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# Novel droop-compensated comb decimation filter with improved alias rejections

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#### ABSTRACT

This paper presents a novel two-stage comb decimator with the improved magnitude characteristic. Simple multiplierless corrector filters, which are designed using the frequency sampling and IFIR methods, are introduced. The proposed filters compensate the comb passband droop in the wideband passband region and increase the attenuations in the folding bands. Using the multirate identity the filters may be moved to a lower rate. The filter design depends only on the number of the cascaded comb filters and do not depend on the decimation factor *M*.

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#### 1. Introduction

CIC (cascaded-integrator-comb) filter [1] is widely used as the decimation filter in oversampled analog-to-digital converters (ADC) due to its simplicity which requires no multiplication or coefficient storage. This filter consists of two main sections, cascaded integrators and combs, separated by a down-sampler, as shown in Fig. 1.

The transfer function of the resulting decimation filter, also known as a RRS (recursive running sum) or comb filter is given by

$$H_{\text{comb}}(z) = \left[ \frac{1}{M} \left( \frac{1 - z^{-M}}{1 - z^{-1}} \right) \right]^K, \tag{1}$$

where *M* is the decimation factor, and *K* is the number of the stages. The integrator section works at the higher input data rate thereby resulting in higher chip area and higher power dissipation for this section. In order to reduce complexity, the non-recursive structure of Eq. (1) can be used [2–5],

$$H_{\text{comb}}(z) = \left[\frac{1}{M}\right]^{K} \left[1 + z^{-1} + z^{-2} + \dots + z^{-(M-1)}\right]^{K}.$$
 (2)

The filtering at the high input rate can be moved to the lower rate by implementing H(z) of Eq. (2) in a polyphase form [2–5].

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Magnitude response of a comb filter has a low attenuation in the folding bands (bands around zeros of comb filter), and a droop in the desired passband, that depends upon the decimation factor M and the number of the cascaded sections K.

Several schemes have been proposed to design comb filters with an improved magnitude response. Methods [6–12] design compensator filters to improve the passband characteristic. The zero rotation is introduced in [13], and later generalized in [14] in order to improve the attenuations in the folding bands. Method [15] introduces novel two-stage nonrecursive architecture for the design of generalized comb filters with an improved passband characteristic. Methods [16–21] deal with a simultaneous improvement of both the passband and the stopband characteristics, using the sharpening technique.

In this paper we propose the simple multiplierless corrector filter to improve the passband as well as the attenuation in the folding bands. To this end we assume that the decimation factor M is an even number, which can be presented in the form:

$$M = 2M_1. (3)$$

The passband is defined by the passband frequency:

$$\omega_p = \frac{\pi}{2M},\tag{4}$$

where *M* is the decimation factor.

The folding bands are defined as:

$$\left[\frac{2\pi i}{M} - \omega_p; \frac{2\pi i}{M} + \omega_p\right]; i = 1, 2, \dots, \lfloor M/2 \rfloor.$$
 (5)

Generally speaking, the corrector filter is a multi-band filter. The ideal magnitude characteristic of the filter in the passband is the

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$$Input \longrightarrow \left[\frac{1}{M}\frac{1}{I-z'}\right]^{K} \longrightarrow \downarrow M \longrightarrow \left[I-z'\right]^{K} \longrightarrow Output$$

Fig. 1. CIC filter.

inverse of that of the comb filter and it is equal zero in the folding bands (5):

$$\left|G_{\mathrm{ideal}}(e^{j\omega})\right| = \left\{ egin{array}{ll} rac{1}{\left|H_{\mathrm{comb}}(e^{j\omega})
ight|} & ext{for} & 0 \leq \omega \leq \omega_{p} \end{array} 
ight. \\ 0 & ext{for} & rac{2\pi i}{M} - \omega_{p} \leq \omega \leq rac{2\pi i}{M} + \omega_{p}; i = 1, 2, \ldots, \left\lfloor M/2 
ight
floor 
ight.$$

This filter can be designed using the frequency sampling method. However the resulting filter works at high input rate and usually has a high order.

The main contribution of this paper is the proposal of five novel simple multiplierless corrector filters, each for the given value of K,  $K = 1, \ldots, 5$ , and arbitrary values of M. The corrector filter simultaneously decreases the passband droop and increases attenuations in odd folding bands of the comb filter. Additionally, it works at the lower rate.

The rest of the paper is organized in the following way. Next section discusses the filter specification. The third section describes the model filter design using the frequency sampling method. The following section then introduces five corrector filters, illustrated with one example. Section 5 discusses the attenuation in the even folding bands and proposes the structure. Last section presents the discussion and the comparisons with some existing methods.

#### 2. Filter specification

In this paper we impose the following conditions for the design of the correction filter:

- The filter must be moved to a lower rate,
- Low order of the filter,
- Multiplierless design, and
- The filter can be used for any value of M, for a given value of K.

To this end we apply the IFIR (interpolated finite impulse response) filter [22]. As it is well known, the IFIR filter has a model and interpolation filters. The interpolation filter is needed to attenuate the images of the expanded model filter. However, we do not need here the interpolation filter, because the images of the expanded model filter are attenuated by the comb filter, as explained in the continuation.

The specification for the model filter is obtained from (6) as:

$$\left| G(e^{j\omega}) \right| = \begin{cases} \frac{1}{\left| H_{\text{comb}}(e^{j\omega}) \right|} & \text{for } 0 \le \omega \le L\omega_p \\ 0 & \text{for } \omega \ge L\omega_s \end{cases} , \tag{7}$$

where  $\omega_p$  is defined in (4) and  $\omega_s$  is defined as:

$$\omega_{\rm s} = \frac{2\pi}{M} - \frac{\pi}{2M} = \frac{3\pi}{2M}.\tag{8}$$

The parameter L is an integer and according to (8) and [22] its maximum value is:

$$L = M/2 = M_1. \tag{9}$$

Thus, considering (9) we impose the condition (3) that the decimation factor is an even value.

Substituting (9) into (8) and (4) from (7) we arrive at:

$$\left| G(e^{j\omega}) \right| = \begin{cases} \frac{1}{\left| H_{\text{comb}}(e^{j\omega}) \right|} & \text{for } 0 \le \omega \le \pi/4 \\ 0 & \text{for } 3\pi/4 \le \omega \le \pi \end{cases}$$
 (10)

Note that the passband and stopband frequencies do not depend on M.

In the following we demonstrate that also the characteristic (10) does not depend on M. To this end, we find  $G(e^{j\omega_1})$ , where  $\omega_1$  is an arbitrary frequency point in the passband,  $\omega_1 \in [0, \omega_p = \pi/4]$ ,

$$\omega_1 = a\pi/4; a \le 1. \tag{11}$$

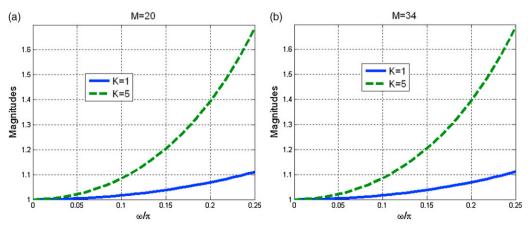
From (10) and (11) we have:

$$\frac{1}{|H_{\text{comb}}(e^{j\omega_1})|} = \left| \frac{M \sin(\omega_1/2)}{\sin(\omega_1 M/2)} \right|^K = \left| \frac{M \sin(a\pi/8)}{\sin(a\pi M/8)} \right|^K. \tag{12}$$

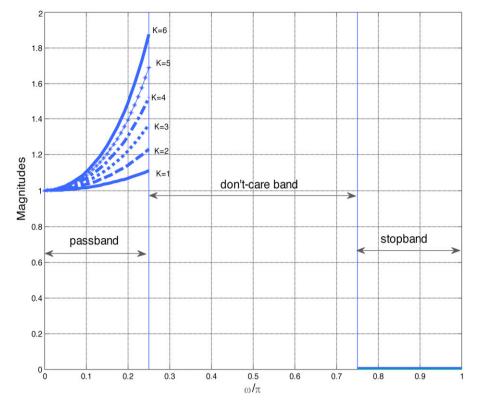
Using  $sin(\beta) \approx \beta$  for a small value of  $\beta$ , we arrive at:

$$\frac{1}{\left|H_{\text{comb}}(e^{j\omega_1})\right|} \approx \left|\frac{M \sin(a\pi/8)}{a\pi M/8}\right|^K = \left|\frac{\sin(a\pi/8)}{a\pi/8}\right|^K. \tag{13}$$

The relation (13) demonstrates that the corrector filter G(z) depends on the comb parameter K and it does not depend on M. The same is confirmed in Fig. 2 where the filter passband specifications for two values of M = 20 and 34, and two values of K = 1 and 5, are presented.



**Fig. 2.** Passband specification for different values of M. (a) M = 20. (b) M = 34.



**Fig. 3.** Filter specification for different values of *K*.

The above mentioned characteristic is very important because, for a given value of K, we design the filter only once, for different values of M.

The specifications (10) for different values of K are shown in Fig. 3.

# 3. Frequency sampling method

Using the specification defined in the section 2 we design the model filter G(z) using the frequency sampling method (MATLAB file fir2). In the next step the designed filter is expanded by  $M_1$ . The proposed filter is the cascade of the comb and the designed expanded model filter:

$$H(z) = H_{\text{comb}}(z)G(z^{M_1}). \tag{14}$$

**Example 1** (:). We design the model filter for M=12 and K=1. Fig. 4a and b show the magnitude responses of the model and the corresponding expanded model filters along with that of the comb filter, respectively.

Note the following:

- The expanded filter has zero magnitude around the comb odd folding bands. This property increases the attenuations in odd comb folding bands.
- The introduced replicas of the expanded filter coincide with the even comb folding bands and are attenuated by the zeros of the comb filter (see Fig. 4b). That is why we do not need the interpolation filter in the IFIR structure. However, as a result, the even comb

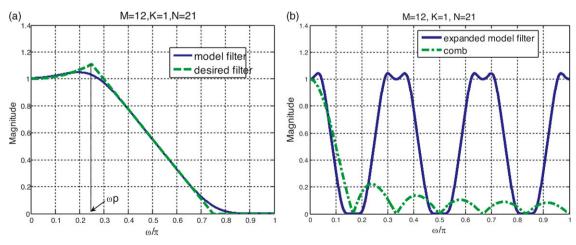
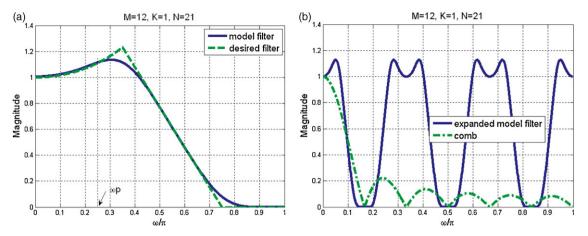
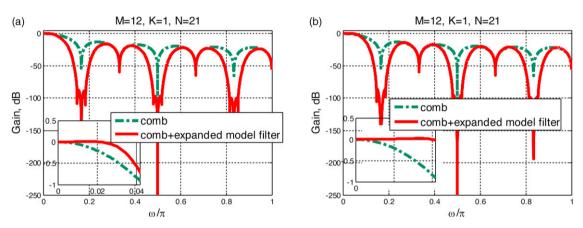


Fig. 4. Illustrations of Example 1. (a) Model filter and desired filter. (b) Expanded model filter and comb.



**Fig. 5.** Illustrations of Example 1 taking  $\alpha$  = 1.4 and  $\beta$  = 3. (a) Model filter and desired filter. (b) Expanded model filter and comb.



**Fig. 6.** Gain responses of Example 1 for original and modified specifications. (a) Original specification. (b)  $\alpha = 1.4$ ,  $\beta = 3$ .

folding bands are not improved. This problem will be considered in Section 5.

 The passband characteristic at the end of normalized passband frequency 0.25 in Fig. 4a exhibits the decaying behavior, which results in a low compensation at the end of the passband. To solve this problem we propose to slightly increase the passband frequency.

Considering the above observations we propose to change the specification (10) with the one shown in (15),

$$\left| G(e^{j\omega}) \right| = \begin{cases} \frac{1}{\left| H_{\text{comb}}(e^{j\omega}) \right|} & \text{for } 0 \le \omega \le \alpha \pi/4 \\ 0 & \text{for } \beta \pi/4 \le \omega \le \pi \end{cases}$$
(15)

where  $\alpha$ , and  $\beta$ , and are the parameters to be defined.

Fig. 5a shows the model filter and the desired filters for  $\alpha$  = 1.4 and  $\beta$  = 3. Fig. 5b shows the expanded model filter and comb.

Fig. 6a and b compare corresponding gain responses for the specifications (10) and (15), respectively, taking the same order of N=21.

Note that the passband of the comb filter is improved taking the filters specification (15) and keeping the same order.

We have three parameters:  $\alpha$ ,  $\beta$ , and the order N, which is approximately proportional to the difference  $\beta - \alpha$ . The increase of  $\alpha$  will result in the passband improvement, while decrease of  $\beta$  will result in the improvement in odd folding bands.

In order to decrease the order of the filter, as much as possible, we fix the order N to be at most 22,  $N \le 22$ . Additionally, in order to

get better compensation, than that provided by the compensation filters [6–10], we impose the following:

$$\min_{\alpha} \max_{\omega \in [0, \alpha\pi/4]} \left| H_{\text{comb}}(e^{j\omega}) - G(e^{j\omega}) \right|. \tag{16}$$

As a result we get the following values of  $\alpha$ ,  $\beta$ , and N, obtained in MATLAB simulations, shown in Table 1, for K = 1, ....5.

Fig. 7a and b show the gain responses for M = 20, taking the values from fourth and fifth rows from Table 1: K = 4,  $\alpha$  = 1.4,  $\beta$  = 2.8, N = 21 and K = 5,  $\alpha$  = 1.4,  $\beta$  = 2.8, N = 21, respectively.

The next issue is to express the coefficients of the filter G(z) by the additions and shifts using the rounding. Here, the rounding constant  $r = 2^{-L}$ , where L is an integer, so that

$$g_r(n) = r \times round(g(n)/r),$$
 (17)

where the rounding round(x) means the rounding of x to the nearest integer.

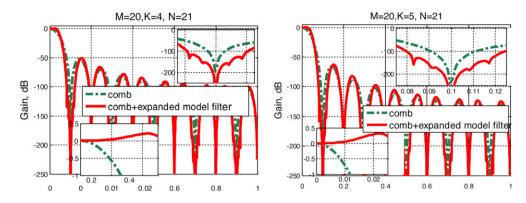
The overall filter is the cascade of the comb filter and the corrector filter, as indicated in,

$$H_r(z) = H_{\text{comb}}(z)G_r(z^{M_1}),$$
 (18)

where  $G_r(z)$  is the rounded filter G(z).

**Table 1** The choice of parameters.

K	α	β	N
1	1.4	3	21
2	1.4	2.8	21
3	1.43	2.8	22
4	1.4	2.8	21
5	1.4	2.8	21



**Fig. 7.** Gain responses for M = 20 and K = 4 and 5. (a) K = 4. (b) K = 5.

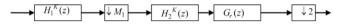


Fig. 8. Two-stage structure.

Considering that the transfer function of comb filter (1) can be rewritten as,

$$H_{comb}(z) = \left[\frac{1}{M} \left(\frac{1 - z^{-M}}{1 - z^{-1}}\right)\right]^{K}$$

$$= \left[\frac{1}{M_{1}} \left(\frac{1 - z^{-M_{1}}}{1 - z^{-1}}\right)\right]^{K} \left[\frac{1}{2} \left(\frac{1 - z^{-M}}{1 - z^{-M_{1}}}\right)\right]^{K}$$

$$= H_{1}^{K}(z)H_{2}^{K}(z^{M_{1}}), \tag{19}$$

where

$$H_1^K(z) = \left[\frac{1}{M_1} \left(\frac{1 - z^{-M_1}}{1 - z^{-1}}\right)\right]^K, \quad H_2^K(z^{M_1}) = \left[\frac{1}{2} \left(\frac{1 - z^{-M}}{1 - z^{-M_1}}\right)\right]^K, \tag{20}$$

we have the structure shown in Fig. 8. From (20) the filter  $H_2^K(z)$  is:

$$H_2^K(z) = \left[ \frac{1}{2} \left( \frac{1 - z^{-2}}{1 - z^{-1}} \right) \right]^K = \left[ \frac{1}{2} (1 + z^{-1}) \right]^K. \tag{21}$$

Note that the corrector filter works at the rate which is  $M_1$  times less than the high input rate. The filter does not depend on the decimation factor but only on the number of the cascaded comb filters K.

#### 4. Five correction filters

The design procedure is given in the following steps:

- 1. For a given *K* design the filter using Frequency Sampling method taking the specification (15).
- 2. Round the filter coefficients using (17).
- 3. Expand the filter by M/2.

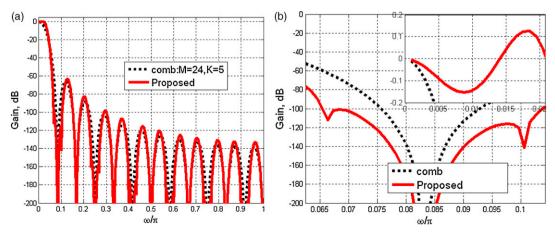
The result of the design is shown in Table 2, taking  $r = 2^{-5}$ .

**Example 2** (:). In this example we illustrate how the filters from the Table 2 can be used when the parameter K is constant and parameter M varies.

- a. Consider M=24 and M=16 for K=5. For K=5 we use the filter No. 5. The overall magnitude characteristics along with the passband, and the first folding band zooms are shown at Fig. 9a and b, respectively, for M=24. Similarly, Fig. 10a and b show the magnitude characteristics, the passband and the first folding zooms, respectively, for M=16.
- b. Next we consider M = 24 and M = 16 for K = 3. The filter No. 3 from Table 2 is used and the corresponding magnitude responses are shown in Figs. 11 and 12.

#### 5. Improving the even folding bands

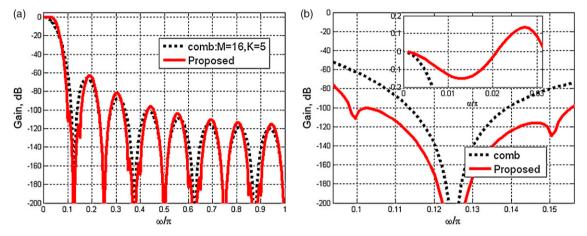
As we can note from Figs. 9–12 the filters from Table 2 improve attenuations in odd folding bands. However, note that the filter  $H_1(z)$  from (19) has the zeros at  $2\pi i/M_1 = 2\pi (2i)/M$ , i.e., in the even folding bands of the comb filter (1). Consequently, cascading  $K_1$ 



**Fig. 9.** Comb and proposed filters: M = 24, K = 5. (a) Overall gain response. (b) Zooms.

**Table 2**Corrector filters.

Filter	K	Non-normalized coefficients, $g_r(n)$				Number of adders/substractors
1	1	$g_r(0) = g_r(5) = -(2^1 + 2^0)$	$g_r(1) = g_r(4) = 2^1;$	$g_r(2) = g_r(3) = 0$	$2^4 + 2^0$ );	7
2	2	$g_r(0) = g_r(9) = 2^0;$ $g_r(1) = g_r(8) = -2^0;$	$g_r(2) = g_r(7) = -(2^2 + 2^0);$	$g_r(3) = g_r(6) = 2^1 + 2^0;$	$g_r(4) = g_r(5) = (2^4 + 2^1);$	12
3	3	$g_r(0) = g_r(9) = 2^0;$ $g_r(1) = g_r(8) = -2^0;$	$g_r(2) = g_r(7) = -(2^2 + 2^1);$	$g_r(3) = g_r(6) = 2^1$ $g_r(6) = 2^1$	4) = $g_r(5) = (2^4 + 2^2 + 2^0);$	12
4	4	$g_{r}\left(0\right)=g_{r}\left(11\right)=2^{0};  g_{r}\left(1\right)=g_{r}\left(10\right)=2^{0};  g_{r}\left(2\right)=g_{r}\left(9\right)=-2^{1};  g_{r}\left(3\right)=g_{r}\left(8\right)=-2^{3};  g_{r}\left(4\right)=g_{r}\left(7\right)=2^{0};  g_{r}\left(5\right)=g_{r}\left(6\right)=\left(2^{4}+2^{3}\right);  g_{r}\left(9\right)=\left(2^{4}+2^{4}\right)=\left(2^{4}+2$				
5	5	$g_r(0) = g_r(11) = 2^0; g_r(1) = g_r(10) = 2^1; g_r(2) = 2^n; g_r(2) =$	$= g_r(9) = -2^1; g_r(3) = g_r(8) = -(8)$	$2^3 + 2^1 + 2^0$ ; $g_r(4) = g_r(7) = 0$	$G_r(5) = g_r(6) = (2^5 - 2^2 - 2^0);$	16



**Fig. 10.** Comb and proposed filters: M = 16, K = 5. (a) Overall gain response. (b) Zooms.

filters  $H_1(z)$  with the filter (18) will result in the improved attenuations in all folding band as shown in Fig. 13.

Using (18) and (19) the proposed filter is:

$$H_p(z) = H_{\text{comb}}(z)H_1^{K_1}(z)G_r(z^{M_1}) = H_1^{K+K_1}(z)H_2^{K}(z^{M_1})G_r(z^{M_1}).$$
 (22)

Denoting

$$H_1(z) = H_1^{K+K_1}(z); \quad H_2(z) = H_2^{K}(z),$$
 (23)

we have the proposed structure shown in Fig. 14.

In the first stage is the cascade of  $K + K_1$  comb filters while in the second stage is the cascade of K combs and the corrector filters.

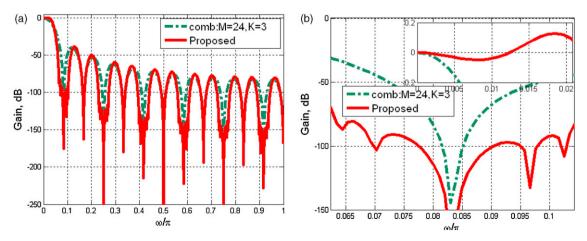
The comb filters in the first stage can be implemented in a nonrecursive form and using the polyphase decomposition one can avoid the filtering at the high input rate.

The parameter  $K_1$  increases the stopband attenuation and does not significantly affect the passband of the filter (18) for  $K_1$  = 1 (see Fig. 13). Therefore for a given value of K and  $K_1$  = 1, one can use the same corrector filter from Table 2, as shown in next example.

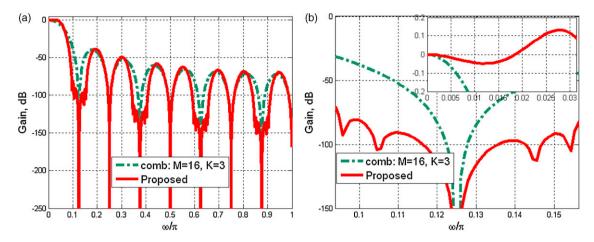
**Example 3** (:). We design the decimation filter for M = 18, K = 4 and taking  $K_1 = 1$ . We take the filter 4 from the Table 2. Fig. 15 presents the corresponding gain responses along with the passband and the first folding band zooms.

## 6. Discussion

In this section we compare the proposed method with different methods proposed in literature.



**Fig. 11.** Comb and proposed filters: M = 24, K = 3. (a) Overall gain response. (b) Zooms.



**Fig. 12.** Comb and proposed filters: M = 16, K = 3. (a) Overall gain response. (b) Zooms.

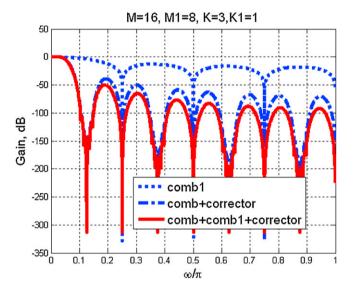


Fig. 13. Improvement in all folding bands.

### **Example 4** (:). Comparison with Method [6]

The coefficients of the compensator [6] are given by

$$[-a/(1-2a), 1/(1-2a), -a/(1-2a)]$$
 (24)

where the parameter a is obtained by minimizing the corresponding error function for the given values of M and K. We compare

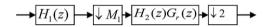


Fig. 14. Proposed structure.

the performance of our proposed filter with that of [6] for K=5 and the decimation factors M=14 and 20. Method [6] requires design of two different compensation filters, one for M=14 with the obtained value a=0.1803, and another for M=20 (a=0.1806), both with two multipliers and two adders. However, we use the same compensation filter 5 from Table 2 which requires 16 adders and no multipliers. The corresponding magnitude responses are shown in Fig. 16.

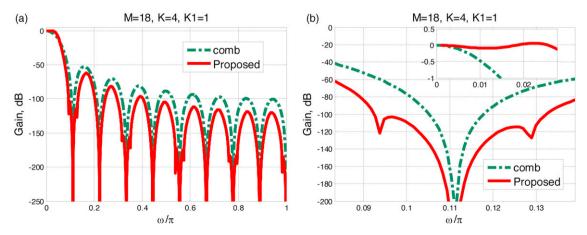
Note that the proposed filter provides better characteristics in the passband as well as in the odd folding bands.

#### **Example 5** (:). Comparison with Method [9]

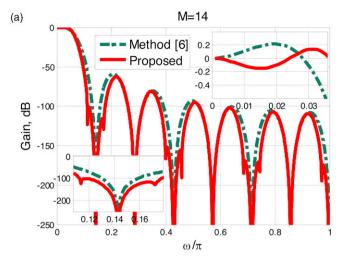
Consider M = 14 and K = 4. The wideband compensator from [9] is cascaded 3 times. The proposed filter uses the filter 4 from Table 2. The overall magnitude responses and the passband zooms are shown in Fig. 17. The proposed method provides better passband compensation and better alias rejection with the similar filter complexity.

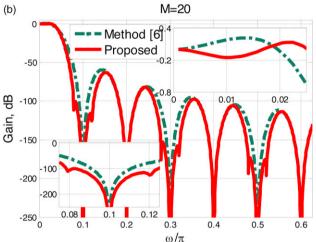
#### **Example 6** (:). Comparison with Method [11]

Authors in [11] proposed closed-form design of CIC compensators based on maximally flat error criterion. As a result the compensation filter of the order N is obtained. Considering K=4 and M=30, the filter from [11] has N=9 and the corresponding



**Fig. 15.** M = 18, K = 3,  $K_1 = 1$ . (a) Overall gain responses. (b) Zooms.





**Fig. 16.** Comparison with Method [6]. (a) M = 14. (b) M = 20.

#### coefficients:

$$[0.0008, -0.0112, 0.0800, -0.3987, 1.6582, -0.3987, 0.0800, -0.0112, 0.0008].$$
 (25)

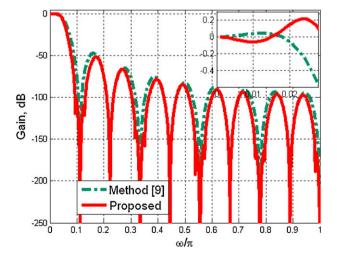


Fig. 17. Comparison with Method [9].

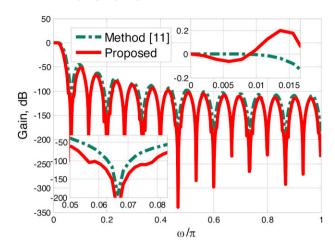


Fig. 18. Comparison with Method [11].

We compare the filter 4 from Table 2 with the filter (25) from [11]. Fig. 18 shows the overall magnitude responses along with the passband and the first folding band zooms.

The proposed filter has better alias rejection while the filter [11] has better passband compensation. However, the filter [11] requires 5 multipliers.

#### **Example 7** (:). Comparison with GCF Method [14]

We consider a 3rd order GCF filter

$$H_{\text{GCF3}}(z) = \frac{1 - z^{-M}}{1 - z^{-1}} \frac{1 - z^{-M} e^{j\alpha M}}{1 - z^{-1} e^{j\alpha M}} \frac{1 - z^{-M} e^{-j\alpha M}}{1 - z^{-1} e^{-j\alpha M}},$$
 (26)

where  $\alpha$  is the extent of the rotation undergone by the zeros of a classical comb filter, which is computed to be  $\alpha$  = 0.03828 [14]. Fig. 19 compares the GCF filter (26) and the proposed filter 3 from Table 2. Note that the proposed filter exhibits better passband characteristic and better alias rejections in odd folding bands, while the GCF filter has better alias rejection in even folding bands. However, the GCF needs 2 multipliers and the values of  $\alpha$  must be calculated for new parameters.

#### **Example 8** (:). Comparison with Sharpening Method [16]

We compare the proposed method with the sharpening method in [16]. The decimation factor M=32. The sharpening polynomial  $3H^2$ – $2H^3$ , where H is the cascade of two comb filters is applied. The proposed method uses K=5 and the filter 5 from Table 2. The overall magnitude responses and the passband zooms are shown in Fig. 20. The proposed method has better alias rejection and the

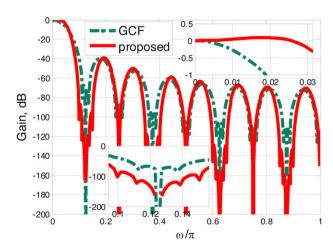


Fig. 19. Comparison with Method [14].

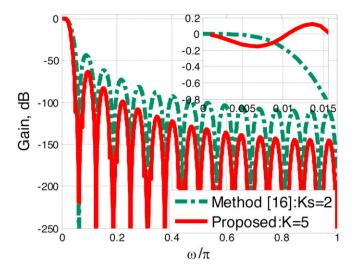


Fig. 20. Comparison with Method [16].

compensation in the higher band of the passband. However, the method [16] has better compensation in the lower band. Additionally, the proposed structure is less complex than the sharpening structure.

#### **Example 9** (:). Comparison with Sharpening Method [18]

We compare the proposed method with the sharpening method in [18]. The parameters from [18] are M = 20, Ks = 2, and b = 1, while in the proposed method we have K = 2 and the corrector filter 2 from Table 2. The proposed filter exhibits better attenuations in the folding bands while the method [18] has slightly better passband characteristic. However, the structure [18] is more complex. The corresponding overall magnitude responses and the passband zooms are shown in Fig. 21.

### **Example 10** (:). Comparison with Sharpening Method [20]

The transfer function of the double sharpened CIC filter [20] is defined as:

$$H_{\text{DSCIC}}(z) = [H_{D_1}(z)]^L [H_{D_2}(z)] [H_{D_3}(z^{M_3})]^{4K}$$

$$\left\{ 3z^{-\frac{M_1 M_2 (M_3 - 1)K}{4}} - 2[H_{D_3}(z^{M_3})]^K \right\},$$
(27)

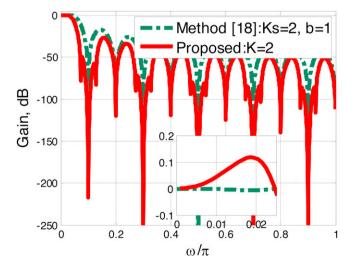


Fig. 21. Comparison with Method [18].

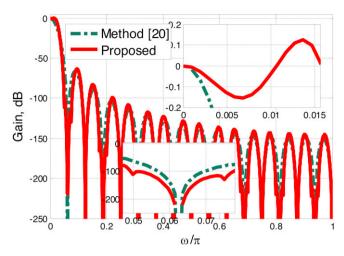


Fig. 22. Comparison with Method [20].

where L and K, are integers and  $M = M_1 M_2$ , and

$$H_{D_1}(z) = \frac{1}{M_1} \frac{1 - z^{-M_1}}{1 - z^{-1}}; \quad H_{D_2}(z) = \frac{1}{M_2} \frac{1 - z^{-M_2}}{1 - z^{-1}};$$

$$H_{D_3}(z^{M_3}) = \frac{1}{M_3} \frac{1 - z^{-M_1 M_2 M_3}}{1 - z^{-M_3}}.$$
(28)

In this example we consider the following parameters for the Method [20]: M = 32 and  $M_1 = 8$ ,  $M_2 = 2$ , and  $M_3 = 2$ . The values of L and L are 5 and 2. The corresponding proposed filter 5 from Table 2 is cascaded with the comb filter of order 5. Fig. 22 shows the overall magnitude responses and passband and the first folding band zooms indicating that the proposed method has better characteristics in both, passband as well as, in the odd folding bands.

#### 7. Conclusion

We present simple multiplerless corrector filters which improve the passband, as well as, the attenuations in the folding bands of the comb filter. In the majority of the existing methods it is necessary to redesign filter for each new value of M and K, whereas the proposed corrector filters may be used for any value of M. The choice of the corrector filter depends on the number of stages K of the comb filter,  $K = 1, \ldots, 5$  and does not depend on the decimation factor. The maximum number of adders is 16. The proposed filters provide a good passband compensation (less than 0.25 dB), and a high attenuation in odd folding bands. In order to further increase attenuations in even folding bands, the comb with M = M/2 may be cascaded with the proposed filters. However, this will result in an increased complexity. Lastly, the proposed method is limited to even decimation factors.

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