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Sharpening minimum-phase filters

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The minimum-phase requirement restricts that filter has all its zeros on or inside the unit circle. As a result the filter does not have a linear phase. It is well known that the sharpening technique can be used to simultaneous improvements of both the pass-band and stop-band of a linear-phase FIR filters and cannot be used for other types of filters. In this paper we demonstrate that the sharpening technique can also be applied to minimum-phase filters, after small modification. The method is illustrated with one practical examples of design.

1. Introduction

There are some applications where high delay introduced by linear-phase finite impulse response (FIR) filters is not permitted like in communication data systems. In order to obtain much lower delay, the solution is a minimum-phase filter (MP) design which preserves desired magnitude response. There have been proposed different methods to obtain minimum-phase filters.

The existing methods can be classified into two principal groups of methods: methods based on linear-phase (LP) filter design, and methods based on complex cepstrum [1-5]. We consider here the FIR MP design method based on the corresponding LP filter. As a difference to LP filter which generally have zeros in quadruplets [6], a MP filter has all its zeros inside or on the unit circle.

The simplest method to design a MP filter for a given specification consists of two following steps:

1. Design the LP filter which satisfies the specification.
2. Reflect all zeros of LP filter which are outside the unit circle to their reciprocal positions inside the unit circle.

The obtained MP filter has the same magnitude response as the original LP filter [6]. However, the MP filter does not have the linear phase and, as a result, its unit sample impulse response does not have the symmetry.

It is well known that the sharpening technique can be used to simultaneous improvements of both the pass-band and stop-band of a linear-phase FIR filter and is not used for other types of filters including MP filters.

The main objective of this work is to propose a novel procedure which make possible the sharpening of minimum phase filters.

2. Sharpening technique

The sharpening technique is introduced in [7] for simultaneous improvements of both the pass-band and stop-band of a linear-phase FIR filter. The sharpening technique uses the amplitude change function (ACF) which is a polynomial relationship of the form $H_{sh} = f(H)$ between the amplitudes of

the sharpened and the prototype filters, H_{sh} and H , respectively. The improvement in the gain response near the passband edge $H=1$, or near the stopband edge $H=0$, depends on the order of tangencies m and n of the ACF at $H=1$, or at $H=0$, respectively.

We consider here the most important polynomials:

$$\mathbf{P}_1: H_{sh}=2H-H^2, (m=1, n=0); \mathbf{P}_2: H_{sh}=3H^2-2H^3, (m=1, n=1); \mathbf{P}_3: H_{sh}=4H^3-3H^4, (m=1, n=2). \quad (1)$$

The polynomial \mathbf{P}_1 provides the best passband improvement, while \mathbf{P}_3 results in best stopband improvement. However, the sharpening technique cannot be applied to minimum phase filters. Next section describes the procedure to apply sharpening also to MP filters.

3. Description of the procedure

The system function of sharpening polynomials (1) can be rewritten in the form:

$$H_{sh}(z) = H^k(z)H_s(z), \quad (2)$$

where $H(z)$ is the original LP filter, k is an integer, z is complex value, and $H_s(z)$ is a LP filter in the form:

$$H_s(z) = K_1z^{-N/2} - K_2H(z), \quad (3)$$

where K_1 and K_2 are constants. Note that the delay $z^{-N/2}$ must be introduced in order that the filter $H_s(z)$ impulse response has a symmetry, i.e. the filter $H_s(z)$ is a LP filter. This implies that the order of the filter must be even i.e. the filter must be the Type 1 filter [8]. The values k , K_1 and K_2 for the polynomials (2) are given in Table 1.

From (2) we write the corresponding sharpening polynomials for the corresponding minimum phase filter (MP filter that has the same magnitude response):

$$H_{shm}(z) = H_m^k(z)H_{sm}(z), \quad (4)$$

where $H_m(z)$ and $H_{sm}(z)$ are obtained from the corresponding LP filters $H(z)$ and $H_s(z)$ by reflecting the zeros from the outside of unit circle to the reciprocal positions inside the unit circle. As a result, the LP filters $H(z)$ and $H_s(z)$ have equal magnitude responses as MP filters $H_m(z)$ and $H_{sm}(z)$, respectively.

Table 1. Parameters of sharpening polynomials.

Polynomial	k	K_1	K_2
P1	1	2	1
P2	2	3	2
P3	3	4	3

The procedure is given in the following steps:

1. Design the LP filter for a given specification.
2. Transform LP filter into MP filter.
3. From (3) find $H_s(z)$ and transform it into MP filter $H_{sm}(z)$.
4. The sharpened MP filter is given in (4).

Using Rouché's theorem [9] we can find that LP filter $H_s(z)$ has $N/2$ zeros inside the unit circle. Being a LP filter, it has other $N/2$ zeros outside the unit circle. As a consequence, the zeros of the MP filter $H_{sm}(z)$ can be easily find by doubling the zeros of $H_s(z)$ which are inside the unit circle, and eliminate that which are outside the unit circle.

The procedure is illustrated in the following example.

Example 1:

Using polynomials (1) sharpen the MP filter satisfying the following specification:

$$\omega_p = 0.1; \omega_s = 0.3; \delta_p = 0.1dB; A_s = 50dB, \quad (5)$$

where ω_p and ω_s are normalized passband and stopband frequencies, δ_p is a maximal passband ripple and A_s is a minimum stopband attenuation.

Step 1: We design a LP filter $H(z)$ using the minmax equiripple method. The order of the filter $N=24$.

Step 2: We transform the filter $H(z)$ into a MP filter $H_m(z)$ by reflecting zeros of $H(z)$ from their outside position inside the unit circle and converting the roots to polynomial. Figure 1 shows the positions of zeros and impulse responses.

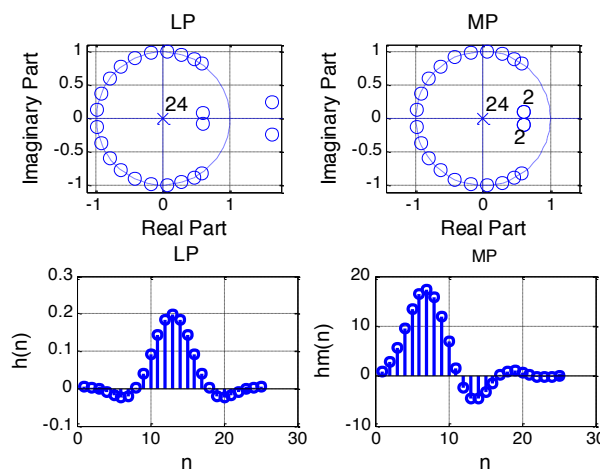


Figure 1. Positions of zeros and impulse responses.

Step 3: From (3) we have:

$$H_{s_i}(z) = K_1 z^{-13} - K_2 H(z) \quad . \quad i=1, 2, 3. \quad (6)$$

Denoting the impulse responses of $H(z)$ and $H_s(z)$, as $h(n)$ and $h_s(n)$, respectively, we have:

$$h_{s_i}(n) = K_1 h(12) - K_2 h(n), \quad n=0, \dots, 24 \quad , \quad i=1, 2, 3. \quad (7)$$

Taking the values from Table 1 we get:

$$\mathbf{P1:} \quad h_{s1}(n) = 2h(13) - h(n); \quad \mathbf{P2:} \quad h_{s2}(n) = 3h(13) - 2h(n); \quad \mathbf{P3:} \quad h_{s3}(n) = 4h(13) - 3h(n) \quad . \quad (8)$$

The filter $H_{s_i}(z)$ is transformed into a minimum system $H_{sim}(z)$ by doubling the zeros which are inside the unit circle, and deleting all others.

Step 4: The corresponding MP sharpened polynomials are:

$$H_{shm1}(z) = H_m(z)H_{sm1}(z); \quad H_{shm2}(z) = H_m^2(z)H_{sm2}(z); \quad H_{shm3}(z) = H_m^3(z)H_{sm3}(z). \quad (9)$$

Figure 2 shows the corresponding magnitude responses of original MP and the sharpened MP filters. The passband zooms are also shown.

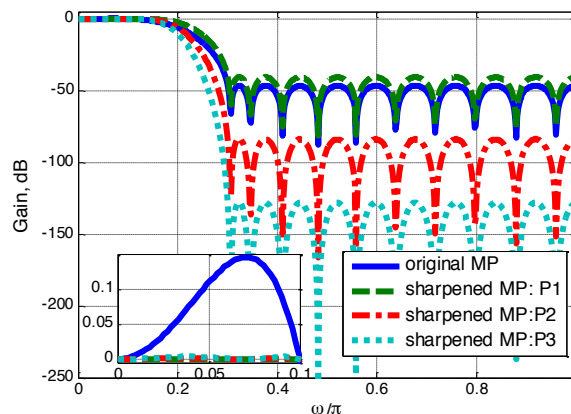


Figure 2. Magnitude responses of MP filters.

4. Conclusion

This paper presents the procedure for the sharpening MP filters. In that way the useful technique, used for FIR filters, can also be used for MP filters. We considered three most important sharpening polynomials. However, the same procedure can also be applied for any sharpening polynomial.

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