) and synthetic Ronchigram generation

The original wavefront calculus is realized with the differences between the sagitta for the aspheric surface to be tested $[z_c(x,y)]$ and its corresponding osculating sphere $[z_e(x,y)]$[1] then

$$2W(x,y) = z_c(x,y) - z_e(x,y), \qquad (1)$$

where the sagitta is defined by the next relation,[1]

$$z = \frac{c\rho^2}{1+\sqrt{1-(k+1)c^2\rho^2}} + A_1\rho^4 + A_2\rho^6 + A_3\rho^8 + A_4\rho^{10}, \qquad (2)$$

where c is the inverse of the radius of curvature, ρ^2 is the spatial coordinates of the pupil (x^2+y^2), and k is the conic constant for the surface, whose values are shown in Table 1; A_1, A_2, A_3, and A_4 are the aspheric deformation constants.

To generate a synthetic Ronchigram, the Ronchi test results can be analyzed as a lateral shear interferometer,[6–8] where the optical path difference is given in one dimension by

$$\mathrm{OPD} = \frac{\partial W(x,y)}{\partial x}\Delta x \cong W(x,y) - W(x+\Delta x, y), \qquad (3)$$

where $W(x,y)$ and $W(x+\Delta x, y)$ are the original and shear wavefronts given by Eq. (1), respectively, while Δx is the shear displacement between the diffraction orders generated by the Ronchi ruling[1] as

$$\Delta x = \frac{\lambda r}{d}; \qquad (4)$$

where λ is the wavelength used, d is the Ronchi ruling period, and r is the curvature radius of the surface under test.

If we place a light detector at an observation plane, a distorted irradiance pattern[9] will be approximately given by

$$I(x,y) = a(x,y) + b(x,y)\cos\left(\frac{2\pi\,\mathrm{OPD}}{\lambda} - \beta\right), \qquad (5)$$

where $a(x,y)$ and $b(x,y)$ are the background illumination and local contrast coefficients, respectively, and the parameter β is determined by the lateral displacement of the ruling position.

Table 2. Correlation values over entire image and within the exit pupil.

Correlation values	Ray tracing vs experimental values	Proposed algorithm vs experimental values
Over entire image	0.7486	0.8771
Within exit pupil	0.0890	0.8585

Step 3: Correlation between the simulated and experimental Ronchigrams

Now, we proceed to the comparison of the simulated Ronchigram with the experimental Ronchigram under the criterion of the two-dimensional correlation between two images. Initially a routine was used for the correlation between the full images of the Ronchigrams; however, this analysis was incorrect, since the correlation was determined from the entire images, i.e., taking into account the areas outside of the exit pupil.

To solve the above problem we generate an algorithm for the calculation of the correlation within the exit pupil of the system, with the equation

$$C = \frac{\sum_m \sum_n (A_{mn} - \bar{A})(B_{mn} - \bar{B})}{\sqrt{\left(\sum_m \sum_n (A_{mn} - \bar{A})^2\right)\left(\sum_m \sum_n (B_{mn} - \bar{B})^2\right)}}, \quad (6)$$

where A and B are the matrices of the intensity values of the simulated and experimental Ronchigrams, respectively; while $\bar{A}$ and $\bar{B}$ are the average intensity levels of their respective Ronchigrams.[10] C takes values between *0* and *1*, where *0* means that they are not equal and *1* means that both images are equal.

We take the Ronchigram images in Fig. 7(a) for the calculation of the correlation over the entire image and the results obtained are shown in Table 2. This demonstrates an inconsistency in the correlation values for the ray tracing versus experimental and therefore the correlation is only calculated within the exit pupil of the system.

3. Experimental Results

To verify the performance of the proposed algorithm two aspherical surfaces and one spherical surface were analyzed. Experimental Ronchigrams with different number of fringes were acquired and compared with simulated Ronchigrams using the algorithm described in previous sections, as well as with one obtained by ray tracing. As can be seen in the next sections, better results are derived by applying the algorithm proposed in this paper, than by applying the ray tracing for the simulated Ronchigrams. The Ronchi ruling used to acquire the Ronchigrams has $\approx$3.15 lines per millimeter (80 lines per inch).

3.1 Spherical surface

The spherical surface being tested has a diameter of 12.80 cm and a curvature radius of 49.85 cm. The Ronchi ruling positions in Figs. 5(a) and 5(b) are 49.65 and 49.30 cm, respectively. These positions were measured with a commercial measuring tape.

In an experiment, several Ronchigrams were simulated at different positions of the Ronchi ruling, to verify the reliability of measurements. By observing the variation of the obtained correlation coefficient, the values measured with the commercial measuring tape were erroneous, that is, a measured value of 49.65 cm was obtained with a correlation coefficient of 0.8126, whereas for a value of 49.63 cm the correlation coefficient was 0.8277 [Fig. 5(a)]. This means an error of 0.02 cm; thus, one conclusion is that the Ronchi test with the proposed algorithm is insensitive to an error in the Ronchi ruling position. For an experimental Ronchigram, several simulated Ronchigrams were calculated for different ruling positions; therefore different correlation values were derived. The best correlation value in the experimental and simulated Ronchigrams is associated with the ruling position closest to the experimental position. With this technique we can eliminate the errors introduced by our experimental measurement of the position of the ruling.

3.2 Parabolic surface

The parabolic surface being tested has a paraxial curvature radius of 273.10 cm, a diameter of 20.50 cm, and a conic constant of -1. The images shown in Fig. 6 were taken while the surface was in the manufacturing process; this is the reason for the deformations on the experimental Ronchigram fringes. The Ronchi ruling positions in Figs. 6(a) and 6(b) are 272.35 and 271.45 cm, respectively.

From Fig. 6 we can see that the value of correlation obtained by comparing ray tracing Ronchigrams with the experimental ones remains low; again, the value increases for the comparison between the Ronchigrams generated with our algorithm and the experimental ones. The value remains low because the Ronchigrams were taken during the manufacturing process and the surface was slightly deformed.

3.3 Hyperbolic surface

The hyperbolic surface being tested has a paraxial curvature radius of 53.30 cm, a diameter of 7.32 cm, and a conic constant of -3.65. The Ronchi ruling positions in Figs. 7(a) and 7(b) are 53.25 and 52.90 cm, respectively.

From Fig. 7, we can observe that the correlation value is very high, since for this case the Ronchigrams were taken when the technician indicated that the polishing process of the surface was finished. We can see a good quality of the fringes except for the problems at the edge of the surface that occur in the conventional polishing process.

3.4 Increasing spatial resolution

One of the objectives of this study is to increase the spatial resolution of the analyzed Ronchigrams. In the previous subsections, three optical surfaces were analyzed. In Figs. 5–7, we can see the Ronchigrams with a few fringes,

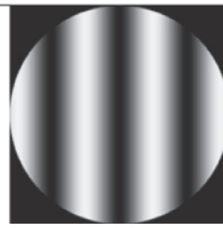
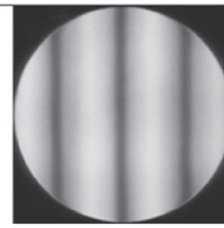
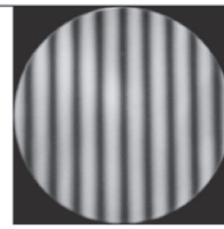
	Ronchigrams with ray tracing	Simulated ronchigrams	Experimental Ronchigrams	Correlation coefficient	
				Ray tracing vs Experimental	Proposed algorithm vs Experimental
(a)				0.1596	0.8126
(b)				0.1886	0.7648

Fig. 5. Ronchigrams of a spherical surface. Ruling positions: (a) 49.65 cm; (b) 49.30 cm.

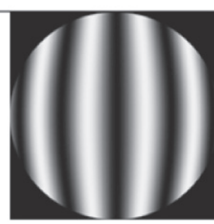
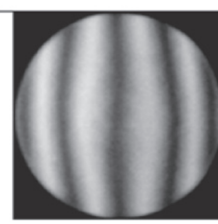
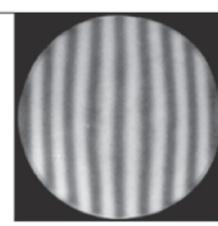
	Ronchigrams with ray tracing	Simulated ronchigrams	Experimental Ronchigrams	Correlation coefficient	
				Ray tracing vs Experimental	Proposed algorithm vs Experimental
(a)				0.1304	0.7868
(b)				0.1695	0.8280

Fig. 6. Ronchigrams of a parabolic surface. Ruling positions: (a) 272.35 cm; (b) 271.45 cm.

i.e., low spatial resolution. This is because the technician usually uses a simulated Ronchigram, scale-printed as a mask and since it is easier for the technician to visually analyze few fringes; a small number of fringes are used to fabricate a surface.

Figures 8(a) and 8(b) show the experimental and simulated Ronchigrams with the proposed algorithm for a spherical surface of 9.0 cm in diameter and a curvature radius of 60.1 cm. The Ronchigrams have 7 and 17 fringes, respectively. Note that the correlation coefficient remains high even when the number of fringes is increased. This validates the usefulness of the proposed algorithm for reproducing Ronchigrams with a high spatial resolution.

4. Conclusions

An algorithm was developed that simulates cosine profile Ronchigrams for aspheric and spherical surfaces, these simulated Ronchigrams to match cosine profiles observed in experimental Ronchigrams. This algorithm allows us to determine a correlation parameter, and subsequently obtain the surface quality desired in the polishing process. To evaluate the algorithm we developed, the same comparison

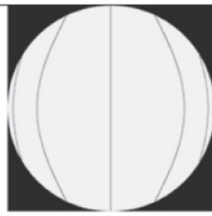
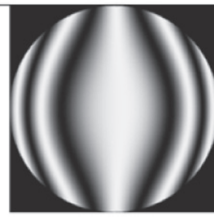
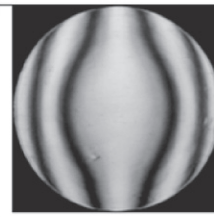
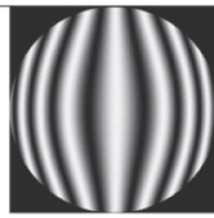
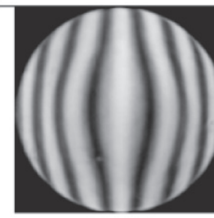
	Ronchigrams with ray tracing	Simulated ronchigrams	Experimental Ronchigrams	Correlation coefficient	
				Ray tracing vs Experimental	Proposed algorithm vs Experimental
(a)				0.0890	0.9061
(b)				0.1220	0.9017

Fig. 7. Ronchigrams of a hyperbolic surface. Ruling positions: (a) 53.25 cm; (b) 52.90 cm.

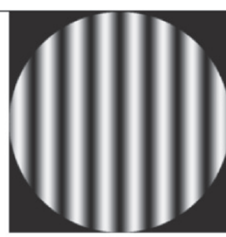
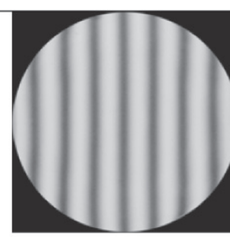
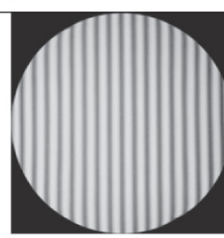
	Simulated Ronchigram	Experimental Ronchigram	Correlation coefficient
(a)			0.8741
(b)			0.8537

Fig. 8. Correlation coefficient values: (a) low spatial resolution; (b) high spatial resolution.

was carried out for experimental Ronchigrams against ray tracing-simulated Ronchigrams. As can be seen from Figs. 5–7, better results are derived from the algorithm proposed here.

This proposed method eliminates the systematic errors introduced by the technician in the surface evaluation during the fabrication process, since the proposed method presented in this paper is quantitative. In this comparison, a correlation operation was established between experimental and simulated Ronchigrams, where you can use the correlation coefficient as a reference to monitor the fabrication of an optical surface, and provide the technician a new tool that can increase the accuracy of the fabrication of optical surfaces.

We found that the correlation value is low by comparing experimental and ray tracing Ronchigrams. This finding is quite reasonable owing to the way of generating the Ronchigram images, in which only the central position of each fringe in the Ronchigram is calculated and plotted. On the other hand, the correlation is high when comparing experimental Ronchigrams and those generated with the proposed algorithm, because both have cosine profiles. Since no other method of generating Ronchigrams with the cosine profile exists in the literature, we compared our algorithm with ray tracing because this is the way the technicians generate reference Ronchigram masks in optical shops. An important result is that the proposed algorithm gives us a computational method to compare Ronchigram images, without the need of scale-printed masks.

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