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Richardson extrapolation-based sensitivity analysis in the multi-objective optimization of analog circuits

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ABSTRACT

The feasible solutions provided by a multi-objective evolutionary algorithm (MOEA) in the optimal sizing of analog integrated circuits (ICs) can be very sensitive to process variations. Therefore, to select the optimal sizes of metal–oxide–semiconductor field-effect-transistors (MOSFETs) but with low sensitivities, we propose to perform multi-parameter sensitivity analysis. However, since MOEAs generate feasible solutions without an explicit equation, then we show the application of Richardson extrapolation to approximate the partial derivatives associated to the sensitivities of the performances of an amplifier with respect to the sizes of every MOSFET. The proposed multi-parameter sensitivity analysis is verified through the optimization of a recycled folded cascode (RFC) operational transconductance amplifier (OTA). We show the behavior of the multi-parameter sensitivity approach versus generations. The final results show that the optimal sizes, selected after executing the sensitivity approach, guarantee the lowest sensitivities values while improving the performances of the RFC OTA.

1. Introduction

To have a general idea on analog integrated circuit (IC) sizing strategies developed by researchers and companies during the last 20 years, an overview on the classification and a brief description of the majority of them can be found in [1]. Although these works and other recently published strategies [2–4] provide good sizing solutions, still the analog design community deals with the hard open problem related to process variations [5,6]. In this manner, we propose to perform multi-parameter sensitivity analysis to the feasible solutions provided by a multi-objective evolutionary algorithm (MOEA), with the goal to select the optimal sizes of an analog IC but with low sensitivities. Because very often, the best feasible solutions meeting extreme performance requirements are located at some peripherals of the feasible solution space, but some variability in the design parameters can transform a best solution to a worst one [7,6,8,5].

Since our proposed multi-parameter sensitivity analysis is performed from numerical data instead of using explicit equations, we propose to apply numerical finite differences and Richardson extrapolation [9–12], to approximate the partial derivatives associated to the sensitivities of the sizing relationships W/L (width/large) of the MOSFETs. These processes are performed in two domains: variables W/L (design parameters) and objectives, where both are evaluated by linking HSPICE®.

The first step of our proposed approach consists on conventional optimization by applying the MOEA called non-dominated sorting genetic algorithm (NSGA-II) [13]. The second step is devoted to perform multi-parameter sensitivity analysis

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for all feasible solutions in the Pareto front. The goal is to discriminate those feasible solutions located in a delicate point that does not support the natural variations of the fabrication processes, i.e. those having large sensitivities.

2. Multi-objective optimization

The optimization stage is performed by applying the MOEA NSGA-II, to minimize a problem of the form [14]:

$$\begin{aligned} &\text{minimize } \mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})]^T \\ &\text{subject to } h_k(\mathbf{x}) \geqslant 0, \quad k = 1 \ldots p, \end{aligned} \tag{1}$$

where function $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$, $\mathbf{x} = [x_1, \ldots, x_n]^T$ is the decision vector and n is the number of variables; $\mathbf{x} \in X$, where $X \subset \mathbb{R}^n$ is the decision space for the variables. Every objective function $f_j(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R},\ j = 1 \ldots m\ (m \geqslant 2)$ and $h_k(\mathbf{x}),\ k = 1 \ldots p$ are performance constraints.

Very often, since the objectives in (1) contradict each other, no point $\mathbf{x} \in X$ minimizes all the objectives simultaneously. The best tradeoffs among the objectives can be defined in terms of Pareto optimality [15–17].

The NSGA-II Algorithm is based on Pareto ranking. First, two populations (P_o and Q_o) are generated, each one of size N. The NSGA-II procedure in each generation consists of rebuilding the current (t) population (R_t) from the two original populations (P_t and Q_t). Next, through a nondominated sorting procedure all solutions in R_t are ranked and classified in a family of sub-fronts [13]. In the next step, a new offspring (P_{t+1}) is created from the current population R_t (previously ranked and ordered by sub-front number), with the goal to choose from a population of size $2N, N$ solutions belonging to the first sub-fronts. Besides, the last sub-front could be greater than necessary, and then a measure ($i_{distance}$) is used to preserve diversity by selecting the solutions that are far from the rest [18]. To build new generations we use differential evolution (DE) [19], as genetic operator.

Regarding to circuit sizing, each variable x represents the width (W) or length (L) of the MOSFETs. Usually, those values are integer-multiples of the minimum value allowed by the fabrication processes. In this manner, if the W/L relationship is expressed in multiples of the minimum L, then the DE operator is performed by rounding W/L to the closer multiple of the minimum L.

3. Multi-parameter sensitivity analysis

The relative or normalized sensitivity (S) can be defined as the cause and effect relationship between the circuit elements variations, and the resulting changes in the performances response [20,21]. Furthermore, in the design of analog ICs the lowest sensitivity is very desired.

Let $f_i(\mathbf{x})$ be an objective function (performance response), where $\mathbf{x} = [x_1, \ldots, x_n]^T$ are the design variables. It is possible to relate small changes in the response of the performance (∂f_i, $i \in [1, m]$) to variations in the design variables ($\partial x_j,\ j \in [1, n]$). It leads us to the single parameter sensitivity definition given by,

$$S_{x_j}^{f_i} \simeq \frac{x_j}{f_i} \frac{\partial f_i}{\partial x_j}. \tag{2}$$

According to (2), there is one sensitivity for each objective function in $\mathbf{f}$ (see (1)) and for each variable in $\mathbf{x}$. Then, it is possible to define the multi-parameter sensitivity which sums the different single sensitivities regarding the different variables for each objective as follows [21]:

$$S^{f_j} = \sqrt{\sum_{i=1}^{n} \left| S_{x_i}^{f_j} \right|^2 \cdot \sigma_{x_i}^2}, \tag{3}$$

where $S_{x_i}^{f_j}$ is calculated by (2), σ_{x_i} is a variability parameter of x_i and the square root is used to preserve the same units.

The performances of an analog IC are evaluated using HSPICE®, and they are considered as the objective functions. As one can infer, using a numerical circuit simulator, there is not possibility to derive an explicit equation for each performance or objective function. Therefore, in order to calculate the partial derivative required by (2), the Richardson extrapolation described by (4), is used herein:

$$\frac{\partial f_i}{\partial x_j} \approx \frac{g_i(\mathbf{x}, j, d) - g_i(\mathbf{x}, j, -d)}{2d}, \quad \text{with } d \to 0, \tag{4}$$

where function g_i is defined as:

$$\begin{aligned} &g_i : \mathbb{R}^n \to \mathbb{R}, \\ &g_i(\mathbf{x}, j, d) = f_i(\mathbf{y}) \mid y_k = x_k \text{ for } k \neq j \text{ and } y_j = x_j + d. \end{aligned} \tag{5}$$

In (4), d is a step parameter that is updated in each iteration [22], for this case $d = 2^{-u} d_{u-1}, d_0$ is assigned to an initial value and u is the current iteration. The recursive calculation continues until a tolerance error, as stopping criterion (δ), is reached.

Our proposed multi-parameter sensitivity analysis approach is based on the Richardson extrapolation sketched in Algorithm 1. This algorithm is performed until the stopping criterion (δ) is reached, and according to Fig. 5, only the feasible solutions provided by the MOEA NSGA-II (accomplishing target specifications) are introduced to the multi-parameter sensitivity analysis.

Due to the iterative processes in Algorithm 1, it is possible to have stagnation in the $f'_{u,v}$ evaluation (line 9 of Algorithm 1), when the very small value of d does not produce difference between $f'_{u,v-1}$ and $f'_{u-1,v-1}$. If that happens, the algorithm saves the last value before the stagnation to avoid a wrong derivative value.

Algorithm 1. Richardson Extrapolation

1: $h = h_0$
2: **for** $u = 0; u < MaxLoops\ ; u++$ **do**
3:   **for** $v = 0; v < MaxLoops\ ; v++$ **do**
4:     **if** $v == 0$ **then**
5:       g^+ = Function evaluation with the parameter $+d$
6:       g^- = Function evaluation with the parameter $-d$
7:       $f'_{u,v} = (g^+ - g^-)/(2 * d)$
8:     **else**
9:       $f'_{u,v} = f'_{u,v-1} + (f'_{u,v-1} - f'_{u-1,v-1})/(2 ** (2 * v) - 1)$
10:         **if** $|f'_{u,v} - f'_{u,v-1}| < \delta$ **then**
11:           break
12:     $d \leftarrow d/2;$
13: **return** $f'_{u,v}$

For instance, let $\mathbf{x} = [x_1, x_2, x_3]^T$ be any solution from the feasible solutions set and $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})]^T$ its objective vector. For the objective function f_1, and for a given initial value of d, the first estimations for the partial derivatives

$$\frac{\partial f_1}{\partial x_1} \approx \frac{g1(\mathbf{x},1,d) - g1(\mathbf{x},1,-d)}{2d},$$

$$\frac{\partial f_1}{\partial x_2} \approx \frac{g1(\mathbf{x},2,d) - g1(\mathbf{x},2,-d)}{2d},$$

$$\frac{\partial f_1}{\partial x_3} \approx \frac{g1(\mathbf{x},3,d) - g1(\mathbf{x},3,-d)}{2d}$$

are calculated. Next, the Richardson extrapolation is executed. That way, the multi-parameter sensitivity for function f_1 is calculated as

$$S^{f_1} = \sqrt{\left(S^{f_1}_{x_1}\right)^2 \sigma^2_{x_1} + \left(S^{f_1}_{x_2}\right)^2 \sigma^2_{x_2} + \left(S^{f_1}_{x_3}\right)^2 \sigma^2_{x_3}}.$$

The other multi-parameter sensitivities S^{f_2} and S^{f_3} are calculated with the same procedure. In our implementation of the Richardson extrapolation, the three partial derivatives for all the functions f_1, f_2 and f_3, with respect to a variable x_i (say x_1), are calculated in a single HSPICE simulation, because those values correspond to the variation of $x_1 \pm d$. The Richardson extrapolation is executed until the stop criteria is reached for all the three partial derivatives.

With the aim to highlight the behavior of the Richardson extrapolation, an example on the calculation of the multi-parameter sensitivity of the SRN function described by (6) [13], is exposed herein. It consists to evaluate two objective functions $[f_1(\mathbf{x}), f_2(\mathbf{x})]^T$, with two variables $\mathbf{x} = [x_1, x_2]^T$, and taking into account two constraints h_1 and h_2. This optimization problem is minimized by using the NSGA-II algorithm with a population size of 100, leading to the result shown in Fig. 1, where are depicted the non-dominated solutions after 250 generations. From these numerical feasible solutions we apply the Richardson extrapolation to evaluate the sensitivity of every solution, with $d_0 = 5\%$ for the current variable and $\delta < 1\%$.

$$SRN = \begin{cases} f_1(\mathbf{x}) = (x_1 - 2)^2 + (x_2 - 1)^2 + 2, \\ f_2(\mathbf{x}) = 9x_1 - (x_2 - 1)^2, \\ h_1(\mathbf{x}) = x_1^2 + x_2^2 \leqslant 225, \\ h_2(\mathbf{x}) = x_1^2 - 3x_2^2 \leqslant -10. \end{cases} \tag{6}$$

Left side of Fig. 2 shows the analytical and numerical solution by applying Richardson extrapolation of the partial derivatives for f_1 and f_2 for the variable $x_1 : \partial f_1/\partial x_1 = 2(x_1 - 2)$ and $\partial f_2/\partial x_1 = 9$. The solid line represents the analytical derivative and markers show the Richardson extrapolation derivative. These results demonstrate that the Richardson extrapolation calculated numerically agrees with the analytical evaluation of the derivative of f_1 and f_2 in (6).

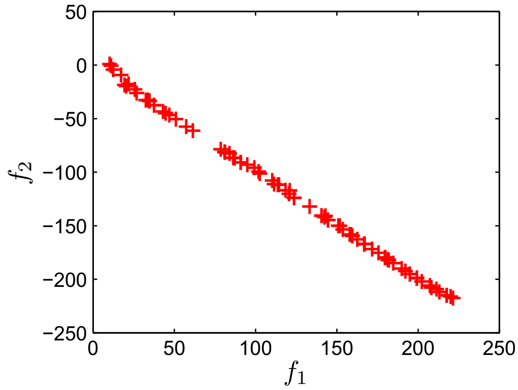

Fig. 1. Optimization of SRN by applying NSGA-II.

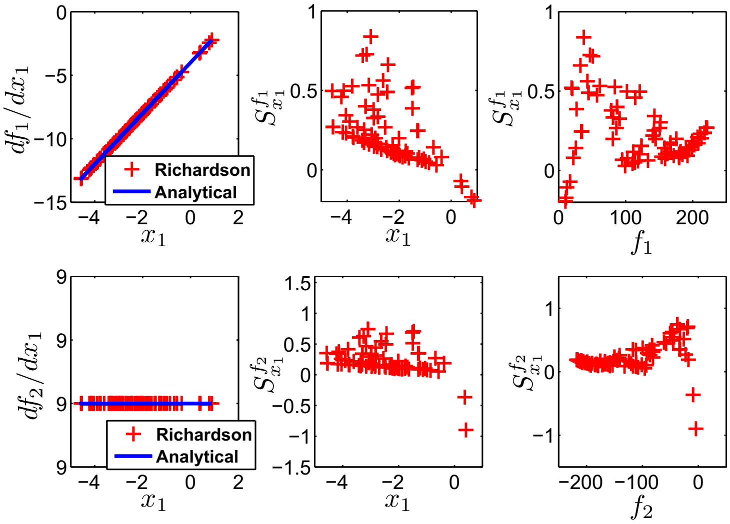

Fig. 2. Sensitivity of SRN with respect to x_1.

The middle and right side of Fig. 2 show the behavior of the single-parameter sensitivity of variable x_1 (calculated as in (2)), and the objective functions f_1 and f_2. On the middle of Fig. 2 are depicted the single-sensitivity results as function of x_1 for f_1 (represented by $S^{f_1}_{x_1}$) and f_2 (represented by $S^{f_2}_{x_1}$). On the right side of Fig. 2, there are depicted again the single-sensitivity results as function of x_1 for f_1 and f_2, but this time versus the objective functions f_1 and f_2.

If we focus on the sensitivity analysis, Fig. 2 shows that for f_1 the closer to zero values of x_1 exhibit low sensitivity, and for f_2 the closer to zero values exhibit the lowest sensitivity too.

Fig. 3 depicts the multi-parameter sensitivity evaluated by (3), with $\sigma = 1\%$ (as variables represent sizes in circuit optimization, we consider the same variability value for them). Upper side of Fig. 3 shows the multi-parameter sensitivity for f_1 (represented by S^{f_1}) and the lower side for f_2 (represented by S^{f_2}). For both cases, the lower values of f_1 and f_2 exhibit low multi-parameter sensitivity.

Finally, from the feasible solution set shown in Fig. 1, it is possible to chose the multi-parameter sensitivities lower than 0.015, considering both objective functions. As a result, the feasible solutions with low sensitivities are shown in Fig. 4. This example demonstrates the usefulness of the Richardson extrapolation to perform multi-parameter sensitivity analysis from numerical data.

4. Proposed optimization system including multi-parameter sensitivity analysis

Our proposed approach to select optimal sizes with low sensitivities has been programmed using MATLAB®, and the circuit under optimization is simulated with HSPICE® through successive simulations [18]. The optimization of the circuit performances is done by modifying the width (W) and length (L) of the MOSFETs.

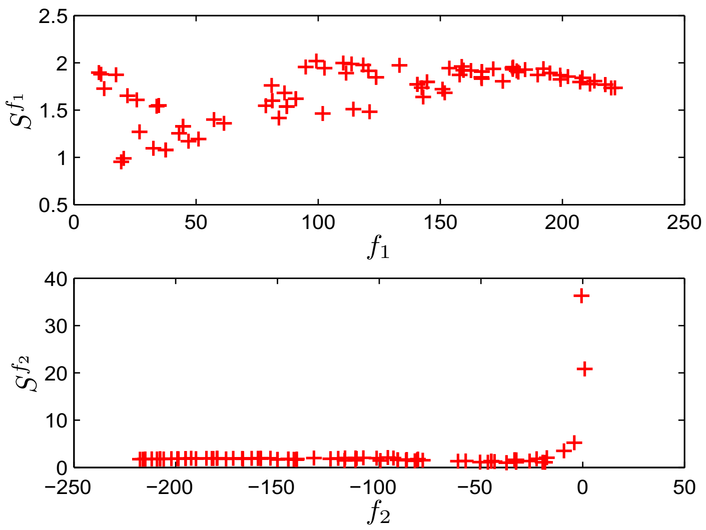

Fig. 3. Multi-parameter sensitivity of SRN function for f_1 and f_2.

In Fig. 5, it can be appreciated that our proposed optimization approach is divided into two general stages: initialization and optimization. In the initialization stage the parameters as maximum number of generations, population size and sensitivity parameters (d_0 and δ), are declared. In the second stage, the NSGA-II algorithm is applied to generate feasible solution sets. In this stage HSPICE® is linked to evaluate the objective functions and constraints. Only the solutions that meet the specifications are introduced to the multi-parameter sensitivity analysis based on the Richardson extrapolation. Afterwards, the non-dominated sort is performed giving priority to the solutions with a measure of multi-parameter sensitivity because are the solutions that accomplish with the target specifications and the constraints. The final solution set contains solutions with low multi-parameter sensitivities.

The efficiency of the procedure depends on the efficiency of both, the NSGA-II and the multi-parameter sensitivity calculation. Regarding to NSGA-II, its efficiency is $O(NM)$, where N is the number of individuals and M is the number of objectives. The multi-parameter sensitivity calculation efficiency is $O(KnM) \approx O(nM)$, where K is the number of iterations in the Richardson extrapolation, n is the number of variables and M is the number of objectives. The worst case for K is the *MaxLoop* constant (Algorithm 1) then in asymptotic notation the effect of such constant is negligible. These efficiencies indirectly depend on the simulator efficiency due that both, the optimization and the multi-parameter sensitivity evaluation are performed from a circuit simulation.

5. Example

The proposed optimization approach including multi-parameter sensitivity analysis is tested on the recycled folded cascode (RFC) operational transconductance amplifier (OTA) shown in Fig. 6. It was taken from [23], but now we include the design of the biasing circuitry shown in the left part. The biasing circuitry consists of {MP1,...,MP4} and {MN1,...,MN4}, to provide two voltages: Vbp and Vbn.

The optimization is executed to accomplish the eight objectives already provided in [23]: DC gain, gain bandwidth product (GBW), phase margin (PM), input referred noise, input offset, settling time (ST), slew rate (SR) and power consumption (PW). This circuit is encoded with ten variables (design parameters) for the MOSFETs, as shown in Table 1:

$$\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_8(\mathbf{x})]^T = \left[\frac{1}{\text{Gain}}, \frac{1}{\text{GBW}}, \frac{1}{\text{PM}}, \text{Noise}, \text{Offset}, \text{ST}, \frac{1}{\text{SR}}, \text{PW}\right]^T. \tag{7}$$

The optimization problem for this circuit is expressed as in (1), with $m = 8,\ n = 10$ and $p = 33$, where $\mathbf{f}(\mathbf{x})$ is the vector formed by the eight objectives, accommodated as shown in (7), to deal with a minimization optimization problem. Therefore, the objectives Gain, GBW, PM and SR have been inverted. X is the search space for the variables listed in Table 1, and the decision space for $\mathbf{x} = [x_1, \ldots, x_{10}]^T$. However, the variables x_1 and x_2 have the domain $[0.18 \ldots 0.72]$ (in µm). The rest of the variables have the domain $[0.18 \ldots 140]$ (in µm). Finally, in a first experiment $h_k(\mathbf{x}),\ k = 1, \ldots, 33$ are performance constraints. In this case, 25 constraints are the saturation conditions in all MOSFETs and 8 target specifications. In a second experiment that includes multi-parameter sensitivity, $h_k(\mathbf{x}),\ k = 1, \ldots, 41$ are the 33 constraints from the first experiment plus eight multi-parameter sensitivities.

The optimization for the first experiment was performed along 250 generations over 10 runs with a population size of 250. For the second experiment, the optimization is stopped after several generations providing the same multi-parameter

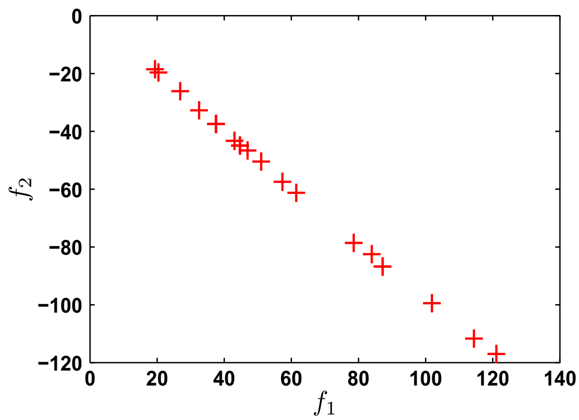

Fig. 4. Feasible solutions for SRN after applying multi-parameter sensitivity analysis.

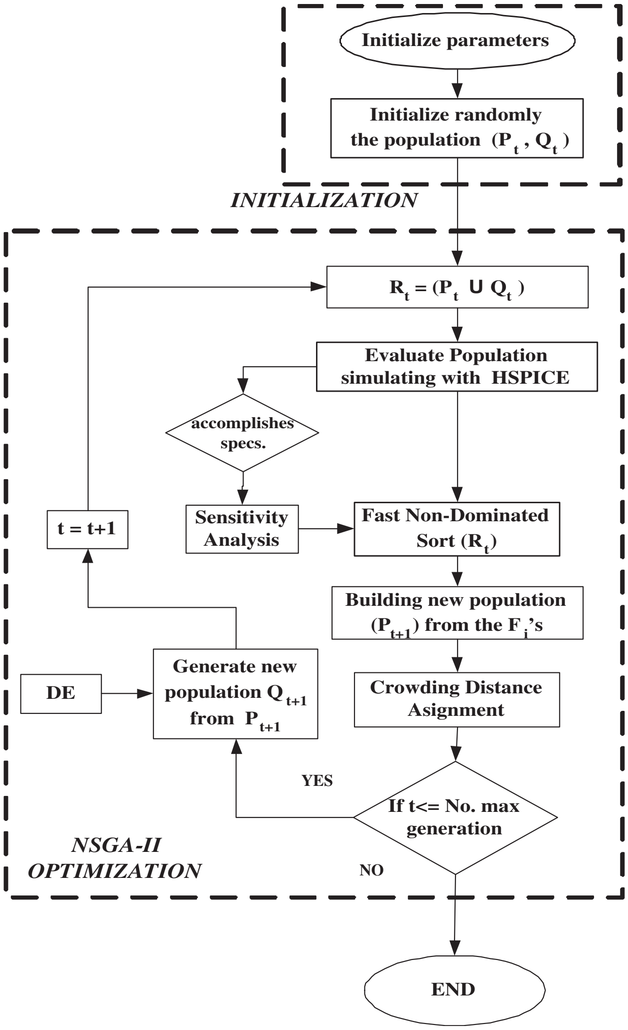

Fig. 5. Flow diagram.

sensitivity because the optimization takes more time than the first experiment. For DE there were arbitrarily selected $\gamma = 1$ and $\eta = 0.4$ for both experiments.

The RFC OTA is biased with $I_{ref} = 400\mu A$ and $V_{DD} = 1.8$ V. The electrical measurements were executed with a load capacitor of 5.6pF and the HSPICE® simulations were performed with a LEVEL 49 standard CMOS Technology of 0.18 μm. The

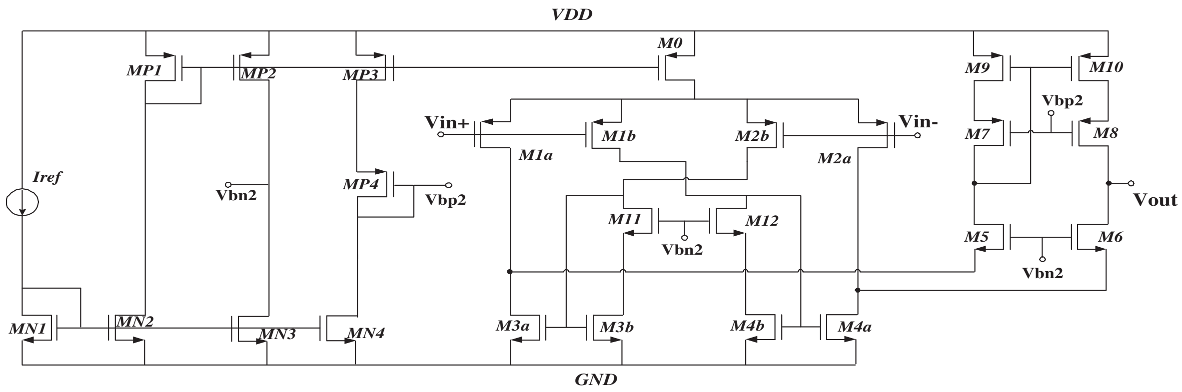

Fig. 6. Recycled folded Cascode OTA.

parameters for the sensitivity analysis are $d_0 = 3\%$ for the design values, and $\delta = 3\%$. The aim to use percentage values for d_0 and δ, is the possibility to manage different design values with different magnitudes. In this example, σ_i is proposed to be 3%.

The size of the final solution set is around 60, in average for all the runs in this experiment. Table 2 shows the target specifications (Specs.), minimum, maximum, average and standard deviation for all the objective functions among the final solution set, however those results do not take into account the sensitivities of every feasible solution. The target specifications to be improved are the values of the objective functions or performances evaluated with the sizes already published in [23]. The application of our optimization stage provides better performances compared to [23], for every objective function. These results are highlighted with bold font in Tables 2 to 5. From Table 2 one can gain an insight on what to expect for this RFC OTA circuit topology working under these design conditions. Later on, this table is an important base line to compare the results that the multi-parameter sensitivity analysis will generate.

The optimization works with a multi-objective problem, then the best performances for the eight objective functions are listed in Table 3, where $\mathbf{x}_1$ is the best solution for gain, $\mathbf{x}_2$ is the best solution for GBW and so on with PM, Noise, Offset, ST, SR and PW. For instance, the maximum gain (solution $\mathbf{x}_1$) is 68 dB; this best point, has GBW = 107.97 MHz, PM = 76.86 deg, Noise = 52.62 μVrms, Offset = 20.04 μV, ST = 17.10 ns, SR = 88.04 V/μs and PW = 3.25 mW. This optimum gain is achieved

Table 1
Encoding for the RFC OTA shown in Fig. 6.

gene	Design variable	Encoding transistors
x_1	L_1	M0,M3a,M3b,M4a,M4b,M9,M10
		MN1, ... ,MN4,MP1, ... , MP4
x_2	L_2	M5, ..., M8
	$2 \cdot L_2$	M1a,M1b,M2a,M2b
x_3	W_1	M0, MP1
x_4	W_2	M1a,M1b,M2a,M2b
x_5	W_3	M3a,M4a
x_6	W_4	M3b,M4b
x_7	W_5	M5, M6,MN3,MN4
	$2 \cdot W_5$	MN1,MN2,MP4
	$4 \cdot W_5$	MP2,MP3
x_8	W_6	M7, M8
x_9	W_7	M9, M10
x_{10}	W_8	M11, M12

Table 2
Best points for the RFC OTA without sensitivity analysis.

Objective	Specs.	MAX	MIN	AVG	STD
Gain [dB]	>65.35	**68.00**	66.03	67.44	0.37
GBW [MHz]	>89.57	**123.14**	96.80	105.15	5.19
PM [deg]	>75.47	**79.45**	75.47	76.76	0.74
Noise [μVrms]	<68.41	69.42	**42.98**	58.63	5.51
Offset [μV]	<206.79	99.90	**0.00**	50.33	27.11
ST [ns]	<20.14	18.50	**15.29**	17.33	0.70
SR [V/μs]	>76.99	**121.11**	77.00	81.38	4.92
PW [mW]	<3.09	3.30	**3.04**	3.22	0.06

Table 3
Best sizing solutions without sensitivity analysis for the RFC OTA.

	Specs.	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$
L_1 [μm]	0.5	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
L_2 [μm]	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
W_1 [μm]	64	132.48	85.32	85.86	130.14	78.3	85.32	178.56	57.78
W_2 [μm]	32	58.5	72	41.94	68.04	49.68	72	49.5	58.14
W_3 [μm]	12	18.9	14.94	17.64	3.96	9.9	14.94	21.24	17.28
W_4 [μm]	4	9.9	7.02	8.1	1.98	4.86	7.02	12.42	6.66
W_5 [μm]	8	16.2	10.62	10.8	16.02	10.08	10.62	21.6	7.2
W_6 [μm]	32	32.76	18	17.1	26.64	69.66	18	34.2	52.92
W_7 [μm]	32	5.22	7.74	8.28	5.94	5.22	7.74	4.5	5.58
W_8 [μm]	32	24.3	10.62	18	9	31.14	10.62	1.8	3.96
Gain [dB]	>65.35	**68.00**	67.57	66.13	66.86	67.79	67.57	67.97	67.69
GBW [MHz]	>89.57	107.97	**123.14**	102.47	116.12	102.79	123.14	102.04	100.04
PM [deg]	>75.47	76.86	75.57	**79.45**	75.47	77.14	75.57	76.99	75.50
Noise [μVrms]	<68.41	52.62	49.23	58.60	**42.98**	51.92	49.23	54.96	61.54
Offset [μV]	<206.79	20.04	60.14	75.42	37.17	**0.00**	60.14	9.75	5.76
ST [ns]	<20.14	17.10	15.29	18.08	16.22	17.51	**15.29**	17.78	18.08
SR [V/μs]	>76.99	88.04	77.96	78.93	78.31	79.66	77.96	**121.11**	79.27
PW [mW]	<3.09	3.25	3.29	3.30	3.28	3.25	3.29	3.25	**3.04**

Table 4
Best points for the RFC OTA including sensitivity analysis.

Objective	Specs.	MAX	MIN	AVG	STD
Gain [dB]	>65.35	**67.83**	66.46	67.01	0.42
GBW [MHz]	>89.57	**106.52**	94.63	96.72	2.97
PM [deg]	>75.47	**77.30**	75.48	75.96	0.45
Noise [μVrms]	<68.41	66.40	**55.27**	63.51	3.27
Offset [μV]	<206.79	96.97	**0.03**	34.84	31.96
ST [ns]	<20.14	18.49	**16.90**	18.25	0.36
SR [V/μs]	>76.99	**79.64**	77.09	77.63	0.56
PW [mW]	<3.09	3.30	**3.24**	3.28	0.02

Table 5
Best sizing solutions including sensitivity analysis for the RFC OTA.

	Specs.	$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$
L_1 [μm]	0.5	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
L_2 [μm]	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
W_1 [μm]	64	125.46	120.06	132.12	132.12	152.82	120.06	125.46	132.12
W_2 [μm]	32	45.18	52.2	54.18	54.18	45.72	52.2	45.18	54.18
W_3 [μm]	12	38.52	42.3	25.56	25.56	30.96	42.3	38.52	25.56
W_4 [μm]	4	19.62	22.14	14.22	14.22	16.92	22.14	19.62	14.22
W_5 [μm]	8	15.66	14.76	16.2	16.2	18.72	14.76	15.66	16.2
W_6 [μm]	32	61.92	25.92	18.72	18.72	59.58	25.92	61.92	18.72
W_7 [μm]	32	8.82	6.84	5.22	5.22	4.5	6.84	8.82	5.22
W_8 [μm]	32	8.28	2.88	3.06	3.06	12.96	2.88	8.28	3.06
Gain [dB]	>65.35	**67.83**	67.40	67.63	67.63	67.45	67.40	67.83	67.63
GBW [MHz]	>89.57	100.06	**106.52**	104.52	104.52	96.01	106.52	100.06	104.52
PM [deg]	>75.47	76.37	75.55	**77.30**	77.30	76.55	75.55	76.37	77.30
Noise [μVrms]	<68.41	61.86	59.07	55.27	**55.27**	59.64	59.07	61.86	55.27
Offset [μV]	<206.79	96.97	60.25	30.83	30.83	**0.03**	60.25	96.97	30.83
ST [ns]	<20.14	18.13	16.90	17.52	17.52	18.40	**16.90**	18.13	17.52
SR [V/μs]	>76.99	79.64	78.32	77.37	77.37	77.87	78.32	**79.64**	77.37
PW [mW]	<3.09	3.29	3.27	3.24	3.24	3.25	3.27	3.29	**3.24**

with L_1 = 0.36 μm, L_2 = 0.18 μm, W_1 = 32.48 μm, W_2 = 58.5 μm, W_3 = 18.9 μm, W_4 = 9.9 μm, W_5 = 16.2 μm, W_6 = 32.76 μm, W_7 = 5.22 μm and W_8 = 24.3 μm.

The next step consisted of performing an optimization including a multi-parameter sensitivity analysis. In such a way that the sensitivity adds eight constraints more: the multi-parameter sensitivities for each one of the eight objectives. Then we have in total 49 constraints for this second experiment, the firsts 33 constraints accomplish with the specifications and the next eight accomplish with the less multi-parameter sensitivity.

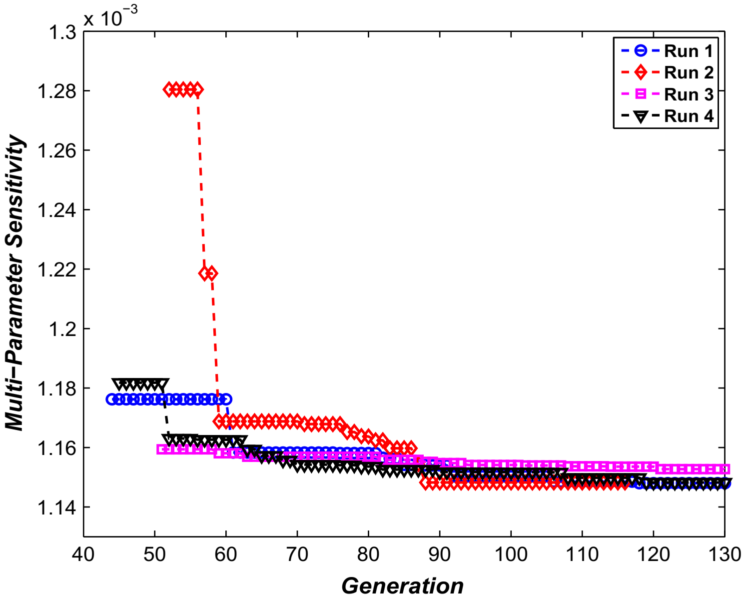

Fig. 7. Behavior of multi-parameter sensitivity vs generations.

In this experiment the size of the final solution set is around the population size, although that within such set, all the solutions accomplish the target specifications, each one has a different value of multi-parameter sensitivity, then it is possible to chose the solutions with the lowest one. By selecting the lowest five solutions in each run, Table 4 shows the target specifications (Specs.), minimum, maximum, average and standard deviation for all the objective functions including multi-parameter sensitivity in the optimization for every feasible solution. As before, the target specifications to be improved are the values of the objective functions or performances evaluated with the sizes already published in [23]. In this second experiment, it is proposed selecting the feasible solutions with the lowest multi-parameter sensitivity, in this case it is necessary to sacrifice some of the objectives with the aim to preserve the best values of the remaining ones. Then it was decided to allow slightly higher values of power consumption.

The application of our optimization stage provides better performances compared to [23], for every objective function except for power consumption that is slightly above the target specification. As before, the best results are highlighted with bold font. By comparing Tables 2 and 4, it is possible to see how the best results from the first experiment were lost, but nevertheless the best results for the second experiment still improve the targets except for the power consumption. Gain, PM and PW are almost in the same value than the first experiment. GBW, Noise and ST are closer to the first experiment values and finally, SR and Offset were decreased significantly compared with the first experiment, but still they are better than targets specs.

The best performances for the eight objective functions in the second experiment are listed in Table 5, where $\mathbf{x}_1$ is the best solution for gain, $\mathbf{x}_2$ is the best solution for GBW and so on with PM, Noise, Offset, ST, SR and PW. In this case, the solution for gain, GBW, PM, Noise, ST and PW is the same ($\mathbf{x}_1 = \mathbf{x}_2 = \mathbf{x}_3 = \mathbf{x}_4 = \mathbf{x}_6 = \mathbf{x}_8$).

At the beginning of the optimization including multi-parameter sensitivity, our system takes into account only the constraints corresponding to the saturation condition of transistors and targets specifications. As soon as a solution accomplishes those constraints, the multi-parameter sensitivity constraints, are included. It allows to categorize those solutions with the less multi-parameter sensitivity among the best ones, to guide the optimization engine. During this processes, it is possible to see how the multi-parameter sensitivity is reduced from the moment when the system found solutions accomplishing the first constraints. Fig. 7 shows how the multi-parameter sensitivities go diminishing over the generations, until the lowest sensitivities values are reached over several generations. For instance, in the four runs, it is possible to see that after the generation 40 several solutions with low multi-parameter sensitivities began to appear, and before the generation 100 the system reaches the lowest sensitivities values.

6. Conclusion

We introduced a multi-parameter sensitivity analysis that selects into the optimization loop, the best feasible solutions generated by the MOEA NSGA-II having the lowest multi-parameter sensitivity. This evolutive algorithm allows us to handle many variables and multiple objectives, both with different magnitudes and only by defining limit values for each one of them. Furthermore, it handles constraints also with different magnitudes. DE was used into the MOEA as a genetic operator but with a discrete approach with the aim to handle multiples of the minimum W/L relationship allowed by the integrated circuit technology.

The use of finite differences and Richardson extrapolation allows us to calculate the partial derivatives for multi-parameter sensitivity analysis, without an explicit mathematical expression. Our proposed approach was tested on the RFC OTA, for which the target specifications were included as constraints in the optimization process, resulting a final solution set that accomplishes simultaneously each objective. In Table 5 are listed the best solutions that accomplish the entire target specifications for each objective and they improve the performances already published in [23] with the lowest multi-parameter sensitivity.

We can point out that the application of our proposed multi-parameter sensitivity analysis shows an important decision point, because before this analysis, all the target specifications are accomplished but if we select those solutions without taking into account their sensitivities, in the fabrication process there is a strong possibility that the designed circuits do not guarantee optimal performances. In summary, we are able to discriminate those solutions that are not really feasible, making easier to select the best solutions having low sensitivities among the final solution sets.

As a conclusion, we believe that our proposed approach is a powerful tool to enhance analog circuit design through generating feasible solutions with low sensitivities to process variations. Also, it is possible to choose among different variables encodings, to explore the best performances of an analog IC, and including multi-parameter sensitivity so that the optimal performance of a circuit design can be guaranteed.

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