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Procedia Technology 7 (2013) 142 – 149

Procedia Technology

The 2013 Iberoamerican Conference on electronics Engineering and Computer Science

Application of generalized sharpening technique for two-stage comb decimator filter design

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Abstract

This paper introduces the generalized sharpening technique to improve the magnitude characteristics of comb decimation filter in passband as well in the folding bands. To this end we design two-stage comb filter. The first stage can be operated at low sampling rate by using polyphase decomposition. A simple compensator is applied in the second stage to improve the passband characteristic of the comb in the second stage. Then the generalized sharpening technique is applied to decrease the passband droop induced by the comb filter placed in the first stage. As a result, a computationally efficient comb-based decimation filter is obtained which presents better magnitude characteristics than previous proposed sharpening methods.

Selection and peer-review under responsibility of CIIECC 2013

Keywords: comb filter, CIC structure, general sharpening, decimation.

1. Introduction

The comb filter is the most popular decimation filter usually used in the first stage of the decimation process. The efficient implementation structure called Cascaded-Integrator-Comb (CIC) was proposed by

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Selection and peer-review under responsibility of CIIECC 2013
doi:10.1016/j.protcy.2013.04.018

Hogenauer (Hogenauer [1]). The popularity of the comb filter is due to its simplicity, its linear phase response and the fact that it does not require multiplications. Nevertheless, comb filters have a high passband droop and a poor attenuation in the so-called folding bands (bands around the zeros of comb filter).

Different works have been proposed to improve the magnitude characteristics of comb filters either in the passband or in the stopband regions or both, passband and stopbands, trying to keep a resulting structure with a low complexity. To decrease the passband droop, compensation filters have been used (Kim et al., [2]), (Dolecek and Mitra, [3]), (Molnar and Vucic, [4]). To improve the stopband characteristic, Lo Presti introduced the comb zero rotation and proposed the Rotated Sinc filter (Lo Presti, [5]). This scheme was generalized by (Laddomada, [6]), where the GCF (Generalized Comb Filter) was introduced. Different approaches were also proposed to improve GCFs (Fernandez and Dolecek, [7]), (Dolecek and Laddomada, [8]).

One of the most useful approaches to improve simultaneously the passband and stopband characteristics of comb filters, consists on using the sharpening technique, originally introduced in (Kaiser and Hamming, [9]) and then extended in (Hartnett and Boudreaux, [10]) and (Samadi, [11]). Kwentus et al. used the sharpening technique of Kaiser and Hamming to improve the magnitude response of the comb filter (Kwentus et al, [12]). Split of the comb filter in two or more stages with the application of the sharpening technique only in the last stage was proposed in (Stephen and Stuart, [13]), (Dolecek and Mitra, [14]). The use of the sharpening technique applied to compensated-comb filters was proposed in (Dolecek and Harris, [15]), (Zaimin He et al., [16]). However, using the sharpening introduced by Kaiser and Hamming can not avoid the passband droop introduced by the first-stage comb filter in a two-stage comb-based scheme.

In this paper we propose a generalized sharpening technique to improve comb filter amplitude characteristics using two-stage comb filter. The simple compensator filter of (Dolecek and Mitra, [3]) is used in the second stage to improve the passband characteristic. Then, the generalized sharpening technique of (Samadi, [11]) is applied to the compensated filter of the second stage with the aim to decrease the droop induced by the first-stage comb filter. As a result, we obtain an efficient filter where both, passband and stopband, are improved.

This paper is organized in the following way. The next section presents the generalized sharpening technique. The proposed method is described in Section III. Section IV shows the discussion of results. Finally, the conclusions are given in section V.

2. Generalized Sharpening Technique

The sharpening technique proposed in (Kaiser and Hamming, [9]) permits simultaneous improvements of both passband and stopband characteristics of linear-phase Finite Impulse Response (FIR) filters. The technique is based on an Amplitude Change Function (ACF) which is a polynomial $P_{m,n}(x)$ that maps the amplitude x into an improved amplitude $y = P_{m,n}(x)$. The improvement in the amplitude near to the passband depends on m, the order of tangency of the ACF at the point $(x, y) = (1, 1)$ to a line with slope equal to zero. Similarly, the improvements in amplitudes near the stopband depend on the order of tangency of the ACF at the point $(x, y) = (0, 0)$ to a line with slope equal to zero, which is denoted as n.

The sharpening technique proposed in (Kaiser and Hamming, [9]) has a limited control of the improvement in both passband and stopband characteristics of the filter. This is because the desired ACF is piecewise constant. In (Hartnett and Boudreaux, [10]) is proposed the generalized sharpening technique, where the desired ACF is piecewise linear. This offers more direct control to change amplitudes in the passband and/or stopband. Besides of the tangencies m and n, the polynomial approximation to the desired ACF is controlled by other two parameters, namely, σ, the slope of a line that passes over the point $(x, y) = (1, 1)$ and δ, the slope of another line that passes over the point $(x, y) = (0, 0)$. The constrains on the approximating polynomial $y = P_{\sigma,\delta,m,n}(x)$ are:

1. The nth-order tangency at $(x, y) = (0, 0)$ to the line of slope δ, i. e., $P_{\sigma,\delta,m,n}(x) \approx \delta x$, for $x \to 0$.
2. The mth-order tangency at $(x, y) = (1, 1)$ to the line of slope σ, i. e., $P_{\sigma,\delta,m,n}(x) \approx \sigma(x-1)+1$, for $x \to 1$.

The desired piecewise linear ACF is illustrated in Figure 1, where x_{pl} and x_{pu} are, respectively, the minimum and maximum amplitude in the passband of the original filter, and x_{sl} and x_{su} are the minimum and maximum amplitude in the stopband of the same filter, respectively. In the same way, y_{pl}, y_{pu}, y_{sl}, and y_{su} are the minimum and maximum amplitudes in the passband and the minimum and maximum amplitudes in the stopband of the sharpened filter, respectively. In (Samadi, [11]) a general formula was deduced to obtain directly the desired amplitude change function from the design parameters. The formula is given by

$$P_{\sigma,\delta,m,n}(x) = \delta x + \sum_{j=n+1}^{R} (\alpha_{j,0} - \sigma\alpha_{j,1} - \delta\alpha_{j,2}) x^j , \tag{1}$$

where $R = n + m + 1$ and

$$\alpha_{j,0} = \sum_{i=n+1}^{j} (-1)^{j-i} \binom{R}{j}\binom{j}{i}, \quad \alpha_{j,1} = \sum_{i=n+1}^{j} (-1)^{j-i} \binom{R}{j}\binom{j}{i}\left(1-\frac{i}{R}\right), \quad \text{and} \quad \alpha_{j,2} = \sum_{i=n+1}^{j} (-1)^{j-i} \binom{R}{j}\binom{j}{i}\frac{i}{R}. \tag{2}$$

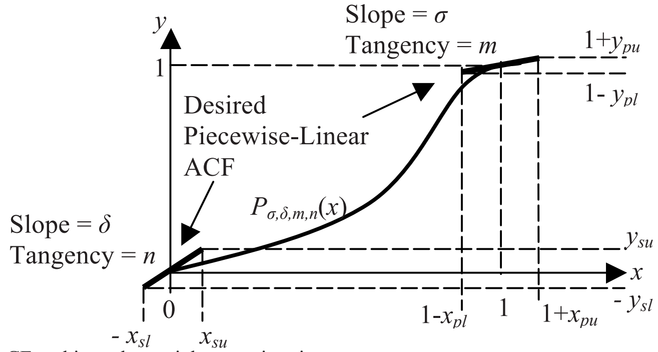


Figure 1. Piecewise linear ACF and its polynomial approximation.

3. Application of the generalized sharpening to the two-stage comb decimator filter

The filter proposed in (Dolecek and Mitra, [14]) takes advantage of two-stage decomposition of the comb filter to apply the sharpening technique only in the second stage. The resulting transfer function is given by:

$$H(z) = [H_1(z)]^L \cdot Sh_1\left\{\left[H_2(z^{M_1})\right]^K\right\}, \tag{3}$$

$$H_1(z) = \frac{1}{M_1}\cdot\frac{1-z^{-M_1}}{1-z^{-1}}, \quad H_2(z) = \frac{1}{M_2}\cdot\frac{1-z^{-M_2}}{1-z^{-1}}, \tag{4}$$

where $M = M_1M_2$ is the decimation factor, L and K are the number of cascaded filters $H_1(z)$ and $H_2(z^{M_1})$, respectively, and $Sh\{H(z)\}$ means that sharpening has been applied to $H(z)$. The value K must be even (Dolecek and Mitra, [14]). The method of (Stephen and Stuart, [13]) is a special case for $M_1 = 2^p$ with p integer and usually $p \le 3$.

The advantages of this approach are the following:

- The down-sampling block M can be divided into two separated down-sampling blocks, M_1 and M_2. Since the first folding band, where the worst case attenuation occurs, is essentially determined by $H_2(z^{M_1})$, it is only required to apply sharpening to this filter. As a result we get better passband

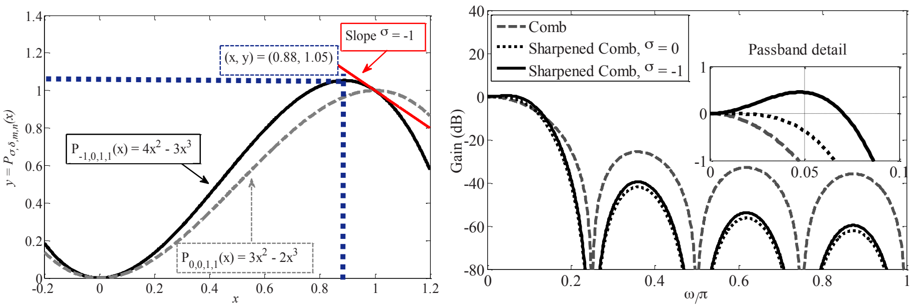
and stopband characteristics with lower complexity than applying sharpening to the original single stage comb filter.

- The filter $H_2(z^{M_1})$ can be moved after the down-sampling by M_1, resulting in lower power consumption because $H_2(z)$ works at a lower rate.
- The filter $H_1(z)$ can work at a lower rate after the down-sampling by M_1 using polyphase decomposition (Aboushady et al., [17]).

However, regardless of the passband improvement by the sharpened filter of the second stage, the resulting filter has always a passband droop that is a consequence of the first-stage comb filter. This can not be solved using the traditional sharpening proposed by Kaiser and Hamming. In this proposal we will apply the generalized sharpening technique to the compensated comb filter of the second stage. As a result, we can take advantage of taking into account the slope parameter σ, and thus correcting the aforementioned effect. The use of the generalized sharpening for this purpose is described in the next sub-sections.

3.1. Sharpening of the second-stage comb filter

Observe in the Figure 2(a) that, by setting a negative slope σ, the amplitude values over the axis x, that are slightly less than one, can be mapped into values greater than one. Since the comb filters have amplitude values slightly less than one in their passband region, they will have values greater than one after being sharpened. Thus, after cascading the sharpened second-stage comb filter with the first-stage comb filter a compensated droop in the passband region can be obtained. On the other hand, knowing that the desired stopband amplitude values are zero, the slope δ has to be equal to zero.



(a) (b)

Figure 2. (a) The traditional sharpening polinomial $P_{0,0,1,1}(x) = 3x^2 - 2x^3$ and the generalized sharpening polinomial $P_{-1,0,1,1}(x) = 4x^2 - 3x^3$. (b) Magnitude responses of a comb filter, a sharpened-comb filter with the traditional polynomial $3x^2 - 2x^3$ and a sharpened-comb filter with the polynomial $4x^2 - 3x^3$, obtained from the generalized approach.

Figure 2(a) shows a comparison of the traditional 3rd-order polynomial of Kaiser and Hamming with parameters $\sigma = 0$, $\delta = 0$, $m = 1$ and $n = 1$, $P_{0,0,1,1}(x) = 3x^2 - 2x^3$, and a polynomial with parameters $\sigma = -1$, $\delta = 0$, $m = 1$ and $n = 1$, $P_{-1,0,1,1}(x) = 4x^2 - 3x^3$, obtained from the generalized sharpening approach. Note that the value 0.88 is mapped to a new value greater than one, 1.05. Figure 2(b) shows a comparison between the magnitude responses of a comb filter, a comb filter sharpened with the polynomial $3x^2 - 2x^3$ and a comb filter sharpened with the polynomial $4x^2 - 3x^3$. Observe that the attenuations around the zeros are very similar for both sharpened comb filters. However, the sharpened comb which uses the generalized approach, has a resulting passband with increased amplitudes over the frequencies $\omega = 0$ to $\omega \approx 0.05\pi$. This characteristic can be used to compensate the droop introduced by the first-stage comb filter.

3.2. Sharpening of the compensated second-stage comb filter

It has been shown recently that applying the Kaiser and Hamming sharpening to a previously compensated comb filter results in a better amplitude characteristic with a less complexity, in comparison with the method where the sharpening is applied to comb filter (Dolecek and Harris, [15]). This also holds for the generalized sharpening technique.

In Figure 3 we have, on the right side, the amplitudes of three filters: a comb filter and two different compensated comb filters. One of them has been compensated with a wideband compensator and the other with a narrowband compensator. On the left side we have the mapping from the original values to new values through the polynomial $4x^2 - 3x^3$. Observe that, at the frequency point ω_p, which represents the upper edge of the passband of interest, the amplitude of the comb filter is mapped to a value that is away from the desired line with slope σ. Moreover, since this line only approximates the necessary values to compensate the droop of the first-stage comb filter, it is not convenient to map values of the original amplitude that are too far from 1. Additionally, it can be seen that the original amplitude values of the comb filter compensated with a wideband compensator (which are greater than one), are mapped to new amplitude values less than one. For this reason it is not convenient to use a wideband compensator. On the other hand, the original amplitude values of the comb compensated with a narrowband compensator are mapped to values greater than one that closely follow the values of the desired line.

A simple multiplierless compensator with only one parameter b, which depends on the number of K stages, was proposed in (Dolecek and Mitra, [3]). This filter has a low complexity and provides a good compensation in a narrow passband. Therefore, we adopt this compensation filter in this proposal. The transfer function of this compensator is

$$G(z^M) = -2^{-(b+2)}\left[1-(2^{b+2}+2)z^{-M}+z^{-2M}\right]. \tag{5}$$

The compensated second-stage filter becomes,

$$H_{2C}(z) = G(z^M)\left[H_2(z^{M_1})\right]^K. \tag{6}$$

Applying the generalized sharpening technique to the compensated filter $H_{2C}(z)$ we obtain the proposed decimation filter whose transfer function is

$$H_P(z) = \left[H_1(z)\right]^L \cdot Sh\left\{H_{2C}(z)\right\}. \tag{7}$$

Using (1), (4), (5) and (6) we arrive at:

$$H_P(z) = \left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^L \cdot \sum_{j=n+1}^{n+m+1}(\alpha_{j,0}-\sigma\alpha_{j,1})\left\{\left[\frac{1}{M_2}\frac{1-z^{-M_1M_2}}{1-z^{-M_1}}\right]^K\left[-2^{-(b+2)}\left[1-(2^{b+2}+2)z^{-M_1M_2}+z^{-2M_1M_2}\right]\right]\right\}^j z^{-(n+m+1-j)\tau} \tag{8}$$

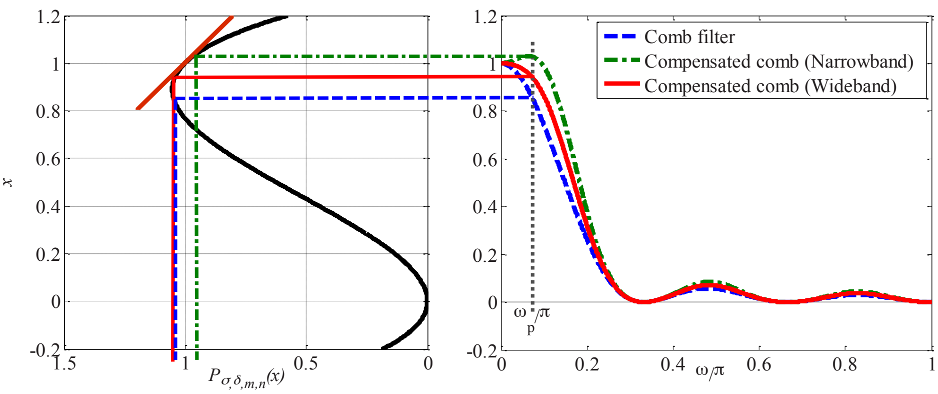


Figure 3. Amplitude changes of a comb filter and two compensated comb filters through the sharpening polynomial $4x^2 - 3x^3$.

where τ is equal to $M_1(M_2 - 1)K/2 + M_1M_2$. The coefficients $\alpha_{j,0}$ and $\alpha_{j,1}$ in (8) are calculated from (2). Thus, the design parameters are the tangencies m and n, the slope σ, and the compensator parameter b, along with the number of cascaded filters L for $H_1(z)$ and K for $H_2(z)$. An efficient structure for decimation is presented in Figure 4, straightforwardly derived from (Saramaki and Ritoniemi, [18]). Note that the filter preceding the down-sampler by M_1 can be decomposed into polyphase components to avoid operations at high rate.

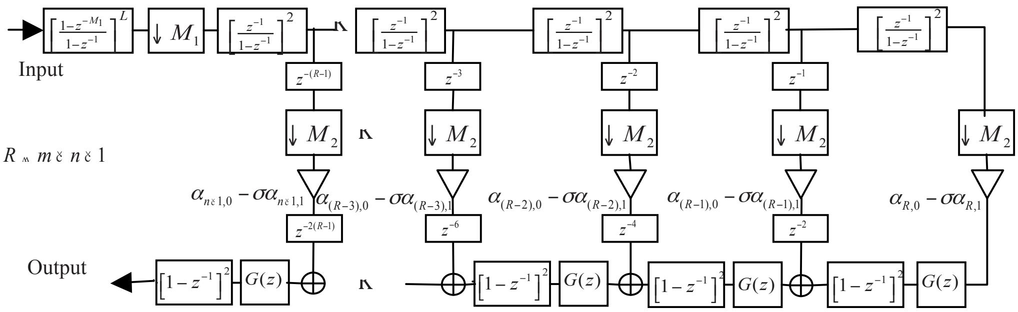


Figure 4. Efficient structure for decimation.

3.3. Choice of design parameters

The parameter K is closely related to the parameter n. By increasing either K or n, the stopband attenuation is enhanced. Nevertheless, it is preferable keeping K constant and as small as possible, whereas n is variable. Considering that K must be an even value, we set $K = 2$. As a consequence, the compensator parameter becomes $b = 2$ (Dolecek and Mitra, [3]). Furthermore, the slope σ controls the values of the ideal ACF that approximate the desired values necessary to compensate the passband droop introduced by the first-stage comb filter, $H_1(z)$. A simple way to assure multiplierless sharpening coefficients is by expressing the slope σ as $\sigma = 2^{-c}$. The constant c must be decreased as the droop introduced by $H_1(z)$ increases. Additionally, the tangency of the sharpening polynomial to the line with slope σ at the point (1, 1) is enhanced by increasing the parameter m. This results in a better passband characteristic but also in a higher complexity of the overall filter. Finally, the parameter L does not have implication in the improvement of the attenuation in the first folding band (where the worst-case attenuation occurs). However, L increases the droop of $H_1(z)$. For this reason, even though it is often considered arbitrary in most two-stage comb-based decimation filters, L should be kept as small as possible.

A simple design procedure for a given stopband specification is presented as follows:

1. Consider the decimation factor as $M = M_1M_2$, and that L and a residual decimation factor v are given. Set $K = 2$, $b = 2$, $\delta = 0$, $n = 0$, $c = 0$ and $m = 1$.
2. Increase n until the stopband requirement is satisfied.
3. Decrease c until an acceptable passband is obtained.
4. Increase m until the passband characteristic in step 2 can not be improved further.

4. Discussion of results

In this section we present design examples to show the effectiveness of the proposal in comparison to other two-stage sharpening-based methods.

Example 1. Consider a decimation process with overall decimation factor $D = M_1 M_2 v = 272$, with $M_1 = 4$, $M_2 = 17$ and $v = 4$. Assume that the passband edge frequency is $\omega_p = 0.9\pi/D$, and a desired stopband attenuation of 100 dB.

The polynomial used in this filter is $P_{\sigma,\delta,m,n}(x) = 5.125x^4 - 4.125x^5$, obtained with $m = 1$, $n = 3$, and $\sigma = -2^{-3}$. On the other hand, Stuart and Stephen use the traditional Kaiser and Hamming polynomial $P_{m,n}(x) = 3x^2 - 2x^3$, obtained with $m = 1$, $n = 1$, and their filter accomplishes the 100 dB attenuation with $K = 4$. Figure 5 shows the magnitude characteristics for both designs. Note that the proposed method achieves a much better passband characteristic.

For both designs, the first-stage comb filter can be decomposed in polyphase components, resulting in the same complexity. The second-stage comb filter of the proposed filter is implemented with the decimation architecture of Figure 5, whereas the one of (Stephen and Stuart, [13]) uses the structure of (Kwentus et al., [12]). Note that the proposed filter has a lower computational complexity, as shown in Table 1.

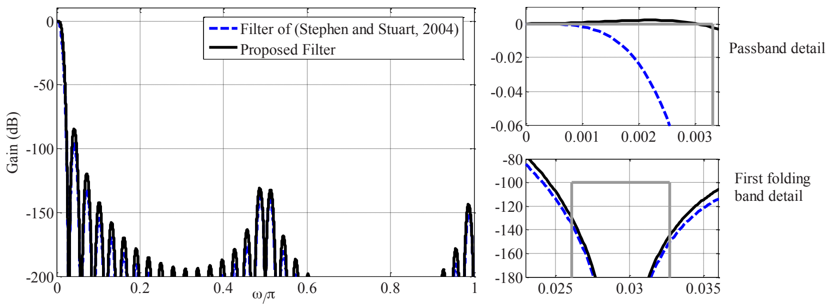


Figure 5. Gain in dB of the Example 1 applying the proposed method and the method of (Stephen and Stuart, [12]).

Example 2. Consider an overall decimation factor $D = 176$, with $M_1 = 8$, $M_2 = 11$ and $v = 2$. Assume the passband edge frequency $\omega_p = 0.9\pi/D$ and the desired stopband attenuation of 60 dB.

The resulting polynomial for this filter is $P_{\sigma,\delta,m,n}(x) = 15.625x^4 - 25.125x^5 + 10.5x^6$, where $m = 2$, $n = 3$, and $\sigma = -2^{-3}$. Figure 6 shows the magnitude characteristics of the proposed design along with the solution of (Dolecek and Harris, [15]), where $P_{m,n}(x) = 2x - x^2$ is used with $K = 8$, and the solution of (Zaimin He et al., [16]), where $P_{m,n}(x) = 3x^2 - 2x^3$ is used with $K = 4$. Note that the proposed method achieves a much better passband characteristic, with only a slight increase of the computational complexity, as shown in Table 1.

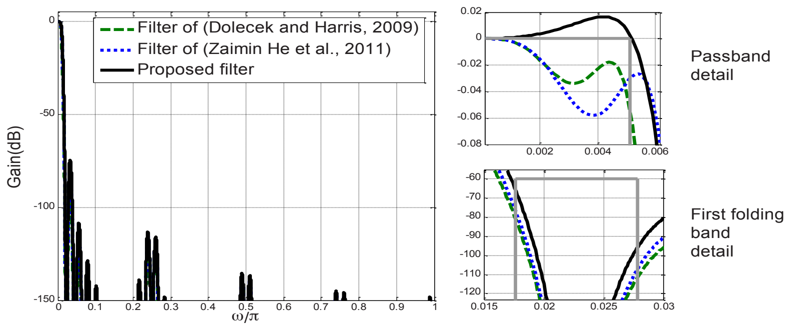


Figure 6. Gain in dB of the Example 2 applying the proposed method, the method of (Dolecek and Harris, [15]) and the method of (Zaimin He, [16]).

5. Conclusion

In this paper we presented the application of the generalized sharpening technique to a two-stage comb-based decimation filter as an alternative method for superior control of the magnitude characteristics of the comb decimator filter. The results show that significant magnitude improvements are obtained with modest or none

Table 1. Comparison of computational complexity of the sharpened filters in Examples 1 and 2

Method	Additions Per Output Sample (APOS) in Example 1	APOS in Example 2
(Stephen and Stuart, [13])	$3KM_2+3K+3 = 219$	—
(Dolecek and Harris, [15])	—	$2KM_2+(3+K)2+1 = 199$
(Zaimin He et al., [16])	__	$3KM_2+3K+12 = 156$
Proposed	$2RM_2+6R-1+$*coefficient adders* $= 202$	175

increase of the computational complexity. The amplitude in the passband region after sharpening consists on values that approximate the values of a line with slope σ that passes through the point (1, 1). However, these values are not exactly equal to the ones required to compensate for the passband droop introduced by $H_1(z)$, even if the ACF becomes ideal. Thus, arbitrary specification in passband can not be met. Additionally, a better improvement in comparison with the traditional Kaiser and Hamming is obtained.

Acknowledgements

This work has been supported by CONACYT Mexico under the project 179587.

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