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# Analysis and Design of Offset QPSK Using Redundant Filter Banks

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**Abstract.** This paper considers the analysis and design of OQPSK digital modulation. We first establish the discrete time formulation, which allows us to find the equivalent redundant filter banks. It is well known that redundant filter banks are related with redundant transformation of the Frame theory. According to the Frame theory, the redundant transformations and corresponding representations are not unique. In this way, we show that the solution to the pulse shaping problem is not unique. Then we use this property to minimize the effect of the channel noise in the reconstructed symbol stream. We evaluate the performance of the digital communication using numerical examples.

## 1. Introduction

Digital communication has been one of the successful technologies during the last decades. The main goal of digital communication is to send binary information from one point to another point. Digital modulation plays a key role in digital communication. Different modulations techniques have been proposed in the literature. In particular, offset quadriphase shift keying (OQPSK) provides better performance than QPSK in presence of jitter at the receiver [1].

Multirate signal processing plays an important role in the analysis and design of digital communications. Particularly, filter banks have been used in both Single Input Single Output (SISO) and Multiple Input Multiple Output (MIMO) channels [2, 3]. As an example, Discrete Multitone Modulation (DMT) used in high speed data communication over the twisted pair telephone line is analyzed by using a filter bank. Generally, filter bank transceivers are important tools in channel equalization applications [2, 3].

The analysis and design of Orthogonal Frequency Division Multiplexing with Offset Quadrature Amplitude Modulation (OFDM/OQAM) systems based on filter banks is proposed in [4]. Thus, the authors proposed a filter bank OFDM/OQAM with an even number of subbands. The conditions of discrete orthogonality are also established. OFDM/OQAM is reduced to OQPSK when a single frequency is used and the number of symbols in OQAM is four with equal energy. Unfortunately, this case is not considered in [4]. Therefore, in this paper, we propose the use of filter banks in the analysis and design of OQPSK. We show that the discrete time formulation of OQPSK involves the use of redundant filter banks, which are particular cases of redundant transformation in the frame theory. The main advantage of

using redundant transformation is that its representation is not unique. We use this interesting property to minimize the effect of the channel noise in the reconstructed symbol stream.

This paper is organized as follows. Section 2 gives the analysis of offset QPSK modulation using multirate systems. Section 3 deals with the design of the pulse shaping filters for OQPSK systems.

## 2. Analysis of Offset QPSK using Redundant Filter Banks

This section introduces the discrete time equivalent of OQPSK signals. We assume that the transmitter OQPSK signal can be written as [1]

$$y(t) = \Re \left\{ \sum_{k=-\infty}^{\infty} \left( a(k)p(t - kT) + jb(k)p(t - T/2 - kT) \right) e^{j2\pi f_0 t} \right\}, \quad (1)$$

where  $a(n)$  and  $b(n)$  are the real and imaginary parts of QPSK code words, respectively,  $f_0$  is the carrier frequency,  $T$  is the symbol duration, and  $\Re\{\cdot\}$  stands for the real part of  $\{\cdot\}$ . The basedband equivalent signal of  $y(t)$  is given by

$$s(t) = \sum_{k=-\infty}^{\infty} \left( a(k)p(t - kT) + jb(k)p(t - T/2 - kT) \right). \quad (2)$$

In order to obtain a maximum spectral efficiency, we critically sample  $s(t)$ , i.e.,  $T_s = T$ , where  $T_s$  is the sample period. Consequently, we have

$$s(n) = \sum_{k=-\infty}^{\infty} c(k)p_d(2n - kT), \quad c(n) = \begin{cases} a(n/2), & n \text{ even,} \\ jb((n-1)/2), & n \text{ odd.} \end{cases} \quad (3)$$

Similarly,  $p_d(n)$  denotes the oversampled version of the the pulse  $p(t)$ , that is,  $p_d(n) = p(nT/2)$ . Using multirate signal processing, we observe that equation (3) represents a decimation filter operating on the input  $c(n)$  [5]. The decimation factor in this case is 2.

We now turn our attention to recover the symbol  $c(n)$  from  $s(n)$ . To do that, we recall that the right inverse of a decimation filter is an interpolation filter [6]. Therefore, the symbol stream  $s(n)$  can be recovered using the following equation:

$$c(n) = \sum_{k=-\infty}^{\infty} s(k)g(n - 2k), \quad (4)$$

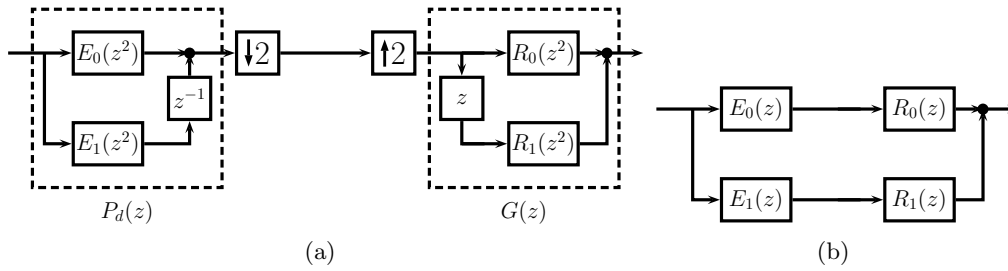
where  $g(n)$  depends on  $p_d(n)$ .

Therefore, the problem to recover  $c(n)$  from  $s(n)$  involves the design of  $g(n)$ . As we shall see,  $g(n)$  is not unique. To do this, consider the Type 1 polyphase components of the  $z$ -transform of  $p_d(n)$ , i.e.,  $P_d(z) = E_0(z^2) + z^{-1}E_1(z^2)$ , where  $E_0(z)$  and  $E_1(z)$  are the Type I polyphase components of  $P_d(z)$ . Additionally, using the Type 2 polyphase components  $R_0(z)$  and  $R_1(z)$  of  $G(z)$ , we have  $G(z) = R_0(z^2) + zR_1(z^2)$ , where  $G(z)$  is the  $z$ -transform of  $g(n)$ .

Cascading the decimation and interpolation filters, and using the polyphase components of  $P_d(z)$  and  $G(z)$ , we arrive at the structure shown in Figure 1(a), which can be redrawn as shown in Figure 1(b). In order to perfectly recover  $c(n)$ , the following equation should be held (see Figure 1(b)):  $R_0(z)E_0(z) + R_1(z)E_1(z) = z^{-\ell}$ , where  $\ell$  is the system delay.

The structure shown in Figure 1(b) is a redundant filter banks. As it is well known the design of redundant filter banks is not unique [7]. From the Frame theory, the polyphase components  $R_k(z)$ ,  $k = 0, 1$ , can be written as [7, 8, 9];

$$\begin{bmatrix} R_0(z) & R_1(z) \end{bmatrix} = \begin{bmatrix} \hat{R}_0(z) & \hat{R}_1(z) \end{bmatrix} + A(z) \begin{bmatrix} E_1(z) & -E_0(z) \end{bmatrix}, \quad (5)$$

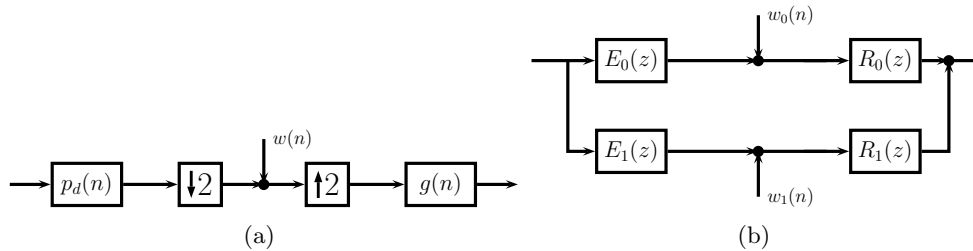


**Figure 1.** a) cascade of decimation and interpolation b) polyphase form of a), and c) Simplification of part a).

where  $A(z)$  is any system function, and  $\widehat{R}_0(z)$  and  $\widehat{R}_1(z)$  are a particular solution of  $\widehat{R}_0(z)E_0(z) + \widehat{R}_1(z)E_1(z) = z^{-\ell}$ . Next section illustrates the design of the filters  $p_d(n)$  and  $g(n)$ .

### 3. Design of pulse shaping filters

Figure 2(a) shows the communication model used to find  $A(z)$ . The signal  $w(n)$  stands for the noise at the communication channel. Using multirate operation, the structure in Figure 2(a) is reduced to the structure shown in Figure 2(b).



**Figure 2.** Communication model.

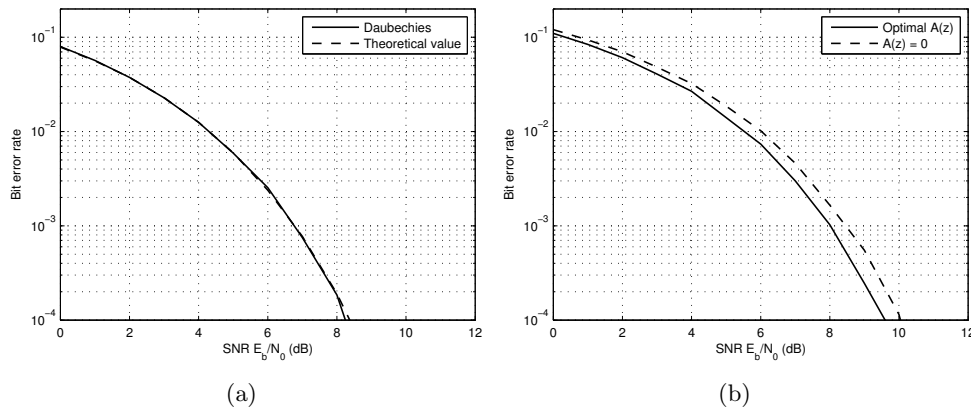
We select the filter  $A(z)$ , such that it minimizes the noise component at the output of the communication system. Consequently, we find that the optimal  $A(z)$  is the Wiener filter given by  $A(z) = -S_{uw}(z)/S_{vv}(z)$ , where  $S_{uv}(z)$  and  $S_{vv}(z)$  are respectively the  $z$ -transforms of the correlations  $R_{uv}(m) = \mathcal{E}(u(n)v^*(n-m))$  and  $R_{vv}(m) = \mathcal{E}(v(n)v^*(n-m))$ , where  $\mathcal{E}(\cdot)$  is the expected value operator. Finally, assuming uncorrelated white noises  $w_0(n)$  and  $w_1(n)$ , we have

$$A(z) = \frac{R_1(z)\widetilde{E}_1(z) - R_0(z)\widetilde{E}_0(z)}{E_0(z)\widetilde{E}_0(z) + E_1(z)\widetilde{E}_1(z)}. \quad (6)$$

We illustrate the design using the following examples.

**Example 1.** We assume that  $p_d(n)$  is the wavelet filter db4, i.e., the number of filter tabs is 6. Since  $p_d(n)$  is orthogonal, we select  $g(n) = p_d(6-n)$  [10]. In this way, we evaluate the performance of the proposed communication system by using the bit error rate (BER), which have a closed form equation given by  $BER = \frac{1}{2}\text{erfc}(\sqrt{E_b/N_0})$ , where  $E_b/N_0$  is the signal to noise ratio (SNR). Figure 3(a) illustrates the comparison between the simulation results and theoretical value.

**Example 2.** Now we consider that  $p_d(n)$  comes from the root Nyquist filter. Additionally, we applied the Euclidean algorithm to find the corresponding polyphase components  $\widehat{R}_0(z)$  and  $\widehat{R}_1(z)$  [2]. Figure 3(b) shows the performance of the communication systems. Observe that the



**Figure 3.** BER performances of OQPSK transmission a) using Daubechies wavelet filter db4 in Example 1 and b) using multirate filter banks in Example 2.

BER is better using an optimal  $A(z)$  than the case where  $A(z) = 0$ , i.e.,  $R_0(z) = \hat{R}_0(z)$  and  $R_1(z) = \hat{R}_1(z)$ .

#### 4. Conclusions

This paper addresses the analysis and design of offset QPSK modulation systems using multirate systems. The discrete analysis of offset QPSK modulation involves redundant filter banks, which are special cases of redundant transformation from Frame theory. The main advantage of redundant transformation is that its representation is not unique. Using this interesting property and Wiener theory, we proposed the design of pulse shaping filter of offset QPSK, which minimize the noise component at the output of the communication system. As a future work, we will extend our result to OFDM/OQAM.

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