Application of amplitude transformation for compensation of comb decimation filters

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The design of compensation filters for comb decimators using amplitude transformation is introduced. It is shown that the transformation of cosine-squared filters provides good compensation characteristics. For a first-degree polynomial, the slope of the transformation line is explicitly set as the unique compensator's multiplierless coefficient. This coefficient changes proportionally with the increase of the comb passband droop. Thus, the proposed approach provides an intuitive and easy way of designing compensation filters.

Introduction: The decimation process is often realised in several stages. Let us consider that M is the decimation factor of the first stage and R is the residual factor. The comb filter, with transfer function $H(z)$ and frequency response $H(e^{j\omega}) = H(\omega)e^{-j\omega(M-1)/2}$, where

$$H(z) = \tfrac{1}{M}\left(1 - z^{-M}\right)/\left(1 - z^{-1}\right) = \tfrac{1}{M}\sum_{i=0}^{M-1} z^{-i} \tag{1}$$

$$H(\omega) = \tfrac{1}{M}\sin(\omega M/2)/\sin(\omega/2) \tag{2}$$

is commonly used in the first stage of the decimation chain because of its simplicity [1]. However, the magnitude response of the comb filter exhibits a passband droop and poor attenuation in the stopbands. The attenuation can be improved by cascading K comb filters. Nevertheless, this results in an increase in the passband droop. The improvement of the passband magnitude characteristic of comb filters is a subject of active research [2–8]. In these methods, the approach consists of designing a low-complexity linear-phase compensation filter, with the transfer function given in general form by $C(z^M)$. By cascading $C(z^M)$ with $H(z)$, the passband droop of the comb filter is decreased, and by applying multirate identities the filter $C(z^M)$ is moved after the downsampling by M. The filters of [5, 6, 8] are multiplierless, hence they have a low complexity. In addition, the filters of [5, 6] can be easily designed because they have only one design parameter that changes with K.

In this Letter, we introduce the design of compensation filters from the perspective of polynomial amplitude transformation. We apply a first-degree amplitude change function (ACF) to a cosine-squared filter. The slope of the transformation line, explicitly presented as the unique multiplierless compensator's coefficient, is expressed in terms of K and R. This provides a simple and intuitive method of design.

Amplitude change of cosine-squared filters for compensation: The amplitude response of an Lth-order linear-phase finite impulse response filter can be changed by using this filter repeatedly. The repetition is dictated by an ACF; this approach was first introduced in [9]. Let us denote the transfer function and frequency response of the filter to be repeated, hereafter referred to as the subfilter, as, respectively, $F(z)$ and $F(e^{j\omega}) = F(\omega)e^{j\omega L/2}$. The resulting transfer function is $F_S(z)$ and the resulting frequency response is $F_S(e^{j\omega}) = F_S(\omega, \boldsymbol{p})e^{-j\omega NL/2}$, where

$$F_S(z) = \sum_{i=0}^{N} z^{-(N-i)L/2} p_i F^i(z) \tag{3}$$

$$F_S(\omega, \boldsymbol{p}) = \sum_{i=0}^{N} p_i F^i(\omega) = \boldsymbol{p}\left[1 \quad F(\omega) \quad \ldots \quad F^N(\omega)\right]^{\mathrm{T}} \tag{4}$$

Note that $\boldsymbol{p} = [p_0\ p_1\ \ldots\ p_N]$ represents the vector of coefficients of the polynomial given as

$$P(x) = \sum_{i=0}^{N} p_i x^i \tag{5}$$

where $P(x)$ is the ACF of degree N.

The amplitude response of the transformed filter, $F_S(\omega, \boldsymbol{p})$, should ideally be equal to $1/H^K(\omega M^{-1})$ for $0 \le \omega \le \pi/R$ in order to compensate for the passband droop of K cascaded comb filters. Moreover, $F_S(\omega, \boldsymbol{p})$ must be monotonically increasing because $H^K(\omega M^{-1})$ is monotonically decreasing in that frequency range. Hence, $F(\omega)$ must be monotonic. For a *cosine-squared* filter, we have

$$F(z) = 2^{-2}\left[1 + 2z^{-1} + z^{-2}\right] \tag{6}$$

$$F(\omega) = \cos^2(\omega/2) \tag{7}$$

$F(\omega)$ is a monotonic function in $0 \le \omega \le \pi/R$ and the cosine-squared filter is a good candidate as the subfilter because of its simplicity. The proposed compensator is $C(z^M)$, with $C(z) = F_S(z)$. The general form to design the compensation filter is finding the vector $\boldsymbol{p}^*$ of optimal coefficients as

$$\begin{aligned} \boldsymbol{p}^* &= \underset{0\le\omega\le\pi/R}{\arg\min}\left\{\left\|\varepsilon(\omega,\ \boldsymbol{p})\right\|_{L_p}\right\} \\ &= \underset{0\le\omega\le\pi/R}{\arg\min}\left\{\left\|1 - F_S(\omega,\ \boldsymbol{p})H^K\left(\omega M^{-1}\right)\right\|_{L_p}\right\} \end{aligned} \tag{8}$$

where $\|\varepsilon(\omega, \boldsymbol{p})\|_{L_p}$ is the L_p-norm of the error $\varepsilon(\omega, \boldsymbol{p})$.

Linearly transformed cosine-squared filter: Instead of directly solving the problem (8), let us look for a practical viewpoint of our transformation-based approach by exploring the first-degree polynomial, i.e. $N = 1$ in (5). Fig. 1 shows the amplitude transformation from $F(\omega)$ to $F_S(\omega, \boldsymbol{p})$ through the line $P(x) = p_0 + p_1 x$. This line (upper-left plot in Fig. 1) has an arbitrary value y_0 for $x = 1$, thus $p_0 = y_0 - p_1$. By substituting (6) in (3) using $p_0 = y_0 - p_1$ and developing the sum we obtain

$$F_S(z) = 2^{-2}\left[p_1 + 2\left(2y_0 - p_1\right)z^{-1} + p_1 z^{-2}\right] \tag{9}$$

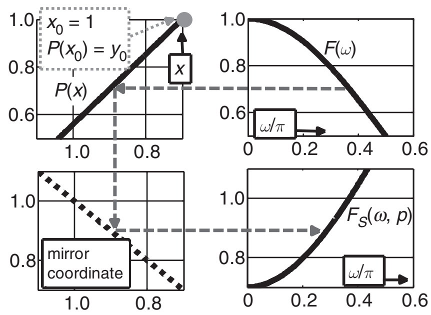

Fig. 1 *Linear transformation of cosine-squared filter*

To see how compensation characteristic arises, proceed counter-clockwise starting from upper right. Follow dashed arrows as a reference

Consider the following characteristics of this second-order filter:

1. To transform the droop of the cosine-squared filter [$F(\omega)$ in the upper-right plot of Fig. 1] into a compensating characteristic [$F_S(\omega, \boldsymbol{p})$ in the lower-right plot of Fig. 1] the slope p_1 of the line $P(x)$ with respect to x must be negative, i.e. $p_1 = -|p_1|$ [see $P(x)$ in the upper-left plot of Fig. 1, being aware that the values x are presented in the vertical axis]. The higher the droop of the comb filter (i.e. the greater K), the greater $|p_1|$. We can implement the filter from (9) with the structure of Fig. 2.
2. The value y_0 is the amplitude response of the compensator at $\omega = 0$. For an L_∞-minimised error, y_0 is less than 1 [8]. However, by setting $y_0 = 1$, the filter becomes simpler and the maximum peak error deviation is just slightly increased. In this way, the compensation characteristic only depends on $|p_1|$.

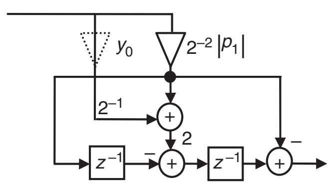

Fig. 2 *Second-order transformation-based compensator*

Usually, $y_0 = 1$

Invariably, any compensator must change its magnitude characteristic as a function of K and R to provide proper compensation in the band of interest. In the proposed scheme, this change is controlled by the coefficient $|p_1|$. To obtain a simple compensator, $|p_1|$ is made multiplierless by simple rounding as follows:

$$|p_1| = 2^{-r}\left|\text{round}\left\{\left(q_2K^2 + q_1K + q_0\right)/2^{-r}\right\}\right| \qquad (10)$$

where r is an arbitrary word-length for the fractional part of the coefficient; in this proposal chosen as $2 \le r \le 6$. Coefficients q_0, q_1 and q_2 in (10) are given as

$$q_2 = 0.0736R^{-2.578} \qquad (11)$$

$$q_1 = 0.1717 \qquad (12)$$

$$q_0 = 0.5438R^{-3.3} - 0.001845 \qquad (13)$$

Comparison: In the following two examples, we compare the proposed method with methods [5, 6, 8].

Example 1 (wideband case): Consider $M = 25$, $R = 2$ and $K = 5$, which ensures an attenuation of at least 45 dB in the stopbands.

From (10), we have $|p_1| = 1 + 2^{-2}$ using $r = 2$. The proposed compensator requires four adders because the coefficient $|p_1|$ in Fig. 2 is replaced with the value $1 + 2^{-2}$. From the method in [6], we have the compensator $C_1(z) = \{-2^{-4} \times [1-(2^4+2)z^{-1} + z^{-2}]\}^4$. The compensator proposed in [8] is $C_2(z) = [(-2^{-2} - 2^{-4} - 2^{-6})(1 + z^{-2}) + (1 + 2^{-1} - 2^{-3} + 2^{-7})z^{-1}]$. Fig. 3 shows the magnitude responses over the band of interest. The magnitude response of the filter [6] is slightly better. However, this filter requires the implementation of four basic three-addition subfilters, which results in 12 additions; three times the number of additions required in the proposed filter. On the other hand, the deviation of the proposed filter in comparison to that in [8] is slightly higher, but that filter requires three more additions and approximately twice the word-length with respect to the proposed filter. Besides, despite the number of additions, the proposed method has the advantage of an easy design procedure with near-optimal solution, whereas method [8] requires a specialised optimisation.

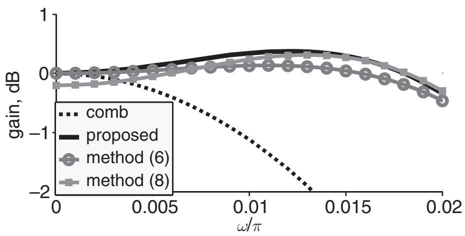

Fig. 3 *Magnitude responses in passband of filters in example 1*

Example 2 (narrowband case): Consider $M = 16$, $R = 4$ and $K = 4$, which ensures an attenuation of at least 65 dB in the stopbands.

From (10), we obtain $|p_1| = 2^{-1} + 2^{-2}$ using $r = 2$. The proposed compensator requires four adders. From method [5], we have the following compensator, $C_1(z) = -2^{-(b+2)} \times [1 - (2^{(b+2)} + 2)z^{-1} + z^{-2}]$, with $b = 1$. The compensator proposed in [8] is $C_2(z) = [(-2^{-3} - 2^{-4} + 2^{-13})(1 + z^{-2}) + (1 + 2^{-1} - 2^{-3} - 2^{-9})z^{-1}]$. Fig. 4 shows the magnitude responses over the band of interest. The filter from [5] is the simplest compensator, and along with the proposed filter it also can be designed with a straightforward method. However, its magnitude characteristic is not near optimal. By using only one extra addition, the proposed method achieves a much better magnitude characteristic.

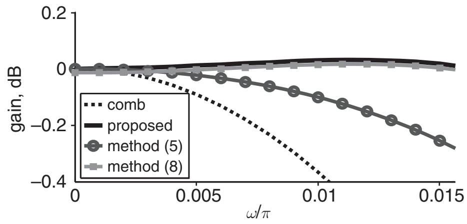

Fig. 4 *Magnitude responses in passband of filters in example 2*

Conclusion: The design of compensation filters has been introduced from the perspective of amplitude transformation, in which the simple cosine-squared filter has been chosen as the subfilter to be transformed. The linear transformation [i.e. $N = 1$ in (5)] results in a second-order compensator. In this case, by ensuring that the magnitude response of the compensator is 1 for $\omega = 0$, the magnitude of the slope of the transformation line becomes the unique coefficient of the compensator. Since this value grows proportionally to K for a given R, we obtain an intuitive and efficient method of design. A formula to obtain the compensator's coefficient in terms of K and R has been presented. The proposed filters have good compensation characteristics, a low complexity, and they can be used for narrowband as well as wideband comb compensation.

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