National Institute of Astrophysics,
Optics and Electronics
Computer Science Department

Integrated Three-dimensional Reconstruction With
Efficiently Acquiring Reflectance Fields

A dissertation submitted by

María Luisa Rosas Cholula

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Supervisor

Dr. Miguel O. Arias Estrada

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Luis Enrique Erro 1
Sta. Ma. Tonantzintla
72840, Puebla, México.

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Reconstrucción Tridimensional Integrada con Adquisición Eficiente del Campo de Reflectancia

Tesis que presenta
María Luisa Rosas Cholula

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Director de tesis
Dr. Miguel O. Arias Estrada

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Abstract

The reconstruction of object surfaces is an important area in computer vision. This area is directed towards the recovery of three-dimensional information of an object placed in a scene. Most classical existing reconstruction methods assume that objects in the scene reflect light equally in all directions. This assumption on surface reflectance is referred as a Lambertian BRDF (Bidirectional Reflectance Distribution Function) however, Lambertian reflectance is not fulfilled for real-world objects, leading to incorrect depth estimates. Several methods propose a partial solution taking advantage of calibrated cameras and lights or some approximation of the surface reflectance.

Reflectance fields are a common image-based model used for rendering the appearance of objects with a high-degree of realism. Reflectance fields are used in a variety of applications. For example, they are used to capture the appearance of real-world objects with complex geometry, like human bodies, furry teddy bears, or bonsai trees. They are also used to represent intricate distributions of light, like the illumination from a flash light. However, despite the increasing popularity of using reflectance fields, fast methods do not exist for acquiring them.

This thesis presents two main contributions towards reflectance field acquisition and three-dimensional reconstruction. The first is a method for acquiring efficiently the reflectance field data to provide the surface reflectance information. This method introduces the use of independent component analysis, it enables fast acquisition of the reflectance field. The prototype acquisition system consists of a pair of projector and camera controlled by a computer. The effectiveness of the system is demonstrated with scenes composed of different material properties where the reflectance quality is maintained.

The second contribution is a three-dimensional reconstruction of real-world objects placed in the scene. Three-dimensional reconstruction is obtained by relating depth estimations with reflectance fields. The reflectance field is integrated with three-dimensional reconstruction equations by considering the system setup as a stereo system, it provides depth estimations of objects exhibiting different material properties with high accuracy.

The reflectance field acquisition and three-dimensional reconstruction proposed are applied to several objects (plane, triangle and sphere) with different material properties from their surface such as: matte, wood, woven and marble. Using these objects we are including all the range of material properties proposed in related
work: opaque homogeneous (matte), textured (woven) and subsurface scattering surface (wood and marble). The results show that the number of images composing the reflectance information is decreased up to 99% in comparison with traditional techniques and the three-dimensional reconstruction is recovered with an error less than 3% of the real-world object measurements.
Resumen

La reconstrucción de las superficies de los objetos es un área importante en visión computacional. Esta área se enfoca en la recuperación tridimensional de objetos situados en la escena. La mayoría de los métodos clásicos de reconstrucción asumen que los objetos en la escena reflejan la misma cantidad de luz en todas direcciones. Esta suposición sobre la reflectancia de las superficies es conocida como BRDF lambertiana (Función de Distribución de Reflectancia Bidireccional Lambertiana) sin embargo, la reflectancia Lambertiana no se cumple para objetos reales, lo cual conduce a estimaciones de profundidad incorrectas. Algunos métodos proponen una solución parcial haciendo uso de cámaras y luces calibradas o de alguna aproximación de la reflectancia de la superficial.

Los campos de reflectancia son modelos basados en imágenes usados para renderizar el aspecto de los objetos con un alto grado de realismo. Los campos de reflectancia se utilizan en una variedad de aplicaciones. Por ejemplo, para capturar la apariencia de objetos del mundo real con geometría compleja, como el cuerpo humano, osos de peluche o árboles bonsái. También son usados para representar distribuciones complejas de luz, como la iluminación de una luz de flash. Sin embargo, a pesar de la creciente popularidad del uso de campos de reflectancia, no existen métodos rápidos para adquirirlos.

En esta tesis se presentan dos contribuciones principales: la adquisición del campo de reflectancia y la obtención de la reconstrucción tridimensional. La primera contribución consiste en un método eficiente para la adquisición del campo de reflectancia que proporcione la información de reflectancia de la superficie. Este método hace uso de análisis de componentes independientes permitiendo una rápida adquisición del campo de reflectancia. El sistema de adquisición consiste en un proyector y una cámara controlados por una computadora personal. La eficacia del sistema se demuestra con escenas compuestas de diferentes propiedades materiales donde la calidad de la reflectancia se mantiene.

La segunda contribución es la obtención de información tridimensional de objetos reales situados en la escena. La reconstrucción tridimensional se obtiene al relacionar las estimaciones de profundidad con los campos de reflectancia. El campo de reflectancia se integra con las ecuaciones de reconstrucción tridimensional al considerar el sistema de adquisición como un sistema estéreo, esto proporciona estimaciones precisas de profundidad de objetos con diferentes propiedades materiales.

El campo de reflectancia y reconstrucción tridimensional se obtienen de varios
objetos (plano, triángulo y esfera) que poseen diferentes propiedades materiales de su superficie tales como: mate, madera, tejido y mármol). Usando estos objetos se incluye todo el rango de propiedades materiales propuestos en el trabajo relacionado: opaco homogéneo (mate), textura (tejido) y superficies con reflejos internos (madera y mármol). Los resultados muestran que el número de imágenes que componen la información de reflectancia es disminuido hasta un 99% en comparación con las técnicas tradicionales y la reconstrucción tridimensional se obtiene con un error menor al 3% con respecto a las dimensiones reales de los objetos recuperados.
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Chapter 1

Introduction

In this chapter, we give a brief explanation about the importance of the work in computer vision and computer graphics and how the thesis provides a solution in the fast acquisition of the reflectance field and the three-dimensional reconstruction recovery.

1.1 Background of the problem

Three-dimensional reconstruction is a discipline of computer vision dealing with the problem of obtaining the object shape or the calculation of distances between the sensor, i.e. the camera, and objects in a scene. There are many different methods for 3D reconstruction, all of them are interested in recovering real information from a particular scene. The most important application areas include: industrial inspection, medical assistance, entertainment industry, archeological finding reconstruction and robot navigation.

Most of the actual methods can be classified as passive and active methods. In general the passive methods use only the information provided by the image obtained with a camera or cameras, under ambient illumination conditions, while the active methods use and manipulate the information provided by the illumination on the scene. Nearly all of these methods rely on the assumption that the objects in the scene have Lambertian reflectance surface (such property is related to the materials that reflect the same amount of incident energy illumination uniformly over all the surface). Unfortunately, this assumption is violated for real world objects, leading to incorrect depth estimates.

Because the different properties of surfaces, ambient illumination is not sufficient to model important characteristics of the object as shape and depth. In the area of computer graphics, the reflection from surfaces is typically described by high dimensional reflectance functions. However, the formulation of analytical models is not always an easy task. An alternative approach to the specification of the reflectance of the surface objects by analytical modeling is the measurement of the reflectance information from real-world surfaces. A camera or array of cameras can obtain a set
of data that describes the transfer of energy between a light field of incoming rays (the illumination) and a light field of outgoing rays (the view). Such set of data is known as the \textit{Reflectance Field}. The reflectance field is composed by thousands of images, depending on material properties of the object and the resolution of the array of cameras. This research proposes to overcome the problem of obtaining three-dimensional reconstruction of objects exhibiting different optical properties from their surface, by obtaining the reflectance information by a novel method. We propose a systematic strategy to compute the reflectance field. This procedure avoids the need of an analytical model of the reflectance information and proposes to use as few images as possible to compose it. In this way, a three-dimensional reconstruction of a real-world object can be obtained without an analytical reflectance model of its surface.

1.2 Problem statement

In order to recover real-world objects, reflectance field measurement technique will be used to define the optical properties of the objects as a multivariable function. These real-world objects exhibit different material properties from their surface. We propose a systematic strategy to compute the reflectance field, it avoids the need of an analytical model and proposes to use as few images as possible to compose the reflectance information. Besides, we propose to obtain the three-dimensional depth map of the object placed in a scene by adding the material properties information.

1.3 Research questions

- Is it possible to relate depth estimation with reflectance field information?
- Is it possible to improve the acquisition of reflectance field by decreasing the number of images needed to compose reflectance information?

1.4 General objective

To obtain a three-dimensional reconstruction of real-world objects by adding their material properties. Material properties are efficiently acquired by decreasing the number of images to compose the reflectance information.

1.5 Particular objectives

- To improve the acquisition of reflectance field by decreasing the number of images needed to compose it.
• To relate depth estimation with reflectance field information for recovering the three-dimensional reconstruction of real-world objects. The material properties of these objects are represented as a multivariable function.

1.6 Methodology

We have identified two main issues to obtain a three-dimensional reconstruction of real-world objects by using reflectance fields: to acquire the reflectance field and to relate three-dimensional reconstruction with reflectance field information.

1.7 Reflectance field acquisition process

Capturing the reflectance field (measure) refers to project patterns towards a scene. The pattern is projected by lighting every single pixel of every light source (projector or array of projectors). Every point of light reflected from the scene is imaged by a sensor (camera or array of cameras). The set of images acquired will compose the reflectance field. It means, if we have a projector of resolution \(m \times n\) shining light onto a scene and a camera of resolution \(p \times q\) capturing the reflected light, we obtain \(m \times n\) images (the projector resolution).

According to [37], the minimum assembly setup to acquire the reflectance field is a camera-projector pair. We propose to improve the capturing process such that the number of images will be decreased. To improve this acquisition process we need to carry out an analysis of the reflectance field, that is, to define the real number of images needed to compute the reflectance information. We explore a method that uses independent component analysis, it will be explained in the proposed method section.

The images which define the reflectance field information compose a matrix extremely sparse (almost all the elements are zero), a simple compression of only storing those matrix elements above a certain threshold allowed us to store this data in less space. Since the camera-projector pair can be considered as a stereo system, a calibration method can be applied. The calibration method comprises two steps: geometric calibration and radiometric calibration. The geometric calibration obtains the extrinsic and intrinsic parameters of the system, such as optical centers, focal distances, rotation and translation matrices. The radiometric calibration obtains the radiometric parameters to ensure that every pixel of the projector illuminates with the same intensity onto the scene. The calibration will be performed with a standard calibration software [3] by introducing the relation between real coordinates, image coordinates and intensities of every pixel. The scene can be composed for objects with different material properties.

**Evaluation:** The authors of the computer graphics area, specifically the authors that image the reflectance field do not evaluate the error for capturing this reflectance information. They only report the number of images and time required to obtain a
CHAPTER 1. INTRODUCTION

complete reflectance field [12, 37, 25]. So, we propose to obtain the reflectance field by illuminating pixel by pixel onto the scene in order to generate ground truth data. After that, to obtain the reflectance field by illuminating the scene with independent component analysis method. The RMS error is computed by comparing both methods. Also, we can evaluate our method in terms of time, size and number of images needed to compose the reflectance field with related work [12, 37, 25].

1.8 3D reconstruction and reflectance fields relation

To relate tree-dimensional reconstruction equations and reflectance fields we start our analysis with the reflectance map equation [43, 42]. From this equation, the normal field of the object can be obtained. However, this equation makes the assumption that is not necessary to obtain the reflectance information and it is eliminated by using reciprocal images. It means that camera and light source are co-located and they are switched. We propose to use the reflectance map equation but including the reflectance field information acquired. Extension of this equation is presented in the proposed reconstruction method and depth maps recovered are presented in results chapter.

Evaluation: Once the 3D model is generated, its dimensions are compared with dimensions of the object recovered and the RMS error is calculated. The RMS error will be compared with other authors of reconstruction methods [19, 20, 38]. It has to be noticed that for almost all the real-world objects is difficult to know their dimensions. So we propose to use those whose dimensions can be directly obtained (plane, triangle and sphere).

1.9 Outline of the thesis

The organization of this thesis is as follows: in Chapter 1 we give a brief explanation about the importance of the work in computer vision and computer graphics and how the thesis provides a solution in the fast acquisition of the reflectance field and the three-dimensional reconstruction recovery. In Chapter 2 we give the background of the field of 3D computer vision and computer graphics methods, and related work in these lines of investigation is covered. In Chapter 3 related theory needed to understand the concepts and terms handled in this thesis are given. Chapter 4 describes the proposed reflectance field acquisition, our technique to decrease the number of images. This chapter also explains the proposed three-dimensional reconstruction method using the reflectance field. In Chapter 5 we present results of the reflectance field acquisition where, the number of images that compose the reflectance data is decreased. Also, we present results of the three-dimensional reconstruction using reflectance field data where, real objects are recovered. Finally, in Chapter 6 the conclusions and future work are presented.
Chapter 2

Background

Our research proposes to relate techniques from two areas: computer vision and computer graphics. In computer vision area we explore different methods to obtain depth information of the objects and their limitations from observations of a scene with a camera, cameras or points of view. In computer graphics we will explore the methods to obtain reflectance data, that is, the optical properties of the objects surface, and their limitations.

2.1 Computer Vision

In computer vision there are two main techniques for 3D recovery (See Figure 2.1): passive and active methods. Passive methods use only the information provided by the image obtained with a camera or array of cameras, under ambient illumination conditions, while active methods use and manipulate information provided by the illumination on the scene.

2.1.1 Passive methods

In passive vision techniques, no energy is emitted for sensing purpose, it is only received. Such passive techniques include stereo vision and shape from X methods [36]. Stereo vision techniques refer to the process of constructing 3D information from at least two slightly different 2D images. Thus, when a point is imaged from two different viewpoints, its image projection undergoes a displacement from its position in the first and second image. The difference of that displacement is known as disparity and, a collection of disparity values for all projected points is known as disparity map that is proportional to the depth up to a constant factor (focal distance).

Passive methods propose two analysis, Static and Dynamic Stereo Analysis: in static stereo analysis no object movements, changes in light or in camera parameters occur in the time interval of taking the images. For a dynamic stereo analysis such changes or motions are possible during a time interval [4, 2, 33].
Reconstruction Methods

Passive methods

Active methods

Stereo Shape from X Structured lighting Photometric

Static Dynamic Shading, Texture, Motion, Focus

Figure 2.1: Reconstruction methods classification. Our research can be classified as an active method, because it adopts light scene manipulation.

On the other hand, shape-from-X methods have been developed for extracting shape information from intensity images. Many of these methods estimate local surface orientation rather than absolute depth at each point. If the actual depth to at least one point on each object is known, then the depth at other positions on the same object can be computed by integrating the local surface orientation. Shape from shading, shape from texture, shape from motion, shape from focus are some techniques included in the shape-from-X approach [33, 41, 11, 10]. All of these passive techniques add some assumptions and heuristics for the scene reconstruction. Some examples include:

1. Ideal reflectance: the surface in the scene are often assumed to satisfy ideal reflectance models. For instance, stereo and shape-from-X techniques generally assume a perfect Lambertian (isotropic) reflection mode. Consequently these techniques have poor performance in the presence of specularities and other deviations from the model.

2. Smoothness: imposing smoothness (the surface is free from irregularities or roughness), or choosing the reconstruction that is smoothest yields some pitfalls, for example the tendency to smooth over sharp edges or miss thin structures in the scene.

3. Ideal projection: simplified projection models like orthographic and ideal pinhole projection are used to make the reconstruction equations more tractable. Consequently, techniques that use these approximations pay a penalty in terms of accuracy and are not well-suited for applications that demand high-accuracy surface measurements.
2.1 COMPUTER VISION

2.1.2 Active methods

In active vision techniques, the illumination is controlled to acquire data that would facilitate the scene interpretation task, for example projecting light into the scene. Two methods can be identified in the active approach: structured lighting and photometric methods.

Structured light techniques refers to the systems in which the scene is illuminated by a known geometrical pattern. The projected pattern on the surface of the objects in the scene is imaged by a camera which is spatially displaced with respect to the source of illumination. The observed image of the patterns contain spatial information that can be used to recover the three-dimensional structure of the objects [5].

In the photometric methods, three images of the same scene are obtained using light sources from three different directions. Both camera and objects in the scene are required to be stationary during the acquisition of the three images. By knowing the surface reflectance properties of the objects in the scene, the local surface orientation at points illuminated by all three light source can be computed [1, 27, 26].

The active methods generally produce more accurate reconstructions than using passive techniques. Some problems of passive vision are simplified, such as, the feature finding by using patterns of light as cues or the overcoming of the correspondence problem. However, active techniques also include some assumptions for the scene recovering process, for example:

1. Specific reflectance model: reflectance properties of the surface in the scene are often assumed to satisfy a specific reflectance model. For instance, structured lighting generally assumes a perfect Lambertian (isotropic) reflection model. The photometric method assumes a Lambertian, Specular or Lambertian-Specular combinations reflectance models. Consequently these techniques perform poorly results when the reflectance properties of the surface have deviations from these models.

2. No ambiguity: The structured lighting method assumes no ambiguities in matching a single stripe, however if multiple stripes of light are projected to acquire the depth map, there would be potential ambiguities in matching stripe segments resulting from object surfaces at different depths.

Figure 2.1 shows the classification of the reconstruction methods. Our research can be located in the active methods line, because it adopts a light scene manipulation. In the last paragraphs some disadvantages of both approaches (active and passive) are exposed. This research tries to overcome some of those limitations. In the next section, a definition of the reflectance field description in the context of computer graphics area is described.
2.2 Computer Graphics

Synthesizing realistic looking images is an important problem in computer graphics. In order to achieve photorealism, the traditional graphics pipeline works by trying to simulate real-world physics accurately. The typical input to such a pipeline is a scene consisting of object shapes, material properties and a set of lights. This means that, the descriptions of object shapes have to be provided (3D model) and material properties have to be modeled. Once available, this representation can be used to render realistic images of the scene from any viewpoint under arbitrary lighting.

2.2.1 Reflectance fields

To obtain the reflectance field for describing the surface properties of the object in the scene is one of the central problems in computer graphics. The material properties are hard to model, as in the case of translucent materials. The surface properties of a scene can be described by a 8D function called the reflectance field which was introduced by Devebec et al [9]. The reflectance field describes the transport of light between the light incident on an object and the light exiting from it. Once available, this representation can be used to obtain realistic images for rendering purposes. The resulting images capture all global illumination effects such as diffuse inter-reflections, shadows, caustics and sub-surface scattering, without the need for an explicit physical simulation.

The optical properties of an opaque homogeneous surface can be characterized by reflectance as a function of incident light direction (two angles) and reflected light direction (two angles). The resulting 4D function is called the bidirectional reflectance distribution function (BRDF) [28]. If the surface is textured rather than homogeneous, then its optical properties depend on position on the surface (two spatial coordinates) as well as direction, leading to a 6D function called the spatially varying bidirectional reflectance distribution function (SBRDF). Dana et al [7] also call it the bidirectional texture function (BTF). Finally, if the object exhibits subsurface scattering, as does marble or human skin, then its reflectance properties depend on the outgoing as well as incoming position, adding two more spatial coordinates. The resulting function is 8D, and is called the bidirectional surface scattering distribution function (BSSRDF). Two ways have been considered in this area for obtaining the reflectance field: analytical description of the reflectance functions and the measurement of the reflection information.

**Analytical model of reflectance:** The analytic descriptions of these functions of some materials such as plastics and metals are known and tractable to compute. In other cases, analytic models are known, but these include free parameters that can be measured from the physical samples of the material. Recent examples of thus are marble [18], human hair [23], wood [24] and smoke [14]. However, some materials are hard to describe using analytical models, for example human skin. Others like woven cloth, are combinations of various subspecies.
2.2. COMPUTER GRAPHICS

Measurement of reflectance: In order to obtain materials hard to describe using analytical models, the bulk optical properties of a physical sample of the material is measured, that is, the reflectance field is computed by capturing a set of images. The most representatives examples are when two-dimensional reflectance fields are acquired by a single moving camera [21] or an array of cameras [40]. These configurations allows to virtually fly around a scene but the illumination cannot be changed. If the viewpoint is fixed and the illumination is provided by a set of point light sources, a 4D reflectance field is obtained. Various researches [44, 6, 9, 22, 34, 15] have acquired such data sets where a weighted sum of the captured images can be combined to obtain relit images from a fixed viewpoint only. However, since point light sources radiate light in all directions, it is impossible to cast sharp shadows onto the scene with this technique. If the illumination is provided by an array of video projectors, and the scene is captured as illuminated by each pixel of each projector, but still as seen from a single viewpoint, one can render views under lighting that includes local light source and shadows, it is a 6D reflectance field is defined. Masselus et al [25] capture such data sets using a single moving projector. At each position of the projector, the scene is captured under a set of illumination basis functions rather than one pixel at a time, to increase speed. Nevertheless, the necessity to move the projector limits the resolution of data that can be captured using such a system. The system described in [12] obtains a 6D function, it has no moving parts, it uses only one projector-camera pair, a beam-splitter and an array of mirrors, but the reflectance field is computed after capturing a huge set of images (thousands of images) to create a high-dimensional lookup table, and this reflection information is only available from some specific views and illumination conditions (where cameras and projectors are physically placed). This means that an image definition from a new and different viewpoint or under other illumination, can not be obtained. Figure 2.2 shows a classification of the reflectance field definition techniques. Our research proposes to obtain
the reflectance field by improving the acquisition, that is, decreasing the number of necessary images to define the reflectance properties of the object material.

2.3 Discussion: several positions of our research in the field

When an image of an object placed in a scene is captured, it is encoded into a set of gray values. These gray values are the result of interactions between light sources and surface materials of the object. Then, all three-dimensional reconstruction methods based on images are influenced by light and material properties of the objects. Estimation of the reflectance properties material of the object is an important issue. Passive methods assume an ideal or known reflectance, so they are no interested about how light can be affecting the object surface leading to incorrect depths estimations. In active methods exist a manipulation of the light sources to facilitate the scene interpretation task, however, in these methods it is assumed a specific behavior of light in contact with the surface of the object (lambertian, specular), this assumption is violated by most real-life objects leading, in the same way, to incorrect recovery calculations. Some works in the computer graphics area are focused in the definition of the interactions between the surface material of the objects and the light sources for rendering purposes, that is, to generate a 2D image from a virtual 3D scene. According to previous works, an analytical model for reflectance fields is difficult to describe. The measurement of the material properties is the method to obtain the reflectance information. However, the acquisition of reflectance field requires capturing a huge set of images. Our research proposes to explore the following aspects:

- To relate depth estimation with the reflectance field information.
- To improve the capturing method for the reflectance field definition by decreasing the number of images needed to compose it.

Figure 2.3 presents where our research in computer vision and computer graphics areas is located. Our research can be classified as an active reconstruction method using the reflectance field measurement technique.

2.4 Related work

In computer vision, there is no reconstruction method that considers the reflectance information. Instead, the methods consider that surface reflectance properties of the objects to be recovered are static. Then, a stereo constraint is introduced by eliminating the BRDF from the equations. In our research, we propose to measure the reflectance data by using the technique of reflectance field measurement, this measurement is not a reconstruction method, but it is used to define the material properties of the objects in computer graphics area for rendering purposes.
2.4. RELATED WORK

Figure 2.3: Location of our research in computer vision and computer graphics areas. Our research can be classified as an active reconstruction method using a 4D reflectance field measurement technique.

2.4.1 Light transport constancy

Light transport constancy is an active method that introduces an invariant for stereo matching which allows to obtain the depth of surfaces. This method is based on the observation that surface reflectance properties are usually static. That is, for a static scene, the percentage of light reflected by a particular surface patch remains constant for a given viewing direction (see Figure 2.4). This constraint allows stereo correspondence to be correctly determined for surfaces with an arbitrary BRDF and does not require calibrated light sources or objects in the scene. However, this method has the disadvantages as other matching techniques based on correspondence and eliminates the reflectance information [8].

2.4.2 Helmholtz reciprocity

Helmholtz reciprocity is an active method that explores the idea that the flow of light can be effectively reversed without altering its transport properties. Zickler et al [43, 42] used reciprocity to reconstruct the geometry of surfaces with arbitrary BRDFs in what they call Helmholtz stereopsis. The authors observed that by interchanging light source and camera position during acquisition, they can use Helmholtz reciprocity to guarantee that points on the surface would have exactly the same transfer characteristic in both configurations. This simplifies stereo matching, even for surfaces with complex BRDFs. In Figure 2.5 the Helmholtz reciprocity geometry is shown. A similar approach was taken by Tu et al [39], who utilized reciprocity for the task of 3D to 2D registration. However, this method needs to have an accurate location of the light source and the camera for the interchanging.
Figure 2.4: A point $x_1$ is illuminated by a light source $L$, the cameras $C_1$ and $C_2$ capture the scene. The light source $L$ is moved, $C_1$ and $C_2$ capture the same scene with different illumination [8].

Figure 2.5: Helmholtz reciprocity geometry. In the left half of this figure, the light source placed in $o_l$ illuminates the object, such object is captured by a camera placed in $o_r$. In the right half of the figure, the light and camera are switched.
2.4.3 Reflectance fields measurement

In [12, 37], the authors capture a set of images to compose the 4D reflectance field. This work uses only a camera-projector pair to obtain the reflectance information and uses a technique known as Dual Photography [37] to exploit the reflectance field reciprocity property. This work also explores the Symmetric Photography technique [12], here the 6D reflectance field is modeled as a transport matrix between the incident light field and the exiting light field. However, these techniques are not a three-dimensional reconstruction method, it is used to obtain the material properties of the object placed in the scene for rendering purposes. The reflectance field is defined only for specific viewpoints (cameras) and under specific illumination conditions (projectors) and a huge set of images (thousands of images) are acquired to compose the reflectance information.

2.5 Discussion

The reflectance field measurement is a technique in the area of computer graphics, to obtain the optical properties of the objects, it is not a three-dimensional reconstruction method. This technique is mentioned as related work because our work will use it to define the material properties of objects which reflectance model does not exist. The idea is that, the real-world objects can be recovered by introducing the reflectance information. However, the reflectance field is composed by thousands of images, depending on the material properties of the object and the resolution of the camera or array of cameras. On the other hand, all reconstruction methods (active and passive) do not consider the reflectance function. Then many real-world objects can have problems or limitations for the 3D recovery. Figure 2.6 presents the

<table>
<thead>
<tr>
<th>Area</th>
<th>BRDF</th>
<th>BTF</th>
<th>BSSRDF</th>
<th>Depth data</th>
<th>Assembly</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helmholtz reciprocity</td>
<td>eliminated</td>
<td>no</td>
<td>no</td>
<td>Normal calculation Reconstruction</td>
<td>2 Camera-light pair co-located</td>
</tr>
<tr>
<td>Light Transport Constancy</td>
<td>eliminated</td>
<td>no</td>
<td>no</td>
<td>Triangulation</td>
<td>Camera-light with intensity variation</td>
</tr>
<tr>
<td>Dual photography</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>Camera-projector pair</td>
</tr>
<tr>
<td>Symmetric photography</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>Camera-projector pair, 4x4 mirrors beam-splitter</td>
</tr>
<tr>
<td>Our method</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>Normal calculation Reconstruction</td>
<td>Camera-projector pair</td>
</tr>
</tbody>
</table>

Figure 2.6: Comparison of related work and our research.
comparison of the related work and our research.
Chapter 3

Reflectance fields and light transport matrix

In this chapter, we formally define the reflectance field and the light transport matrix. We also describe important properties of the transport matrix: symmetry and duality.

3.1 Photometry: terms and quantities

All methods described in this work are influenced by lighting and by reflection characteristics of the observed object. Therefore, in this section we give an introduction to the terms and quantities used in the field of photometric area and the necessary fundamental equations to understand the properties of illumination and the object materials. In Figure 3.1 we give the most common quantities in radiometric and photometric equivalence (for a more deep description of the quantities we refer to the reader at the book by R. Klette [30]).

<table>
<thead>
<tr>
<th>Radiometric quantity</th>
<th>Photometric quantity</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>radian energy</td>
<td>luminous energy</td>
<td>$Q$</td>
</tr>
<tr>
<td>radian power</td>
<td>luminous power</td>
<td>$\Phi$</td>
</tr>
<tr>
<td>radian flux, irradiance</td>
<td>luminous flux, illuminance</td>
<td>$E$</td>
</tr>
<tr>
<td>radiant emittance</td>
<td>luminous emittance</td>
<td>$M$</td>
</tr>
<tr>
<td>radian exitance</td>
<td>luminous exitance</td>
<td>$I$</td>
</tr>
<tr>
<td>radian intensity</td>
<td>luminous intensity</td>
<td>$L$</td>
</tr>
</tbody>
</table>

Figure 3.1: Radiometric and photometric quantities.
3.2 Reflection models

The amount of light encoded into the gray value of a particular pixel of a digital image can be seen as the result of interactions between surface materials and light sources. Some quantities in radiometric and photometric are shown in Figure 3.1.

Formally, the radian power $\Phi$ is the radian energy $Q$ per unit time, so the radian power is given by the formula:

$$\Phi = \frac{dQ}{dt} \quad (3.1)$$

The radian emittance and irradiance is defined in terms of the emittance area $A_1$ and received area $A_2$ given by:

$$M = \frac{d\Phi}{dA_1}, \quad E = \frac{d\Phi}{dA_2} \quad (3.2)$$

The irradiance is important for computer vision since it is the quantity measured by the imaging sensor and is encoded into a gray value for every pixel.

The function that model the behavior of reflectance over a surface is called a Reflectance Distribution Function (RDF). In computer vision as well as in computer graphics the Bidirectional Reflectance Distribution Function (BRDF) is used as a fundamental tool to describe reflection characteristics. The BRDF describes how bright the differential surface $dA$ of a material appears when it is observed from a general direction and illuminated from a particular direction.

**Definition 1.** We define the BRDF as the ratio between the reflected differential radiance $dL_1$ at the viewer direction and the differential irradiance $dE_2$ coming from the illumination direction:

$$fr(\theta_2, \phi_2; \theta_1, \phi_1) = \frac{dL(\theta_2, \phi_2; \theta_1, \phi_1; E_2)}{dE_2(\theta_2, \phi_2)}$$

the term direction should be interpreted as a differential solid angle in a direction given by spherical coordinates, $\theta$ denotes the slant angle and $\phi$ stands for the tilt angle.

Figure 3.2 shows a general representation of the BRDF.

3.3 Lambertian surfaces

A perfectly diffuse reflecting surface appears equally bright when observed from any direction. If a surface emits the entire incoming energy through reflection then is called a Lambertian Surface and neither absorption nor transmission of radiation takes place. It follows that the entire radiance $L_1$ which is reflected over the visible hemisphere is equal to the incoming irradiance $E_2$. A lambertian surface has three properties:
3.4 Reflectance maps

We can define a Reflectance map as a continuous or discrete function such that

\[ \mathcal{R} : \mathbb{R}^2 \rightarrow \mathbb{R} \]  

(3.3)

Usually the gradient space representing the surface gradients \((p, q)\) is chosen for reflectance maps, i.e., \(\mathcal{R}(p, q)\). For instance we can consider linear reflectance maps as follows:

\[ \mathcal{R}(p, q) = E_0 \cdot \rho \frac{p \cdot p + q \cdot q + 1}{\| (p_s, q_s, -1) \|} \]  

(3.4)

where \(s\) is represented by its illumination gradient \((p_s, q_s)\), \(E_0\) is the irradiance of the light source, \(\rho\) is the albedo which describes the relative portion of the radiation which is reflected by the surface.

1. The reflected radiance \(L_1\) does not depend on the direction (isotropic) and is constant, i.e. \(L_1(\theta_1, \psi_1) = \text{constant}\).

2. The BRDF is constant, i.e. \(fr(\theta_2, \phi_2; \theta_1, \phi_1) = \text{constant}\).

3. The radiant emittance \(M\) is equal to the irradiance \(E_2\).
A particular reflectance map is the lambertian reflectance map [29] given by

\[ R(p, q) = E_0 \cdot \rho \left( \frac{1}{\| (p, q, -1) \|} \right) \]  

This model has the following properties: the maximal radiance of the reflectance map occurs for gradient \((p, q) = (0, 0)\), all gradients at distance \(r = \| (p, q) \|\) to the origin has the same radiance value, hence the reflectance map is rotationally symmetric.

### 3.5 Reflectance field and light transport matrix

The light fields are used to describe the radiance at each point \(x\) and in each direction \(\omega\) in a scene. Ignoring wavelength and fixing time, this is a 5D function which we denote by \(E(x, \omega)\). Thus, \(E(x, \omega)\) represents the radiance leaving a point \(x\) in direction \(\omega\) (a complete table of the mathematical terms used in this section is provided in Table 1).

Levoy and Hanrahan [21], observed that if the viewer is moving within the unoccluded space, then the 5D representation of the light field can be reduced to 4D. We can characterize this function as \(L(\psi)\) where \(\psi\) specifies a point and an incoming direction on a sphere [9]. A 4D light field can be used to generate an image from any viewing position and direction, but it will always show the scene under the same lighting. In general, each field of incident illumination on a scene will induce a different field of exiting illumination from the scene. Debevec et al [9] showed that the exiting light field from the scene under every possible incident field of illumination can be represented as an 8D function called the reflectance field:

\[ R(L_i(\psi_i); L_0(\psi_0)) = R(\psi_i; \psi_0) \]

Here, \(L_i(\psi_i)\) represents the incident light field on the scene, and \(L_0(\psi_0)\) represents the exiting light field reflected off the scene.

We have so far described the light field concepts using continuous functions, but for actual measurements, we work with discrete forms of these functions. In order to do so, let us parameterize the domain \(\psi\) of all incoming directions by an array indexed by \(i\). The outgoing direction corresponding to an incoming direction is also parameterized by the same index, \(i\). Now, consider emitting unit radiance along ray \(i\) towards the scene (e.g., using a projector). The resulting light field, which we denote by vector \(t_i\), captures the full transport of light in response to this impulse illumination. This is called the impulse response [13] or the impulse scatter function [35]. We can concatenate all the impulse responses into a matrix \(T\) which we call the light transport matrix:

\[ T = [t_1, t_2, \ldots, t_n] \]

Since light transport is linear, any outgoing light field represented by a vector \(L_0\) can be described as linear combination of the impulse responses, \(t_i\). Thus, for an incoming illumination described by vector \(L_i\), the outgoing light field can be expressed as:

\[ L_0 = TL_i \]
3.5. REFLECTANCE FIELD AND LIGHT TRANSPORT MATRIX

Table 3.1: Table of terms and variables

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>3D space of all points in a volume, domain of functions</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>2D space of all directions at a point, domain of functions</td>
</tr>
<tr>
<td>$x, x'$</td>
<td>two points in domain $V$, e.g., $(x, y, z), (x', y', z')$</td>
</tr>
<tr>
<td>$\omega, \omega'$</td>
<td>two points in domain $\Omega$, e.g., $(\theta, \phi), (\theta', \phi')$</td>
</tr>
<tr>
<td>$dx'$</td>
<td>a differential volume at $x'$, i.e., $dx'dy'dz'$</td>
</tr>
<tr>
<td>$d\omega'$</td>
<td>a differential direction at $\omega'$, i.e., $d\theta'd\phi'$</td>
</tr>
<tr>
<td>$\bar{L}(x, \omega)$</td>
<td>5D light field function on domains $V$ and $S$, radiance at $(x, \omega)$</td>
</tr>
<tr>
<td>$\bar{L}_i(x, \omega)$</td>
<td>5D incoming light field</td>
</tr>
<tr>
<td>$\bar{L}_0(x, \omega)$</td>
<td>5D outgoing light field</td>
</tr>
<tr>
<td>$K(x, \omega; x', \omega')$</td>
<td>the direct light transport from $(x', \omega')$ to $(x, \omega)$</td>
</tr>
<tr>
<td>$\bar{L}_i$</td>
<td>5D discrete incoming light field</td>
</tr>
<tr>
<td>$\bar{L}_0$</td>
<td>5D discrete outgoing light field</td>
</tr>
<tr>
<td>$K$</td>
<td>matrix of the direct light transport coefficients</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>4D space of all incoming directions on all points of a sphere, domain of functions</td>
</tr>
<tr>
<td>$\psi$</td>
<td>a point in domain $\Psi$, e.g., $(u, v, \theta, \phi)$</td>
</tr>
<tr>
<td>$\psi_i, \psi_0$</td>
<td>two points in domain $\Psi$, e.g., $(u_i, v_i, \theta_i, \phi_i), (u_0, v_0, \theta_0, \phi_0)$</td>
</tr>
<tr>
<td>$L(\psi)$</td>
<td>4D light field function on domain $\Psi$, radiance at a point $\psi$</td>
</tr>
<tr>
<td>$L_i(\psi_i)$</td>
<td>4D incoming light field</td>
</tr>
<tr>
<td>$L_0(\psi_0)$</td>
<td>4D outgoing light field</td>
</tr>
<tr>
<td>$R(\psi_i, \psi_0)$</td>
<td>8D reflectance field mapping $L_i(\psi_i)$ to $(L_0(\psi_0))$</td>
</tr>
<tr>
<td>$L_i$</td>
<td>4D discrete incoming light field</td>
</tr>
<tr>
<td>$L_0$</td>
<td>4D discrete outgoing light field</td>
</tr>
<tr>
<td>$t_i$</td>
<td>impulse response to unit illumination along a ray $i$</td>
</tr>
<tr>
<td>$\mathbf{T}$</td>
<td>matrix of the light transport coefficients for 4D representation, symmetric</td>
</tr>
<tr>
<td>$\hat{\mathbf{T}}$</td>
<td>a sub-block of $\mathbf{T}$, not necessarily symmetric</td>
</tr>
</tbody>
</table>
The light transport matrix $T$, is thus the discrete analog of the reflectance field $R(L_i(\psi_i); L_0(\psi_0))$.

![Diagram showing incoming and outgoing transport of light](image)

**Figure 3.3:** The symmetry of the radiance transfer between incoming and outgoing directions implies that the transport of light between a ray $i$ and a ray $j$ is equal in both directions as the diagram shows.

### 3.6 Symmetry of the transport matrix

The idea that the flow of light can be effectively reversed without altering its transport properties was proposed by von Helmholtz in his original treatise in 1856 [16]. He proposed the following reciprocity principle for beams traveling through an optical system (i.e., collections of mirrors, lenses, prisms, etc.):

*Suppose that a beam of light $A$ undergoes any number of reflections or refractions, eventually giving rise (among others) to a beam $B$ whose power is a fraction $f$ of beam $A$. Then on reversing the path of the light, an incident ray $\bar{B}$ will give rise to a beam*
A whose power is the same fraction of beam $\hat{B}$.

In other words, the path of a light beam is always reversible, and furthermore the relative power loss is the same for the propagation in both directions. For the purpose of a reflectance field generation, this reciprocity can be used to derive an equation describing the symmetry of the radiance transfer between incoming and outgoing directions $\psi_i$ and $\psi_0$:

$$R(\psi_i; \psi_0) = R(\psi_0; \psi_i)$$ (3.9)

where $R$ is the reflectance field. For the light transport matrix defined in Section 3.5, this implies that the transport of light between a ray $i$ and a ray $j$ is equal in both directions, i.e.

$$T[i, j] = T[j, i] \Rightarrow T = T^T$$ (3.10)

Therefore, $T$ is a symmetric matrix (see Figure 3.3).

Figure 3.4: Duality of the light transport matrix. Because $T = T^T$, therefore, if $\hat{T}$ describes the light transport from $\hat{p}$ to $\hat{c}$, then $\hat{T}^T$ describes the light transport from $c''$ to $p'$. 

### 3.7 Duality of the Transport Matrix

The symmetry property of the transport matrix holds when our acquisition system can measure the outgoing ray corresponding to each incoming ray. If the acquisition setup is such that the source of radiation (e.g. a projector) and the sensor (e.g. a camera) sample different subsets of incoming and outgoing rays respectively, then the transport matrix which describes this light transport is not symmetric. It turns out that the transport matrix exhibits an interesting duality property in this case.

We explain this with reference in Figure 3.3. We have a projector of resolution $p \times q$ shining light onto a scene and a camera of resolution $m \times n$ capturing the reflected light. Since the light transport is linear, we can express the light transport from the projector through the scene and into the camera with the following equation:

$$\hat{c} = \hat{T} \hat{p}$$ (3.11)
The column vector $\tilde{p}$ is the projected pattern (size $pq \times 1$), and $\tilde{c}$ (size $mn \times 1$) represents the image captured by the camera. Matrix $\mathbf{T}$ (size $mn \times pq$) is the transport matrix that describes how light from each pixel of $\tilde{p}$ arrives at each pixel of $\tilde{c}$.

We use the prime subscript ($'$) to indicate that we are working in the primal space to distinguish it from its dual counterpart, which we will introduce in a moment. Then, by using the principle of Helmholtz reciprocity as described in the previous section, we can represent the dual of equation 3.11 as follows:

$$p'' = \mathbf{T}^T \tilde{c}''$$

In this equation, the transport matrix $\mathbf{T}$ of the scene is the same as before except that we have now transposed it to represent the light going from the camera to the projector.

We shall refer to equation 3.11 as the primal equation and equation 3.12 as the dual equation. In the dual space, $p''$ represents the virtual image that would be visible at the projector if the camera were projecting pattern $\tilde{c}''$. This is described in Figure 3.4.

Thus, because of the duality, the $\mathbf{T}$ matrix can be acquired in either space and then transposed to represent the light transport in the other space. It is important to note that the two equations are not mathematical inverses of each other (i.e. $\mathbf{T}^T \mathbf{T} \neq \mathbf{I}$). This is because energy is lost in any real system through absorption or scattering. Therefore, if we measure $\tilde{c}$ after applying $\tilde{p}$, we cannot put this back in $\tilde{c}''$ and expect the resulting $p''$ to equal the original $\tilde{p}$.

### 3.8 Discussion

The estimation of reflectance properties of the objects requires the definition of reflectance field and light transport matrix. Light transport matrix can be used to efficiently represents reflectance data. Besides it provides an efficient representation for storage. Also we need to understand some important properties of the transport matrix as symmetry and duality to develop our method of acquisition and add reflectance information to our 3D reconstruction method.
Chapter 4

Proposed Method

The first contribution of this thesis is a method to efficiently acquire the reflectance field, by reducing the number of images required to describe the field. The second contribution is related to the 3D reconstruction by using the reflectance field, both methods are described below.

4.1 Proposed reflectance field acquisition

To obtain the reflectance field for describing the surface properties of an object in a scene is one of the central problems in computer graphics. However, the formulation of analytical models for complex surfaces is not always an easy task.

An alternative approach is to acquire the reflectance information from real-world surfaces. This acquisition is carried out by capturing with a camera or array of cameras a set of data that describes the transfer of energy between a light field of incoming rays (the illumination) and a light field of outgoing rays (the view). Such set of data is known as the Reflectance Field [9].

In order to obtain the reflectance field of a scene, thousands of images are acquired depending on the optical properties of the object placed in the scene and how much variation is permitted in the illumination and the viewer position [21, 8].

The traditional technique to acquire the reflectance field of an object consists in illuminating and capturing point by point the object placed in the scene using a video projector. In order to accelerate the acquisition, some algorithms are devoted to parallelize the capture. To illuminate multiple pixels at the same time, it is possible only with the assumption that each projected pixel affects a small and localized region of the scene. Even so, the number of images that composes the reflectance field is extremely large (thousands of images) [37, 25].

In this research we propose a systematic strategy that uses independent component analysis (ICA) to acquire the reflectance field. Our method takes advantage of the fact that the pixels illuminated affect local regions of the scene. We consider the transfer of energy between the incoming and outgoing light fields as signal mixtures in order to use an ICA approach to decompose the signal mixtures into statistically
independent signals. Our procedure avoids the need of any analytical model of the reflectance field, it reduces the images required to describe the field (up to 99%) and our strategy keeps the reflectance field quality very close to the traditional approach.

Independent component analysis (ICA) belongs to a class of blind source separation (BSS) methods for separating data into underlying informational components, where such data can take the form of images, sounds, telecommunication channels or stock market prices. The term "blind" is intended to imply that such method can separate data into source signals even if very little is known about the nature of those source signals [17]. Some application of ICA are the separation of artifacts in magnetoencephalography (MEG) data, finding hidden factors in financial data, reducing noise in natural images and telecommunications.

ICA is based on the assumption that if different signals are from different physical processes, then those signals are statistically independent. ICA takes advantage of the fact that the implication of this assumption can be reversed, leading to a new assumption: if statistically independent signals can be extracted from signal mixtures then these extracted signals must be from different physical processes. Accordingly, ICA separates signal mixtures into statistically independent signals. If the assumption of statistical independence is valid then each of the signals extracted by independent component analysis will have been generated by a different physical process, and will therefore be a desired signal.

4.1.1 Independent Component Analysis

Independent component analysis is a method for separating a multivariate signal into additive subcomponents supposing the mutual statistical independence of the source signals [17]. Assume that we observe \( n \) linear mixtures \( x_1, \ldots, x_n \) of \( n \) independent components

\[
x_i = a_{i1}s_1 + a_{i2}s_2 + \cdots + a_{in}s_n, \quad \text{for all } i.
\]

In the ICA model, it is assumed that each mixture \( x_i \) as well as each independent component \( s_k \) is a random variable. The observed values \( x_i \) are a sample of this random variable. It is convenient to use vector-matrix notation instead of the sums like in the previous equation. Let us denote by \( \mathbf{x} \) the random vector whose elements are the mixtures \( x_1, \ldots, x_n \) and likewise by \( \mathbf{s} \) the random vector with elements \( s_1, \ldots, s_n \). Let us denote by \( \mathbf{A} \) the matrix with elements \( a_{ij} \). Using the vector-matrix notation, the above mixing model is written as

\[
\mathbf{x} = \mathbf{A}\mathbf{s}
\]

The statistical model in equation 4.2 is called independent component analysis, or ICA model. The ICA model is a generative model, which means that it describes how the observed data are generated by a process of mixing the components \( s_i \). The independent components are latent variables, meaning that they cannot be directly observed. Also the mixing matrix is assumed to be unknown. All we observe is the random vector \( \mathbf{x} \), and both \( \mathbf{A} \) and \( \mathbf{s} \) have to be estimated by using such vector.
The starting point for ICA is the assumption that the component $s_i$ are statistically independent. Then, after estimating the matrix $A$, we can compute its inverse $W$, and obtain the independent component simply by

$$s = Wx$$  \hspace{1cm} (4.3)$$

ICA is very closely related to the method called blind source separation (BSS). A "source" means here an original signal. "Blind" means that the mixing matrix is unknown. Figure 4.1 shows the mixing (top) and unmixing (bottom) process.

4.1.2 ICA of the reflectance field

The idea to apply the method of ICA for acquiring the reflectance field is originated from an usage example. Imagine two or more people talking at the same time. Their speech are the sources. The mixtures of their speech are recorded by appropriate number of microphones. Now, the task is to find the original sources (independent components) with no prior information about the mixing information (see Figure 4.2).

Imagine now that the people talking at the same time are the projector pixels lighted up over the scene. The camera pixels represent the microphones. All the information of light (interaction between the rays of light and the scene) is mixed.
and it can be imaged by a camera. The task is to find the original sources that is, the amount of light resulting to illuminate the scene with every projector pixel independently.

To acquire the reflectance field by ICA method we have to consider the scene configuration in Figure 4.3. The scene is illuminated by a light source $L_i$. A particular point in the scene $x_i$ will reflect light to the camera $C$. According to equation 3.8 the outgoing light field represented by the vector $L_0$ is the reflected intensity in the direction of $C$ from the point $x_i$ and it can be considered as a signal mixture of the impulse responses, $T_i$.

According to the ICA model, these independent components $t_i$ cannot be directly observed. $L_i$ is the incident light intensity at point $x_i$. Considering equation 4.1, the observed values from the point $x_i$ are samples of $L_0$ and can be expressed as:

$$
L_0(x_1) = (L_0(x_1^1), L_0(x_1^2), \ldots, L_0(x_1^N))
$$
$$
L_0(x_2) = (L_0(x_2^1), L_0(x_2^2), \ldots, L_0(x_2^N))
$$
$$
\vdots
$$
$$
L_0(x_i) = (L_0(x_i^1), L_0(x_i^2), \ldots, L_0(x_i^N))
$$

(4.4)
4.1. PROPOSED REFLECTANCE FIELD ACQUISITION

Figure 4.3: The scene is illuminated parallel by a light source $L_i$. A particular point in the scene $x_i$ will reflect light to the camera $C$. The outgoing light field $L_0$ is the reflected intensity in the direction of $C$ from the point $x_i$.

where the superscripts specify the identity of the intensity level of the $L_0$ sample and the subscripts specify the identity of the reflectance field element.

Following the ICA model (see equation 4.3), we can calculate $T_e$ as an estimated of $T$ (light transport matrix), such as

$$T_e = WL_0$$ \hfill (4.5)

4.1.3 Evaluation

The estimation of the reflectance field with our method is evaluated by following the next steps.

1. The scene is illuminated pixel by pixel by the projector (brute-force-scan).
2. The reflectance field of the scene is composed for every value of intensity captured by the camera.

3. The scene is parallel illuminated by the projector.

4. The estimation of the transport matrix $T_e$ is computed by using our proposed method.

5. Brute-force-scan reflectance field and the reflectance field computed with our method is compared, that is, the RMS error is calculated.

The RMS error is computed as:

$$\text{RMS} = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{n}}$$  \hspace{1cm} (4.6)

where $\hat{y}_i$ represents every intensity that composes the reflectance field. This reflectance field was acquired by brute-force-scan method. The variable $y_i$ represents every intensity of the reflectance field acquired by our method. Squaring the subtractions, averaging the squares, and taking the square root give us the RMS error. The evaluation and the RMS calculation will be detailed in the experiments chapter.

### 4.1.4 Discussion

We have proposed a method to obtain the reflectance field of objects with an anisotropic BRDF (4D) from their surface. The method is intended to accelerate the acquisition of the reflectance information using independent component analysis approach. We proposed a method that considers the outgoing rays of the light field as independent signals. These independent signals are obtained from the decomposition of a set of signal mixtures. These signal mixtures are acquired by taking images of the scene when it is illuminated by a projector with all its pixels turned on and when the illumination undergoes amplitude variations. In Chapter 5 we demonstrate the ability to decrease the number of images for the reflectance field composition and the reflectance quality is maintained.

### 4.2 3D from reflectance field

Three-dimensional reconstruction is a discipline of computer vision dealing with the problem to obtain the object shape or the calculation of distances between the sensor, i.e. the camera, and objects in a scene. Three-dimensional reconstruction is interested in recovering real information from a particular scene. Most of the reconstruction methods use the information provided by the image, under ambient illumination conditions. Also, nearly all the methods rely on the assumption that the objects in the scene have Lambertian or known reflectance surface.
Because the different optical properties of the object surface placed in the scene, ambient illumination and the assumption that the object surface reflect the same amount of light uniformly over all the surface is not sufficient to model some important characteristic of the object as shape and depth.

An alternative approach to the specification of the reflectance or optical properties of the surface objects by analytical modeling is the capturing of the reflectance information from real-world surfaces. The acquisition is carried out, for capturing with a camera or array of cameras a set of data that describes the transfer of energy between a light field of incoming rays (the illumination) and a light field of outgoing rays (the view). Such set of data is known as the Reflectance Field [9].

Our method explores the problem of obtaining the three-dimensional reconstruction of objects exhibiting an anisotropic BRDF (the objects material have the property that their reflection characteristics vary to rotations of the surface about its normal) by using a 4D slice of the 8D reflectance field. The 4D slice of the reflectance field is obtained by a camera-projector pair. Our method exploits the property of reciprocity of the reflectance field to impose the epipolar constraint by considering the camera-projector pair as a stereo system. In Chapter 5, we show how our method can be used to recover objects with an anisotropic BRDF of their surface. This procedure avoids the need of an analytical model of the reflectance data.

### 4.2.1 BRDF

The Bidirectional Reflectance Distribution Function (BRDF) is a projection of the 8D reflectance field into a lower dimension. From equation 3.9, the 4D reflectance field can be represented as

$$f_r(L_i(\Omega_1); L_0(\Omega_2)) = f_r(\Omega_1; \Omega_2)$$

where $L_i(\Omega_1)$ represents the incident light field on the scene, and $L_0(\Omega_2)$ represents the exiting light field reflected off the scene and $\Omega_1, \Omega_2$ are incoming and outgoing directions, e.g., $(\theta_1, \phi_1), (\theta_2, \phi_2)$. The BRDF describes how bright the differential surface $dA$ of a material appears when it is observed from a certain direction and illuminated from a certain direction. The reciprocity exposed in Section 3.6 for the 4D reflectance field can be written as

$$f_r(\Omega_1; \Omega_2) = f_r(\Omega_2; \Omega_1)$$

Some materials have the property that their reflection characteristics are invariant to rotations of the surface about its normal. Such materials are called isotropic. Materials not having this characteristic are called anisotropic. The BRDF covers anisotropic reflection.

As equation 3.10 shows, to discretize equation 4.8, all incoming and outgoing directions in domain $\Omega$ can be parameterized by an array indexed by $i$. We denote the resulting 4D light field by vector $\hat{t}_i$ and this 4D light field is concatenated as:

$$\hat{T} = [t'_1 t'_2 \ldots t'_n]$$
For each incoming illumination described by vector $L'_i$, the outgoing light field can be expressed as

$$L'_0 = \hat{T}L'_i \quad (4.10)$$

Matrix $\hat{T}$ is the discrete analog of the 4D reflectance field. The reciprocity exposed in Section 3.6 implies that the transport of light between a ray $i$ and a ray $j$ is equal in both directions, i.e.

$$\hat{T}[i,j] = \hat{T}[j,i] \implies \hat{T} = \hat{T}^T \quad (4.11)$$

### 4.2.2 Depth recovery from the 4D reflectance field

Consider the scene configuration in Figure 4.4 (left). The scene is illuminated by a light source $L'_i$. A particular point in the scene $p$ will reflect light to the camera $C$ according to equation 4.10, the outgoing light field represented by vector $L'_0$ is the reflected intensity in the C direction from point $p$. Let $o_1$ and $o_2$ denote the positions of the light source and camera, respectively. The unit vectors $\Omega_1 = \frac{1}{|o_1 - p|}(o_1 - p)$ and $\Omega_2 = \frac{1}{|o_2 - p|}(o_2 - p)$ denote directions from $p$ to the light source and camera, respectively. Given this configuration, the image irradiance (see [30, 43, 42]) at the projection of $p$ is

$$e = f_r(\Omega_2, \Omega_1) \frac{\hat{n} \cdot \Omega_2}{|o_2 - p|^2} \quad (4.12)$$

where $f_r$ is the BRDF (4D function).
4.2. 3D FROM REFLECTANCE FIELD

In the above equation is assumed that every ray light $\Omega_2$ from the light source illuminates the scene and the number of rays reflected is just one, it can be considered true for those objects with 4D material properties. Now, we add the transport matrix $\hat{T}$ to the equation 4.12, so we have,

$$e = \hat{T}(p) \frac{\hat{n} \cdot \Omega_2}{|o_2 - p|^2}$$

(4.13)

where $\hat{T}(p)$ is the 4D transport matrix that corresponds to a point $p$ of the scene, $\hat{n}$ can be expressed as $(\frac{dz}{dx}, \frac{dz}{dy}, -1)$, the ray from the camera can be expressed as $\Omega_2(p) = (\Omega_{2x}, \Omega_{2y}, \Omega_{2z})$.

Figure 4.5: Block diagram of the reconstruction process of our method. The final step of the diagram is the recovery of the depth information.

Taking advantage of the symmetry of the transport matrix we can impose the epipolar constraint to provide a solution to equation 4.13. Consider the scene configuration in Figure 4.4 (right). The scene is illuminated by a camera $L_0''$. A particular point in the scene $p$ will reflect light and it is captured by a light source. The vector $\Omega_2(p)$ and the denominator $|o_2 - p|^2$ can be determined when the setup is calibrated [42]. Imposing the epipolar constraint we can express the normal $\hat{n}$ as $(\frac{dz}{dx}, 0, -1)$. The point $p(x, y, z)$ will have projections in the camera and the light
source (considered as a second camera) established by calibration parameters of the system. Expressing the depth as \( z(x, y) \), we rewrite equation 4.13 as

\[
\frac{dz}{dx} = \frac{e|o_r - p|^2\hat{T}(p) + \Omega z}{\hat{T}(p)\Omega x}
\] (4.14)

This can be numerically integrated as

\[
z(x, y) = \int_{x_0}^{x} \frac{dz}{dx} dx + z(x_0, y)
\] (4.15)

For each epipolar line \( y \), this integral provides the depth across the epipolar line. We can determine for each epipolar line \( y \) the \( z(x_0, y) \), since the point \( p(x, y, z) \) have projections in the camera and we know the corresponding projections to the light source when the 4D transport matrix is acquired.

Figure 4.5 shows the block diagram of the reconstruction process of our method.

### 4.2.3 Evaluation

The estimation of three-dimensional reconstruction for real objects is obtained with our method and it is evaluated by following the next steps.

1. By using the reflectance field and reconstruction equations, the 3D model of the scene is generated.
2. Dimensions of the objects are known.
3. Dimensions of the object recovered with our method is calculated.
4. Both dimensions are compared, that is, the RMS error is calculated.

The RMS error is computed as:

\[
\text{RMS} = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{n}}
\] (4.16)

where \( \hat{y}_i \) represents every depth data that composes the 3D model. This 3D model is generated by using the dimension of the object. The variable \( y_i \) represents every depth data of the 3D model generated by our method. Squaring the subtractions, averaging the squares, and taking the square root give us the RMS error. The evaluation and the RMS calculation will be detailed in the experiments chapter.
4.2.4 Discussion

The estimation of reflectance properties of the object is important for a correct 3D measurement. In computer graphics, the material properties of such object materials are measured and they are described as a dimensional reflectance functions (8D function). Theory shows how to obtain the three-dimensional reconstruction of objects exhibiting an anisotropic BRDF by integrating a 4D slice of the 8D reflectance field information by using a camera-projector pair and it will be validated in the experiments section.
Chapter 5

Experiments and Results

In this chapter we present results of the reflectance field acquisition where, the number of images that compose the reflectance data is decreased. Also, we present some results of the three-dimensional reconstruction using the reflectance field data where, real objects are recovered.

5.1 System setup

The methods presented in this thesis require capturing images under patterned illumination from a projector. Therefore, we need a capture setup composed by a projector and a camera. It is desirable to use the most simple and non expensive assembly setup, it is the case of the assembly studied by [37]. This system improves another systems [44, 6, 9, 22, 34, 15] because it is the first that obtains the reflectance field with only a camera and a digital projector.

We use a camera-projector pair which is computer controlled and we use the system in a dark room without any interference. The scene is fixed and is placed in the field of view of the camera and projector as Figure 5.1 shows. The distance between camera and projector is about 0.3m and the distance from the system setup to the scene is about 0.5m. Figure 5.2 shows the configuration setup. The setup is composed by a Samsung digital projector with a resolution of 640x480 pixels, and a Canon PowerShot-G5 camera with a resolution of 640x480 in B/W.

5.2 Reflectance field acquisition

The acquisition process consists in obtaining the reflectance field. In order to accelerate the acquisition we would like to parallelize the patterns, illuminating multiple pixels at the same time. This is possible only if we make the assumption that each projector pixel affects a small, localized region of the scene from the point of view of the camera.

The acquisition method (ICA-capture) was applied to several objects (plane, triangle and sphere) with different material properties from their surface such as:
matte, wood, woven and marble (see Figure 5.3). Using these objects we are including all the range of material properties proposed in related work: opaque homogeneous (matte) [28], textured (woven) [7] and subsurface scattering surface (wood and marble) [21].

In related work [12, 37, 25], the authors do not measure the quality of the reflectance field acquired, they show the image synthesized by using such reflectance information and give relevant data as size, time required and number of images. In our work, besides showing the image and obtaining data as size, time and number of images, we define the reflectance field quality acquired by comparing it to ground truth reflectance data. For the creation of ground truth reflectance, we use brute-force scan technique [37]. Brute-force scan technique is the traditional form to acquire the reflectance field of an object placed in the scene. In the next section, brute-force scan technique is described.

Figure 5.1: Camera and projector compose the capture setup. The scene is placed in the field of view of the system setup.
5.2. REFLECTANCE FIELD ACQUISITION

5.2.1 Brute-force scan

Acquiring reflectance field with brute-force scan technique refers to project patterns towards a scene. The pattern is projected by lighting every single pixel of the light source (projector). Every point of light reflected from the scene is imaged by a sensor (camera). The set of images captured will compose the reflectance field.

The reflectance field by illuminating pixel by pixel (brute-force scan) onto the scene is acquired as in [37].

The projector of the system setup with resolution \( p \times q \) shine light onto a scene. Every point of light reflected from the scene is imaged by the camera of resolution \( m \times n \). The number of images captured by the camera is: \( p \times q \) images.

The images are stored in a transport matrix \( T \) (see equation (4.9)) of size \( mn \times pq \). However, the matrix acquired is extremely sparse, a simple compression of only storing the \( T \) matrix elements above a certain threshold allowed us to store this data in a matrix of size \( p \times q \) approximately. This matrix size depends on the kind of object to be recovered. Nevertheless, the number of images acquired is still large.

Given a reflectance field acquired using a camera and projector setup, the reflectance field that describes the transport between the camera and a projector can be written as the transport matrix \( T \) (see equation (4.9)). Let \( L_0 \) denote the camera image that is observed when projecting the illumination pattern \( L_i \). Every pixel projected of \( L_i \) onto the scene has associated projector coordinates \((x_p, y_p)\). The pixel reflected from the scene has associated a set of camera coordinates \((x_c, y_c)\).

In order to show that the reflectance field has been acquired adequately we have
Figure 5.3: ICA-capture was applied to several objects (plane, triangle and sphere) with different material properties from their surface: matte, wood, woven and marble.
Figure 5.4: The top half shows the system setup for reflectance field capturing. The bottom half shows the virtual configuration for implementing the Dual Photography technique.
Figure 5.5: Some examples of the images captured that compose the reflectance field of the scene. It was captured with the camera-projector assembly, the distance between the system setup and the scene is about 0.5m. In the left the projection area is illustrated, in the middle we can see the scene and in the right we can see the images captured.
5.2. REFLECTANCE FIELD ACQUISITION

Figure 5.6: Image generated from the point of view of the camera by obtaining the reflectance field of the scene.

Figure 5.7: Image generated from the point of view of the projector by considering the projector like a virtual camera and the camera like a virtual projector.
implemented the technique called Dual Photography [12]. In [12], the symmetry property of the reflectance field expressed as a transport matrix is carried out (See equation 4.11). This technique consists in obtaining the reflectance field between the camera and projector. The reflectance field can be transformed to obtain a new view, that is, the point of view of the projector.

Every \((x_p, y_p)\) projector coordinate has a direct relationship with every \((x_c, y_c)\) camera coordinate. Figure 5.4 shows the system configuration of Dual Photography technique, it is composed by a digital camera and a projector. To show that the reflectance field has been acquired we obtain the view perceived by the projector by following the next steps:

1. The scene is illuminated pixel by pixel by the projector.
2. The value captured by the camera is stored as a function of pixel location.
3. It is assumed that the information of light captured by the camera is the same than the light perceived by the projector.
4. The projector is considered like a virtual camera and the camera is considered like a virtual projector.
5. The stored values are placed in the correct position of this virtual camera. Then the image from the point of view of the projector is generated.

Some examples of the images captured that compose the reflectance field of the scene are showed in Figure 5.5, it has to be noticed that the window projected (set of pixels lighted up) is larger than one pixel, the figure has the purpose to illustrate the capture. The scene is composed of objects with different material properties.

Once the reflectance field of the scene is acquired, we can generate the image from the point of view of the camera as Figure 5.6 shows. Also, the reflectance field can be transformed to generate the image from the point of view of the projector as Figure 5.7 shows. We can see that perspective and direction of the light onto the scene correspond a new point of view (point of view of the projector).

In the next section we apply independent component analysis capture to obtain the reflectance field for decreasing the number of images needed to compose it.

### 5.2.2 Independent Component Analysis method

According to Section 4.1.2, the scene is parallel illuminated by the projector, that is, every pixel of the projector or points of light with resolution of 640x480 represented by \(L_i\) are lighted up.

The scene will reflect light to the camera with resolution of 640x480 that is, 640x480 signal mixtures of the impulse responses, \(T\) are received. The illumination undergoes amplitude variations by projecting \(N\) different levels of gray of sequential patterns. Figure 5.8 shows an example of scene illuminated with 4 different levels of gray.
According to equation 4.4, every vector \( \mathbf{L}_i(x_i) \) is composed by \( N \) amplitude variations and \( 0 \leq i \leq 640 \times 480 \). This means that we can obtain the 4D reflectance field with only \( N \) images.

The experiments were carried out with \( N = 26 \) that is, 26 images with amplitude variations of 10 levels of gray were captured, the value of \( N \) will be defined below.

Figure 5.9 shows the results of applying the method on the objects: plane, triangle and sphere, we indicated important data such as, the window size, the number of pixels illuminated at the same time, the number of images captured and the RMS error for every case. In the figure we can observe that with the traditional technique of acquisition of the reflectance field, a total of 307200 images compose the reflectance field. With our method the number of images needed to compose the reflectance field is of 26. It is meaning that the number of images is decreased up to 99\% in all the objects.

Figure 5.10 shows relevant data (size, time, number of images and RMS) for the objects captured using ICA method.
CHAPTER 5. EXPERIMENTS AND RESULTS

Figure 5.9: Results of applying the ICA-capture on the objects: plane, triangle and sphere. All the objects have different material properties: matte, wood, woven and marble. We indicated important data such as, the number of pixels illuminated at the same time, the number of images captured and the RMS error for every case.

**Definition of N**

The value of N is defined by taking into account the next considerations:

- **Camera resolution.** Sequential amplitude variations are projected (starting from one level of gray) onto the scene, images from the scene were acquired with the camera, if it is possible to distinguish on the images the amplitude variations, we define such amplitude variation as the minimum step of level of gray.

- **Projector resolution.** Sequential amplitude variations are projected onto the scene. The projector is able to project amplitude variations of one level of gray. However, the amplitude variation is dependent of the camera resolution, explained in the last step.

- **RMS error.** To obtain the RMS error, the reflectance with brute-force scan method is acquired, the reflectance with our method is acquired also. We have to acquire 255 reflectance fields with our method. Every reflectance field is composed by 0 to 255 images, it depends of the amplitude variation that we are projecting. For example, the last reflectance field is composed by 255 images with an amplitude variation of one level of gray. Once we obtain both reflectance fields, we calculate the RMS error.
5.2. REFLECTANCE FIELD ACQUISITION

Figure 5.10: Relevant data (size, time, number of images and RMS) for the objects (plane, triangle and sphere) captured using the ICA-method. All the objects have different material properties: matte, wood, woven and marble.

The value of $N$ is defined by the minimum step of level of gray that the projector and camera are able to project and capture respectively, and the minimum value of the RMS error. Figure 5.11 shows the definition of $N$.

Figure 5.13 and Figure 5.14 show the image synthesized by illuminating pixel by pixel the scene (first column), the image synthesized by the ICA-method (middle) and RMS of the example, it is calculated by doing a comparison between the image synthesized by illuminating pixel by pixel and the image synthesized by using ICA-method (third column). Figure 5.15 shows relevant data (size, time, number of images and RMS) for the scenes captured using ICA-method. We can see that with 26 images, the RMS is maintained under the 2%.
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Figure 5.11: Definition of N by minimizing the RMS error from the comparison of the reflectance field acquired pixel by pixel (pbp) and the reflectance field obtained with our method (om) by doing amplitude variations of levels of gray (1 to 255 amplitude variations, that is, 1 to 255 images) for the materials: matte, wood, woven, marble.

5.3 Comparison with related work

The authors of related work do not measure the quality of the reflectance field acquired, they only show the image synthesized by using the reflectance information and give relevant data as size, time required and number of images. In our work, besides showing the image and obtaining data as size, time and number of images, we define the reflectance field quality acquired by comparing it to ground truth reflectance data. Ground truth reflectance data is obtained by using brute-force scan technique[37]. Brute-force scan technique is the traditional form to acquire the reflectance field of an object placed in the scene. The RMS of each example is less than 2% as Figure 5.9, 5.10 and 5.15 show.

We compare our method [32] in terms of time, size and number of images needed to compose the reflectance field with related work [12, 37, 25] (for a more deep description of related work methods see section 2.4), as Figure 5.12 shows. It has to be noticed, the difficulty of comparing the results with other methods, since the objects in the scene, materials properties of the objects, conditions of acquisition and setup configuration are different. However, this may give us an idea of the location of our method. In the figure we can observe that our method requires less images, time and storage size.
Figure 5.12: Comparison of our method in terms of time, size and number of images needed to compose the reflectance field with related work.

5.4 Discussion

When the projection and capturing are performed by using the brute-force scan and the setup described, 640x480 images were captured. In our capture method, only 26 images were acquired for all the objects, it means that with our method we were able to decrease the number of images for the reflectance field composition up to 99%. The RMS of each example (less than 2%) shows that our strategy keeps the reflectance field quality very close to the traditional approach. Also, time is reduced from days to minutes.

We assume that every point projected onto the scene has local influence, if this assumption is not met, our method will fail. For example, if the object were a diffuse concave sphere pointed toward the camera, there would be a lot of interreflection in the sphere. The pixel values will have the effect of this indirect light. In this case, our technique will fail and one would be forced to project each pixel independently (pixel by pixel).

5.5 3D recovery and reflectance fields relation

To obtain the three-dimensional reconstruction of the object placed in the scene using the reflectance field data, we are using equation 4.15 formulated in Section 4.2.2. We consider the geometry where the scene is illuminated by an isotropic point source and observed by a camera. The scene is located in the field of view of the camera and light source. In the setup is assumed that every ray light from the light source illuminates the scene and the number of rays reflected is just one, it can be considered true for
5.5.1 Depth recovery integrating the reflectance field

In order to obtain the three-dimensional reconstruction of the object placed in the scene, first the calibration parameters have to be computed. To do that we use the Dual Photography [37] technique to use the camera-projector assembly as a stereo system for enabling the projector to capture images like a camera, thus making the calibration of a projector essentially the same as that of a camera, which is well established.

A standard black-and-white checkerboard is used. The flat checkerboard positioned with different poses is imaged by the camera and poses from the point of view

Figure 5.13: Images synthesized by illuminating pixel by pixel the scene (top), images synthesized by the ICA-method (bottom) and RMS of the example, it is calculated by doing a comparison between the image synthesized by illuminating pixel by pixel and the image synthesized by using ICA-method (right).

those objects with 4D material properties.
Figure 5.14: Images synthesized by illuminating pixel by pixel the scene (top), images synthesized by the ICA-method (bottom) and RMS of the example, it is calculated by doing a comparison between the image synthesized by illuminating pixel by pixel and the image synthesized by using ICA-method (right).

of the projector are generated. Figure 5.16 shows examples of the checkerboard images captured from the point of view of the camera and Figure 5.17 shows images synthesized from the point of view of the projector.

Once, these poses are obtained the intrinsic and extrinsic parameters of the stereo system using the Matlab toolbox provided by Bouguet [3] are computed.

The three-dimensional reconstruction using the reflectance field was implemented in Matlab and empirically validated its effectiveness in the following experiments. The reconstruction method was applied to several objects: plane, triangle and sphere, with different material properties from their surface such as: matte, wood, woven and marble. In Matlab, the model of every object was created by using the measurements taken directly from every object (plane, triangle and sphere) as Figure 5.18 shows.

We obtain the reconstruction of all the objects and we compute the RMS error.
CHAPTER 5. EXPERIMENTS AND RESULTS

Figure 5.15: Relevant data (size, time, number of images and RMS) for the composed images captured using ICA-method.

<table>
<thead>
<tr>
<th>Scene</th>
<th><strong>Brute-Force Scan</strong></th>
<th><strong>Our method</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Size (TB)</td>
<td>Time (Days)</td>
</tr>
<tr>
<td>Example 1</td>
<td>0.6</td>
<td>107</td>
</tr>
<tr>
<td>Example 2</td>
<td>0.4</td>
<td>107</td>
</tr>
</tbody>
</table>

The error was calculated by comparing the reconstructed object and the model of the object created in Matlab.

*Plane object:* The first object recovered is a plane. We place the object with an inclination of 40 degrees. The plane has different material properties from its surface: matte, wood, woven and marble. Figures 5.20, 5.21, 5.22 and 5.23 show the three-dimensional reconstruction of the plane.

*Sphere object:* The second object recovered is a sphere. The diameter of the sphere is of 4.5cm. In the experiments we were able to reconstruct about half of the sphere since we can not recover what is not seen by the camera. Figures 5.24, 5.25, 5.26 and 5.27 show the three-dimensional reconstruction of the sphere with matte, wood, woven and marble material properties.

*Triangle object:* The last object reconstructed by our method is a triangle, it is 10x8x5cm in size and exhibits different material properties: matte, wood, woven and marble, from their surface. Figures 5.28, 5.29, 5.30 and 5.31 show the three-dimensional reconstruction of the object.

We can observe that the error in the reconstruction of the matte object is smaller than the error in the reconstruction of the objects with other material properties. This is because the matte material has not interreflections [28], it means that every point projected onto the scene has local influence. In the case of wood, woven and marble objects, they have texture and subsurface scattering surface, it means that the materials present interreflections [7, 21]. We can observe also that in marble material for all the objects, the shape of the plane, sphere or triangle is recovered by our method but finer surface detail is lost. Nevertheless, with our method we were able to obtain the three-dimensional reconstruction with an error less than 3% of the real-world object measurements. To define the percentage of error in the reconstructions, we establish the relation of proportion between the depth value to be recovered and
5.6. COMPARISON WITH RELATED WORK

Figure 5.16: Different positions of the checkerboard for calibration from the point of view of the camera.

the RMS error for every example.

5.6 Comparison with related work

The same situation of the reflectance field acquisition method, the difficulty of comparing the reconstruction results with other methods exists, since the objects in the scene, materials properties of the objects, conditions of acquisition and setup configuration are different. However, we compare the error of our method with related work. Figure 5.32 shows the comparison of our method [31] with two important and actual techniques: 3D based on Digital Fringe Projection (academic [19] and industrial [38] configurations) and 3D with Kinect [20]. Our method has higher accuracy than 3D based on Digital Fringe Projection [19] when matte and wood objects are recovered and lower accuracy when marble objects are reconstructed. Our reconstruction method has higher accuracy than 3D with Kinect for all objects.

We also show an image (see Figure 5.33) that compares the reconstruction of a teddy bear obtained with our method and the reconstruction of the teddy bear obtained with the Kinect depth sensor. Unfortunately, the free libraries of Kinect do not include the way to obtain measurements. We can see that with our method, finer surface detail is recovered, while with Kinect method the surface detail of the teddy
CHAPTER 5. EXPERIMENTS AND RESULTS

5.7 Discussion

We have verified experimentally that three-dimensional reconstruction objects exhibiting an anisotropic BRDF is possible using a 4D slice of the 8D reflectance field. We show that the property of reciprocity of the reflectance field can be used to impose the epipolar constraint by considering the camera-projector pair as a stereo system. With our method, the three-dimensional reconstruction is recovered with an error less than 3% of the real-world object measurements. To define the percentage of error in the reconstructions, we establish the relation of proportion between the depth value to be recovered and the RMS error for every example.

In the reconstruction method is assumed that every ray light from the light source illuminates the scene and the number of rays reflected is just one, if this assumption is not met, our method will fail. Using the same example of Section 5.4, if the object to be recovered were a diffuse concave sphere pointed toward the camera, there would be a lot of interreflection in the sphere. The number of rays reflected is more than one, due to the effect of the indirect light. In this case, the technique will fail and one would be forced to reformulated the reconstruction equation for considering all the
5.7. DISCUSSION

Figure 5.18: Models created in Matlab by using the measurements taken directly from every object (sphere, triangle and plane).

rays of light caused by the interreflection and the reflectance field used would have to be acquired pixel by pixel. For example, in the cases of wood, woven and marble objects, the RMS in the reconstructions is larger than the RMS of the matte object, it is caused by some interreflections in wood, woven and marble objects.
Figure 5.19: Three-dimensional reconstruction of a plane that exhibits matte material properties from their surface.
Figure 5.20: Three-dimensional reconstruction of a plane that exhibits matte material properties from its surface.
Figure 5.21: Three-dimensional reconstruction of a plane that exhibits woven material properties from its surface.
5.7. DISCUSSION

Figure 5.22: Three-dimensional reconstruction of a plane that exhibits wood material properties from its surface.
Figure 5.23: Three-dimensional reconstruction of a plane that exhibits marble material properties from its surface.
Figure 5.24: Three-dimensional reconstruction of a sphere that exhibits matte material properties from its surface.
Figure 5.25: Three-dimensional reconstruction of a sphere that exhibits woven material properties from its surface.
Figure 5.26: Three-dimensional reconstruction of a sphere that exhibits wood material properties from its surface.
Figure 5.27: Three-dimensional reconstruction of a sphere that exhibits marble material properties from its surface.
Figure 5.28: Three-dimensional reconstruction of a triangle that exhibits matte material properties from its surface.
Figure 5.29: Three-dimensional reconstruction of a triangle that exhibits woven material properties from its surface.
Figure 5.30: Three-dimensional reconstruction of a triangle that exhibits wood material properties from its surface.
Figure 5.31: Three-dimensional reconstruction of a triangle that exhibits marble material properties from its surface.
Figure 5.32: Comparison of our method with two important and actual techniques: 3D based on Digital Fringe Projection (academic and industrial configurations) and 3D with Kinect.

<table>
<thead>
<tr>
<th>Method</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D based on Digital Fringe Projection [19]</td>
<td>1-2mm</td>
</tr>
<tr>
<td>3D based on Digital Fringe Projection (3D scanners) [38]</td>
<td>0.05mm</td>
</tr>
<tr>
<td>3D with Kinect [20]</td>
<td>5-10mm</td>
</tr>
<tr>
<td>Our method [31]</td>
<td>0.1-3mm</td>
</tr>
</tbody>
</table>
Figure 5.33: Three-dimensional reconstruction of a real-object exhibiting a non-Lambertian material from its surface. In the top, we are using 3D with Kinect. In the bottom, we are using our method. In the right, the image of the object.
Chapter 6

Conclusions and future work

In this chapter we describe the main contributions of the thesis. We identify some benefits resulting from this research and finally we discuss the future research directions.

6.1 Contributions

This thesis explores a system for efficient acquisition of reflectance fields. The key challenge in acquiring reflectance fields efficiently is that a reflectance field is extremely large. Even if state of the art techniques are used for acquisition, the time required to acquire a reflectance field is still intractable. Our system reduces the storage and time requirements up to 99%. This reduction is possible by using independent component analysis. Based on this method, we have developed algorithms for acquiring the transport matrix data and represent it as a signal mixture of impulse responses.

This thesis explores also that the real three-dimensional reconstruction from a scene is generated by adding the measurement of the material properties of the objects that compose the scene. The work presents the integration of the reflectance field and the three-dimensional reconstruction equations. Based on this integration, an algorithm for recovering the depth map from the scene is developed. The scene is composed by objects exhibiting an anisotropic BRDF from their surface. The implementation validates that the method can be applied to objects with anisotropic BRDF, the three-dimensional reconstruction is recovered with an error less than 3% for the real-world objects measurements.

The research results reported in this thesis can be used to improve the performance and features of existing 3-D recovery systems and reflectance field acquisition systems. Specific improvements that can be made and the resulting benefits are outlined below:

- Faster acquisition speed of the reflectance field. With our acquisition method, the speed in the acquisition of the material properties of the objects can be
increased. As a result, our method could meet the speed requirements of more applications, such as on-line scene relighting or rapid prototyping of scenes. For example, film directors or game designers may want to quickly compose a photo-realistic scene, for pre-visualization purposes.

- Better measurement accuracy. Current methods of reconstruction do not consider the reflectance field information, leading to incorrect depth estimates. The integration of reflectance data with industrial reconstruction methods could increase the reconstruction accuracy.

- Lower system cost. The research work conducted in this thesis has a low-cost system configuration, e.g. using commercial computer projectors and low-end industrial digital cameras. The hardware cost of such systems is low and it could ensure the wide deployment of these techniques in industry.

6.2 Future work

Following the research work described in this thesis, new research can be conducted in the following areas:

- Improvement on the algorithm for generation of projection patterns is possible. The projection pattern presented works well. However, there is still room for improvement in this pattern to achieve better performance. In future research, the use of Gaussian or gray code inspired patterns should be explored. For example, the projector could emit a series of white square patterns of constant intensity over some parts of its projection angle or Gaussian patterns could be used, resulting in smoother images.

- In this work, we have demonstrated a technique for acquiring reflectance fields. However, the reflectance fields acquired are incomplete. In order to change the lighting and the viewpoint, the acquired reflectance field needs to be dense. Techniques have been proposed for interpolating slices of the reflectance fields, both from the view direction and from the illumination direction, but the problem of interpolating reflectance fields is still open. One can also sample incoming and outgoing light fields more densely by increasing the number of cameras and projectors. This will increase the number of viewpoints in the light field. It should be noted that faster processing and use of high resolution cameras could reduce the time significantly in the future.

- If we can sample incoming and outgoing light fields more densely, we must be able to reduce the error in the three-dimensional reconstructions by considering different points of view and illumination directions. Also, we can obtain a full 3-D recovery of the object.
6.2. FUTURE WORK

• Although we focus in this work on three-dimensional reconstruction using the reflectance field, our method of reflectance field acquisition has other uses. For example, our method can be applied to the problem of scene relighting, in photo-realistic animation, digital photography, and interactive games. Other potential domains include ophthalmology or surgical simulation. This work has the potential to allow directors to completely synthesize realistic actors, immerse gamers in a completely real environment, or enable doctors to train on photo-realistic simulation imagery.
CHAPTER 6. CONCLUSIONS AND FUTURE WORK
Appendix A

Appendix: Publications

A.1 Publication 1

Acceleration of the reflectance field acquisition using independent component analysis

M.L. Rosas-Cholula, and M.O. Arias-Estrada
Department of Computer Sciences
National Institute of Astrophysics, Optics and Electronics, INAOE
Puebla, México
{mluisa,ariasmo}@inaoep.mx

Abstract
In order to generate photorealistic images, a central problem in computer graphics is the description of an object reflectance model. The reflectance field technique describes the object surface properties and can be used for photorealistic rendering. The reflection of surfaces can be described as a high dimensional reflectance function. For complex surfaces, an analytical model is not always easy to formulate, therefore the direct real-world surface acquisition is preferred. The reflectance is typically acquired with a camera or array of cameras that capture the reflectance field of the object surface but the reflectance information can be composed of thousands of images, depending on the surface material properties and the camera resolution. In this work we propose a systematic strategy that incorporates Independent Component Analysis (ICA) to acquire the reflectance field and reducing by orders of magnitude the required number of captured images and keeping the same reflectance field quality. In our experiments, a reflectance field can be obtained with only 26 images, compared to the classical approach that require thousands of images, with an error less than 2%.

1. Introduction

To obtain the reflectance field for describing the surface properties of an object in a scene is one of the central problems in computer graphics. However, the formulation of analytical models for complex surfaces is not always an easy task. An alternative approach is to capture the reflectance information from real-world surfaces. This acquisition is carried out, for capturing with a camera or array of cameras a set of data that describes the transfer of energy between a light field of incoming rays (the illumination) and a light field of outgoing rays (the view). Such set of data is known as the Reflectance Field [1]. For obtaining the reflectance field of a scene, thousands of images are acquired depending on the optical properties of the object placed in the scene and how much variation is permitted in the illumination and viewer position [2,3,4].

The traditional technique to acquire the reflectance field of an object consists in illuminating and capturing pixel by pixel the object placed in the scene using a video projector. In order to accelerate the acquisition, some algorithms are devoted to parallelize the capture. To illuminate multiple pixels at the same time, it is possible only with the assumption that each projected pixel affects a small and localized region of the scene. Even so, the amount of images that composes the reflectance field is extremely large (thousands of images) [5].

There are some techniques for compression of the reflectance field such as, Vertex Quantization (VQ) and basis decomposition techniques like wavelets or Matrix Factorization (MF) based on SVD or PCA. More recently Local PCA, which can be thought of as a combination of VQ and MF, and Tensor Factorization (TF) have become popular. However, a large amount of images are acquired first to compose the reflectance field and then a compression technique can be applied [6].

This paper proposes a systematic strategy that uses independent component analysis (ICA) to acquire the reflectance field. Our method takes advantage of the fact that the pixels parallel illuminated affect local regions of the scene. We consider the transfer of energy between the incoming and outgoing light fields as signal mixtures in order to use an ICA approach to decompose the signal mixtures into statistically independent signals. Our procedure avoids the need of analytical model of the reflectance field, it reduces the images required to describe the field and our strategy keeps the same reflectance field quality as the traditional approach.

2. Reflectance fields and independent component analysis

The light fields are used to describe the radiance at each point $x$ and in each direction $\omega$ in a scene. Ignoring wavelength and fixing time, this is a 5D function which we denote by $\hat{L}(x, \omega)$. Thus, $\tilde{L}(x, \omega)$ represents the radiance leaving a point $x$ in direction $\omega$. Levoy and Hanrahan [2] observed that if the viewer is moving within the unoccluded space, then the 5D representation of the light field can be
reduced to 4D. We can characterize this function as \( L(\psi) \), where \( \psi \) specifies a point and an incoming direction on a sphere [1]. A 4D light field can be used to generate an image from any viewing position and direction, but it will always show the scene under the same lighting. In general, each field of incident illumination on a scene will induce a different field of exiting illumination from the scene. Debevec et al [1] showed that the exiting light field from the scene under every possible incident field of illumination can be represented as an 8D function called the reflectance field: 

\[ R(L_i(\psi_i); L_0(\psi_0)) = R(\psi_i; \psi_0) \]

Here, \( L_i(\psi_i) \) represents the incident light field on the scene, and \( L_0(\psi_0) \) represents the exiting light field reflected off the scene. In order to work with discrete forms of these functions, the domain \( \psi \) of all incoming directions can be parameterized by an array indexed by \( i \). The outgoing direction corresponding to an incoming direction is also parameterized by the same index, \( i \). Now, consider emitting unit radiance along ray \( i \) towards the scene (e.g., using a projector). The resulting light field, which is denoted by vector \( t_i \), captures the full transport of light in response to this impulse illumination. This is called the impulse response [7] or the impulse scatter function [8]. We can concatenate all the impulse responses into a matrix \( T \) which we call the light transport matrix:

\[ T = [t_1 t_2 \ldots t_n] \tag{1} \]

Since light transport is linear, any outgoing light field represented by a vector \( \mathbf{L}_0 \) can be described as linear combination of the impulse responses, \( t_i \). Thus, for an incoming illumination described by vector \( \mathbf{L}_i \), the outgoing light field can be expressed as:

\[ \mathbf{L}_0 = T \mathbf{L}_i \tag{2} \]

The light transport matrix \( T \) is thus the discrete analog of the reflectance field \( R(L_i(\psi_i); L_0(\psi_0)) \).

In the other hand, the independent component analysis is a method for separating a multivariate signal into additive subcomponents supposing the mutual statistical independence of the source signals [9]. Assume that we observe \( n \) linear mixtures \( x_1, \ldots, x_n \) of \( n \) independent components

\[ x_i = a_{i1} s_1 + a_{i2} s_2 + \cdots + a_{in} s_n \quad \text{for all } i \tag{3} \]

In the ICA model, it is assumed that each mixture \( x_i \) as well as each independent component \( s_k \) is a random variable. The observed values \( x_i \) are a sample of this random variable.

It is convenient to use vector-matrix notation instead of the sums like in the previous equation. Let us denote by \( \mathbf{x} \) the random vector whose elements are the mixtures \( x_1, \ldots, x_n \) and likewise by \( \mathbf{s} \) the random vector with elements \( s_1, \ldots, s_n \). Let us denote by \( \mathbf{A} \) the matrix with elements \( a_{ij} \). Using the vector-matrix notation, the above mixing model is written as

\[ \mathbf{x} = \mathbf{As} \tag{4} \]

The statistical model in 4 is called independent component analysis, or ICA model. The ICA model is a generative model, which means that it describes how the observed data are generated by a process of mixing the components \( s_i \). The independent components are latent variables, meaning that they cannot be directly observed. Also the mixing matrix is assumed to be unknown. All we observe is the random vector \( \mathbf{x} \), and both \( \mathbf{A} \) and \( \mathbf{s} \) have to be estimated by using such vector.

\[
\begin{bmatrix}
\mathbf{x} \\
\mathbf{s}
\end{bmatrix} =
\begin{bmatrix}
\mathbf{A} & \mathbf{0} \\
\mathbf{0} & \mathbf{I}
\end{bmatrix}
\begin{bmatrix}
\mathbf{s} \\
\mathbf{x}
\end{bmatrix}
\]

The starting point for ICA is the assumption that the components \( s_i \) are statistically independent. Then, after estimating the matrix \( \mathbf{A} \), we can compute its inverse \( \mathbf{W} \), and obtain the independent component simply by

\[ \mathbf{s} = \mathbf{Wx} \tag{5} \]

ICA is very closely related to the method called: blind source separation (BSS) or blind signal separation. A “source” means here an original signal. “Blind” means that the mixing matrix is unknown. The Fig. 1 shows the mixing (top) and unmixing (bottom) process.
3. Independent component analysis of the reflectance field

Consider the scene configuration in Fig. 2. All the scene is illuminated parallel by a light source \( L_i \).

A particular point in the scene \( x_i \) will reflect light to the camera \( C \). The outgoing light field \( L_i \) is the reflected intensity in the direction of \( C \) from the point \( x_i \).

According to the ICA model, these independent components \( t_i \) cannot be directly observed. \( L_i \) is the incident light intensity at point \( x_i \).

Considering 3, the observed values from the point \( x_i \) are samples of \( L_i \) and can be expressed as

\[
L_0(x_i) = (L_0(x_i^1), L_0(x_i^2), \ldots, L_0(x_i^N))
\]

\[
L_0(x_i) = (L_0(x_i^1), L_0(x_i^2), \ldots, L_0(x_i^N))
\]

where the superscripts specify the identity of the intensity level of the \( L_0 \) sample and the subscripts specify the identity of the reflectance field element.

Following the ICA model (see 5), we can calculate \( T_0 \) as an estimated of \( T \) (light transport matrix), such as

\[
T_0 = WL_0
\]

4. Test capture and results

The capture setup for the experiments requires a projector and a camera. There is no restriction on the location of the camera and the projector. Also there is no geometric calibration required. Capturing the reflectance field refers to project patterns towards a scene. The pattern is projected by lighting every single pixel of the light source (projector). Every point of light reflected from the scene is imaged by a sensor (camera). The set of images captured will compose the reflectance field.

In the experiments, the reflectance field by illuminating pixel by pixel (brute-force scan) onto the scene is acquired as in [5]. After that, we obtain the reflectance field by our method. To define the reflectance field quality acquired with our method, we compare the images synthesized from both reflectance fields and a RMS error is computed. The system setup is composed by a Samsung digital projector with a resolution of 640x480 pixels, and a Canon PowerShot-G5 camera with a resolution of 640x480 in B/W.
The brute-force-scan method requires that the projector of the system setup with resolution $p \times q$ shine light onto a scene. Every point of light reflected from the scene is imaged by the camera of resolution $m \times n$. The number of images captured by the camera is: $p \times q$ images. The images are stored in a transport matrix $T$ (see 1) of size $mn \times pq$. The matrix size depends on the kind of object to be recovered.

In our method all the scene is parallel illuminated by the projector it is, every pixel of the projector or points of light with resolution $640 \times 480$ represented by $L$, are lighted up. The scene will reflect light to the camera with resolution of $640 \times 480$ that is, $640 \times 480$ signal mixtures of the impulse responses, $T$ are received. We do amplitude variations by projecting $N$ different levels of gray of sequential patterns. The Fig. 4 shows an example of a scene illuminated with 4 different levels of gray.

The experiments were carried out with $N = 26$ that is, 26 images with amplitude variations of 10 levels of gray were captured. The value of $N$ was defined by minimizing the RMS error from the comparison of the reflectance field acquired pixel by pixel and the reflectance field obtained with our method by doing amplitude variations of levels of gray (1 to 255 amplitude variations it is, 1 to 255 images) from the scene. The Fig. 5 shows the definition of $N$.

According to the 7, every vector $L_0(x_i)$ is composed by the $N$ amplitude variations and $0 \leq i \leq 640 \times 480$. This means that we can obtain the 4D reflectance field, $T_o$, with $N$ images. The calculation of $T_o$ is a sequential procedure implemented in Matlab. The Fig. 6 and Fig. 8 show the image synthesized by illuminating pixel by pixel the scene (top) and the image synthesized by using our method (bottom). The Fig. 7 and Fig. 9 show the RMS error calculated by doing a comparison between the image synthesized by illuminating pixel by pixel and the image synthesized by using our method.

5. Discussion

When the projection and capturing are performed by using the brute-force scan and the setup described in the experimental results section, $640 \times 480$ images were captured. In our method, only 26 images were acquired that is, with our method we were able to decrease the amount of images for the reflectance field composition up to 99%. The RMS of the examples (2%) shows that the reflectance field quality is maintained. The Fig. 10 summarizes the results of our method.

6. Conclusions

We have implemented a system setup composed by a camera and a projector to obtain the reflectance field of objects with an anisotropic BRDF (4D) from their surface. We proposed a method for accelerating the acquisition of the reflectance information using independent component analysis approach. We proposed a method that considers the outgoing rays of the light field as statistically independent signals. These independent signals are obtained from the decomposition of a set of signal mixtures. These signal mixtures are acquired by taking images of the scene when it is illuminated by a projector.
with all its pixels turned on and when the illumination suffers amplitude variations. The theory and experiment have demonstrated the ability to decrease the number of images for the reflectance field composition up to 99%.

Figure 6. Example 1: Images synthesized by illuminating pixel by pixel the scene (top) and by our method (bottom).

Figure 7. RMS of the example, it is calculated by doing a comparison between the image synthesized by illuminating pixel by pixel and the image synthesized by using our method.

Figure 8. Example 2: Images synthesized by illuminating pixel by pixel the scene (top) and by our method (bottom).

Figure 9. RMS of the example 2, it is calculated by doing a comparison between the image synthesized by illuminating pixel by pixel and the image synthesized by using our method.
A.2 Publication 2

Integrated three-dimensional reconstruction using reflectance fields

Maria-Luisa Rosas¹ and Miguel-Octavio Arias²

¹,² Computer Science Department, National Institute of Astrophysics, Optics and Electronics, Puebla, 72840, Mexico
{mluisa,ariasmo}@ccc.inaoep.mx

Abstract
A method to obtain three-dimensional data of real-world objects by integrating their material properties is presented. The material properties are defined by capturing the Reflectance Fields of the real-world objects. It is shown, unlike conventional reconstruction methods, the method is able to use the reflectance information to recover surface depth for objects having a non-Lambertian surface reflectance. It is, for recovering 3D data of objects exhibiting an anisotropic BRDF with an error less than 0.3%.

Keywords: Three-dimensional reconstruction, Reflectance fields, Computer Vision, Computer Graphics.

1. Introduction
Many of the current methods in 3D computer vision rely on the assumption that the objects in the scene have Lambertian reflectance surface (such property is related to the materials that reflect the same amount of incident energy illumination uniformly over all the surface). Unfortunately, this assumption is violated for almost all real world objects, leading to incorrect depth estimates [1][2].

In the area of computer graphics, the reflection from surfaces is typically described by high dimensional reflectance functions. However, the formulation of analytical models is not always an easy task. An alternative approach to the specification of the reflectance or optical properties of the surface objects by analytical modeling is the capture of this reflectance information from real-world surfaces. The acquisition is carried out, with a camera or array of cameras to obtain a set of data that describes the transfer of energy between a light field of incoming rays (the illumination) and a light field of outgoing rays (the view). Such set of data is known as the Reflectance Field [3].

This document explores the problem of obtaining the three-dimensional reconstruction of objects exhibiting an anisotropic BRDF (the objects material have the property that their reflection characteristics vary to rotations of the surface about its normal) by using a 4D slice of the 8D reflectance field. The 4D slice of the reflectance field is obtained by a camera-projector pair. Our method exploits the property of reciprocity of the reflectance field to impose the epipolar constraint by considering the camera-projector pair as a stereo system. As an example, we show how our method can be used to recover objects with an anisotropic BRDF of their surface. This procedure avoids the need of an analytical model of the reflectance data.

2. Theory

2.1 Reflectance field and Light transport constancy
Debevec et al [3] showed that the exiting light field from the scene under every possible incident field of illumination can be represented as an 8D function called the reflectance field:

\[ R(\textit{L}_i(\psi_i); \textit{L}_0(\psi_0)) = R(\psi_i; \psi_0) \] (1)

Here, \( L_i(\psi_i) \) represents the incident light field on the scene, and \( L_0(\psi_0) \) represents the exiting light field reflected off the scene. In order to work with discrete forms of these functions, the domain \( \psi \) of all incoming directions can be parameterized by an array indexed by \( i \). The outgoing direction corresponding to an incoming direction is also parameterized by the same index, \( i \). Now, consider emitting unit radiance along ray \( i \) towards the scene (e.g., using a projector). The resulting light field, which is denoted by vector \( \textit{t}_i \), captures the full transport of light in response to this impulse illumination. This is called the impulse response or the impulse scatter function [4].

All the impulse responses can be concatenated into a matrix \( \textbf{T} \) which is called the light transport matrix:

\[ \textbf{T} = [\textit{t}_1 \textit{t}_2 \ldots \textit{t}_n] \]
Since light transport is linear, any outgoing light field represented by a vector $L_0$ can be described as linear combination of the impulse responses, $t_i$. Thus, for an incoming illumination described by vector $L_i$, the outgoing light field can be expressed as:

$$L_0 = TL_i$$

The light transport matrix $T$, is thus the discrete analog of the reflectance field $R(L_i(\psi_i); L_0(\psi_0))$.

2.2 Symmetry of the transport matrix

The idea that the flow of light can be effectively reversed without altering its transport properties was proposed by von Helmholtz in his original treatise in 1856 [5]. He proposed the following reciprocity principle for beams traveling through an optical system (i.e., collections of mirrors, lenses, prisms, etc.):

Suppose that a beam of light $\mathbf{A}$ undergoes any number of reflections or refractions, eventually giving rise (among others) to a beam $\mathbf{B}$ whose power is a fraction $f$ of beam $\mathbf{A}$. Then on reversing the path of the light, an incident ray $\mathbf{B}$ will give rise to a beam $\mathbf{A}$ whose power is the same fraction $f$ of beam $\mathbf{A}$.

In other words, the path of a light beam is always reversible, and furthermore the relative power loss is the same for the propagation in both directions. For the purpose of a reflectance field generation, this reciprocity can be used to derive an equation describing the symmetry of the radiance transfer between incoming and outgoing directions $\psi_i$ and $\psi_0$:

$$R(\psi_i; \psi_0) = R(\psi_0; \psi_i)$$

where $R$ is the reflectance field. For the light transport matrix defined in the last section, this implies that the transport of light between a ray $i$ and a ray $j$ is equal in both directions, i.e.

$$T[i, j] = T[j, i] \implies T = T^T$$

Therefore, $T$ is a symmetric matrix (See work in [6]).

2.2 BRDF

The Bidirectional Reflectance Distribution Function (BRDF) is a projection of the 8D reflectance field into a lower dimension. From equation 1, the 4D reflectance field can be represented as

$$f_r(L_i(\Omega_1); L_0(\Omega_2)) = f_r(\Omega_1; \Omega_2)$$

where $L_i(\Omega_1)$ represents the incident light field on the scene, and $L_0(\Omega_2)$ represents the exiting light field reflected off the scene and $\Omega_1, \Omega_2$ are incoming and outgoing directions, e.g., $(\theta_1, \phi_1), (\theta_2, \phi_2)$. In essence, the BRDF describe how bright the differential surface $dA$ of a material appears when it is observed from a certain direction and illuminated from a certain direction.

The reciprocity exposed in the last section, the 4D reflectance field can be written as

$$f_r(\Omega_1; \Omega_2) = f_r(\Omega_2; \Omega_1)$$

Some materials have the property that their reflection characteristics are invariant to rotations of the surface about its normal. Such materials are called isotropic. Materials not having this characteristic are called anisotropic. As equation 2 shows, in order to discretize the equation 7, all incoming and outgoing directions in domain $\Omega$ can be parameterized by an array indexed by $i$.

We denote the resulting 4D light field by vector $\mathbf{L}_i$, and this 4D light field is concatenated as

$$\mathbf{T} = [t_1 t_2 \ldots t_n]$$

For an incoming illumination described by vector $L_i'$, the outgoing light field can be expressed as

$$L'_0 = \mathbf{T}L'_i$$

The matrix $\mathbf{T}$ is the discrete analog of the 4D reflectance field. The reciprocity exposed in the last section implies that the transport of light between a ray $i$ and a ray $j$ is equal in both directions, i.e.

$$\mathbf{T}[i, j] = \mathbf{T}[j, i] \implies \mathbf{T} = \mathbf{T}^T$$

3. Depth recovery from the 4D reflectance field

Consider the scene configuration in Fig. 1a. All the scene is illuminated by a projector $\mathbf{L}_i$. A particular point in the scene $p$ will reflect light to the camera $C$ according to equation 9, the outgoing light field represented by the vector $L_i'$ is the reflected intensity in the direction of $C$ from the point $p$ with normal vector $n$. Let $\Omega_1$ and $\Omega_2$ denote the positions of the projector and camera, respectively. The unit vectors $\Omega_1 = \frac{1}{|\Omega_1 - p|}(\Omega_1 - p)$ and $\Omega_2 = \frac{1}{|\Omega_2 - p|}(\Omega_2 - p)$ denote the directions from $p$ to the projector and camera, respectively. Given this configuration, the image irradiance (see [7]) at the projection of $p$ is
where \( f_r \) is the BRDF (4D function).

\( e = f_r(\Omega_2, \Omega_1) \frac{\hat{n} \cdot \Omega_2}{|\sigma_2 - p|^2} \)

(11)

where \( \hat{T}(p) \) is the 4D transport matrix that corresponds to a point \( p \) of the scene, \( \hat{n} \) can be expressed as \((\frac{dx}{du}, \frac{dz}{du}, -1)\), the ray from the camera can be expressed as \( \Omega_2(p) = (\Omega_{2x}, \Omega_{2u}, \Omega_{2z}) \)

Taking advantage of the symmetry of the transport matrix we can impose the epipolar constraint to provide a solution to equation 12. Consider the scene configuration in Fig. 1b. All the scene is “illuminated” by a camera \( L'_o \). A particular point in the scene \( p \) will reflect light and it is “captured” by a light source. Then, we can consider the system configuration as a stereo setup such as, it can be calibrated as a stereo system.

The vector \( \Omega_2(p) \) and the denominator \(|\sigma_2 - p|^2\) can be determined when calibrating a stereo setup. Imposing the epipolar constraint we can express the normal \( \hat{n} \) as \((\frac{dx}{du}, 0, -1)\).

The point \( p(x, y, z) \) will have projections in the camera and the light source (considered as a second camera) established by calibration parameters of the system. Expressing the depth as \( z(x, y) \), we rewrite the equation 12 as

\[
\frac{dz}{dx} = \frac{e \Omega_{2z} - p^2 \hat{T}(p) + \Omega_{2z} \Omega_{2x}}{\hat{T}(p) \Omega_{2x}}
\]

(13)

This can be numerically integrated as

\[
z(x, y) = \int_{x_0}^{x} \frac{dz}{dx} dx + z(x_0, y)
\]

(14)

For each epipolar line \( y \), this integral provides the depth across the epipolar line. We can determine for each epipolar line \( y \) the \( z(x_0, y) \) since the point \( p(x, y, z) \) have projections in the camera and we know the corresponding projections to the light source when the 4D transport matrix is captured.

3. Test reconstruction

In order to obtain the three-dimensional reconstruction of the object placed in the scene some calibration parameters have to be computed, to do that we use the Dual Photography [6] technique to use the camera-projector assembly as a stereo system for enabling the projector to “capture” images like a camera, thus making the calibration of a projector essentially the same as that of a camera, which is well established. A standard black-and-white checkerboard is used. The flat checkerboard positioned with different poses is imaged by the camera.
and poses from the point of view of the projector are generated. Once, these poses are obtained the intrinsic and extrinsic parameters of the stereo system using the Matlab toolbox provided by Bouguet [8] are computed. Fig. 2 shows an example of the checkerboard images captured from the point of view of the camera (a) and synthesized from the point of view of the projector (b).

The three-dimensional reconstruction using 4D light field was implemented in Matlab and validated its effectiveness in the following experiment. We obtained the reconstruction of a real object exhibiting a non-Lambertian material which dimensions are known and the RMS error was computed by comparing the reconstructed object and the real object measurements. The Fig. 3 shows the three-dimensional reconstruction of a cube. The RMS error between the cube recovered and the real cube is of 0.3%.

4. Conclusions
All methods of three-dimensional reconstruction in computer vision area are influenced by light and the material properties of the objects. The estimation of the material of reflectance properties of the object is important for a correct 3D measurement. In computer graphics, the
material properties of such objects materials are measured and they are described as a dimensional reflectance functions (8D function). The theory and experiment have demonstrated the ability of obtaining the three-dimensional reconstruction of objects exhibiting an anisotropic BRDF by integrating a 4D slice of the 8D reflectance field information by using a camera-proyector pair with an error less than 0.3% of the real-world object measurements. Also, this procedure represents the first step to extend the formulation to include 6D and 8D surface properties.

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References


First Author obtained her B.Eng. in Computer Science at the UPAEP (University of Puebla) in Puebla, Mexico, in 2002. She obtained a M.Sc. in Computer Science at the INAOE (National Institute of Astrophysics, Optics and Electronics, Puebla, Mexico) in 2004. For two years (2006-2008), she worked in Prefixa Vision Systems (Puebla, Mexico) where she developed a 3D Camera. Since 2008 she is a Ph.D student in the Computer Science department at the INAOE. She is an inventor of the patent: Method and apparatus for rapid three-dimensional restoration. Her current research interests are computer vision, computer graphics, FPGA and CUDA architectures, robotics and genetic algorithms.

Second Author obtained his B.Eng. in Communications and Electronics at the FIMEE (University of Guanajuato) in Salamanca, Gto. in 1990. He also obtained a M.Eng. in Instrumentation and Digital Systems at the FIMEE two years later. In 1997, he finished his Ph.D. degree at the Computer Vision and Systems Laboratory of Université Laval (Quebec city, Canada). He was a professor-researcher at the Computer and Systems Laboratory at Laval University where he worked on the development of a Smart Vision Camera. Since 1998 he is with the Computer Science department of INAOE (National Institute of Astrophysics, Optics and Electronics, Puebla, Mexico) where he continues his research on FPGA architectures for computer vision.
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