Optimization of Analog Circuits by Applying Evolutionary Methods

by

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Abstract

Analog circuit design is a difficult work due to the many parameters which are necessary to take into account and to the different objectives which are necessary to satisfy such as gain, band width , offset, impedances, etc. To achieve optimal performances, it is necessary to deal with all the parameters of an analog circuit in order to know exactly which elements are responsible to improve the design. In this manner, a CAD tool is useful here because it is possible to enhance analog circuit design since one have many solutions from which one is able to select that one accomplishing the desired performances.

This thesis uses two evolutionary algorithms, NSGA-II and MOEA/D, to optimize analog circuits from a selected topology taking into account design constraints. The optimization is performed by sizing each transistor and there is not necessary to set initial conditions, only it is necessary to set boundary limits for each variable. The evolutionary algorithms are tested with two different genetic operators: basic cross-over/mutation and differential evolution. Both evolutionary methods use HSPICE to evaluate performances of the analog circuits by using standard CMOS technology of 0.35 μm.

The evolutionary algorithms are applied first to optimize unity-gain cells with three objectives: gain, bandwidth and offset, with different number of variables, from two to nine variables. Afterwards, the optimization is on current conveyors which are build of more than twenty transistors but this time they are optimized in four objectives: resistances in each port and gain. For all these circuits it is possible to add design constraints such as saturation conditions.

Both evolutionary methods have been tested, and it is shown that the optimization results are into the set limits, while the objective functions are improved. Finally, from the optimization results, an analog designer is able to choose the best one which meet the desired specifications of the circuit.

Resumen

El diseño de circuitos analógicos es una tarea difícil debido a muchos parámetros que son necesarios tomar en cuenta y los diferentes objetivos los cuales son necesarios satisfacer tales como ganancia, ancho de banda, offset, impedancias, etc. Para lograr un óptimo desempeño es necesario contemplar todos los parámetros de un circuito analógico para saber exactamente cuales elementos son responsables de mejorar el diseño. De esta forma, una herramienta de CAD es útil porque es posible mejorar el diseño de circuitos analógicos y obtener muchas soluciones puesto que uno tiene muchas soluciones de las cuáles es posible seleccionar una que cumpla con el desempeño deseado.

En esta tesis se utilizan dos algoritmos evolutivos: NSGA-II y MOEA/D, para optimizar circuitos analógicos a partir de una topología seleccionada tomando en cuenta compromisos de diseño. La optimización se hace a partir del dimensionamiento de cada transistor y no es necesario establecer valores iniciales, solamente es necesario establecer límites para cada variable. Los algoritmos evolutivos son probados con dos diferentes operadores genéticos: cruza y mutación básicas y evolución diferencial. Ambos métodos evolutivos utilizan HSPICE para evaluar los desempeños de los circuitos analógicos usando una tecnología estándar CMOS de 0.35 μm.

Los algoritmos evolutivos son aplicados primero para optimizar celdas de ganancia unitaria con tres objetivos: ganancia, ancho de banda y offset, con diferente número de variables, desde dos hasta nueve variables. Después, la optimización se hace en current conveyors los cuales son construidos con más de veinte transistores pero esta vez son optimizados con cuatro objetivos: resistencias en cada terminal y ganancia. Para todos estos circuitos es posible agregar compromisos de diseño tales como condiciones de saturación.

Ambos métodos evolutivos han sido probados, y es demostrado que los resultados de la optimización estan dentro de los límites establecidos mientras que las funciones objetivo son mejo-

radas. Finalmente, de los resultados de la optimización, un diseñador es capaz de escoger el mejor resultado que cumpla con las especificaciones del circuito.

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Contents

Chapter 1

INTRODUCTION

Nowadays, circuit design has been influenced by a great range of techniques capable to fulfill automatically an entire performance set and even though, automated analog integrated circuit design is becoming a viable solution for increasing design productivity for critical analog components. On one hand, the problem for accomplishing device parameters and performances is, in general, a nonlinear transformation that is not known but it may be simulated [1]. On the other hand, analog synthesis is a challenging problem because the analog design process is characterized by a combination of experience and intuition, and requires creativity to deal with the large number of free parameters and the sometimes obscure interactions between them [2]. Circuit synthesis begins by selecting a specific topology and its design is performed by finding parameter values (e.g. transistor lengths and widths), but this process needs to be improved to satisfy a set of specifications.

Analog synthesis is a complicated task and exist different proposed automated works (topology selection, biasing and sizing devices, layout, optimization or simultaneously more than one of these objectives). Moreover, constraints between design parameters and performances is a difficult work, because it requires the consideration of a variety of technological and structural effects [3]. So a designer could deal with a design problem for which formal design methods and analytical procedures neither can solve. In this manner, the complexity of modern integrated systems has increased, making necessary the development of new CAD tools, so in a next section it will be discussed briefly more about this topic.

For instance, optimization provides a truly general engineering design scheme, different from highly specialized and particular design and synthesis techniques that has begin used in widely disciplines in sciences. Optimization process starts with the formulation of an initial guess solution, normally involving a fixed topology, together with an initial set of values for the design variables, next it is determined the behavior of the system by evaluating the performance features cited in the target specifications. If the specifications are satisfied the process finishes, otherwise , the design variables are adjusted giving rise to a modified circuit and the process is repeated until a satisfactory solution is achieved [4].

There exist a lot of optimization techniques beginning with Pareto Theory (probably the oldest) and continuing with Theory of Games, Neural Networks and Evolutive Algorithms (EA) . From these last, EA's are divided in different kinds or sub-techniques like Ant Colony, Particle Swarm Optimization (PSO) [5] , Non-Dominated Sorting Genetic Algorithm (NSGAII) [6] , Multi-Objective Algorithm Based on Decomposition (MOEA/D) [7] and more. Any of these techniques is useful for optimizing problems and they have been used to simplify and to reduce time of synthesis for constrained non linear circuits [2, 8, 9].

1.1 CAD tools

Today, to keep up with the increasing complexity of electronic systems, researchers have increased analog design automation resulting in the development of different CAD tools like IDAC [10], OASYS [11], OPTIMAN [12], ASTRX/OBLX [13], ANACONDA [14], AMGIE [15], WATSON [16] , CAFFEINE [17] and MOJITO [18] are among the tools which are possible to find in the literature. The most successful tools use optimization techniques to search for the optimal design solution according to some given specifications [19].

A traditional procedure to design analog circuits starts by selecting a topology, then the next step is related to find values for the parameters, finally it is possible to make variations to the schematics, the layout or the optimization. The topology selection could be chosen before, after or during the sizing process and even it is possible to consider a top-down creation or a bottom-up generation by using a hardware description language [20].

It is possible to talk on the Interactive Design for Analog Circuits (IDAC) wich was able to design transconductance operational amplifiers and oversampled A/D converters by sizing a library of more than 40 analog schematics as a function of p-well and n-well CMOS technology. At the beginning, the user has to specify technology, limits values of the electrical parameters of elements (transistors, resistors, capacitors), temperature and supply voltage values. At the end it is possible to compare the different characteristics of various schematics and depending on the kind of device, to compare a frequency or transient response [10].

An analog synthesis system (OASYS) attempts to translate a set of functional specifications into a sized circuit schematic using detailed analog domain knowledge. This could be considered a top-down process due to its three steps algorithm to make a design [11]. ANACONDA [14] combines ideas from evolutionary algorithms and numerical pattern search beginning with a fixed topology the circuit is mapped to a constrained optimization problem using the basic synthesis formulation from ASTRX/OBLX [13]. Then it is invoked a sequence of detailed circuit simulations for each evaluation during the numerical search. Other tool is a fully integrated analog synthesis environment called AMGIE [15] which gathers for cell and tech libraries using engines such as ISAAC [21] for a symbolic analysis of ac equations and DONALD [22] like equation manipulation tool, also, making topology selection, sizing and verification (using SPICE) and optimizing with OPTIMAN [12], and finally generating the layout of the circuit automatically by LAYLA [23] tool.

Following the idea of using genetic algorithms based on transistor-level circuit simulations, WATSON [16] generates new sets of design variables using SPICE as simulator linking it into the framework to evaluate the performance of the given circuit. The performances (operating point, small-signal, noise and/or transient analysis) are extracted from the simulations and passed to a next optimization stage using an accurate reduced-order models which fit to a Pareto front [3]. This tool could be used for a system-level design due to the trade-offs between different blocks into a circuit. Finally this tool is useful also because it is possible to find the values of the design variables of a specific topology such that all specifications are met. In other hand CAFFEINE [17] resume symbolic models but ensuring that such models are compact and interpretable by using genetic programming, linking SPICE simulations and incorporating a multi-objective evolutionary algorithm (NSGA-II [6]). This program was made on MATLAB platform and makes an analysis of the trade-off between complexity of equations and error of approximations, but finally achieve better results than others techniques like posinomial models [24].

A recent multi-topology and multi-objective sizing tool (MOJITO) [18], continuing the last idea, it is able to optimize circuits by using a topologies library considering thousand of topologies simultaneously and applying NSGA-II as searching engine where the individuals are generated randomly choosing a value for each parameter using a uniform distribution, using SPICE for performance calculation and making possible to find trade-offs between different performances.

From the CAD tools described above, it is clear that automatic analog circuit sizing is a problem of major importance to the Electronic Design Automation (EDA) and System-On-Chip design industry [18], and it can be appreciated that EDA have generated best results by incorporating Multi-objective evolutionary algorithms like NSGA-II and others, and all optimization process are into simulated-based circuit optimization methods which main idea is to select a circuit and performance specifications, and further the sizes and biasing of all devices have to be determined such that the circuit meets the specifications [19].

1.2 Objectives

In this work there are used two evolutionary algorithms (NSGA-II and MOEA/D) to optimize analog circuits taking into account constraints and linking SPICE like a simulator to measure performances. In this manner, the proposed optimization system includes a multiobjective optimization . The input stage is introduced by a netlist and the output stage generates a set of optimized solutions which are into feasible limits of target optimization. Then the general objective of this work is focused on the optimization of analog circuits by applying evolutionary methods.
The particular objectives are:

- Use two evolutionary techniques: NSGA-II and MOEA/D with constraints ensuring convergence and making metrics measurement for each one.
- Develop an automatic system able to optimize circuits by applying NSGA-II and MOEA/D.
- Test the optimizers on different circuits such as unity gain cells, over different bias conditions for the same number of generations and perform cross-over/mutation operations among the parameters to compare NSGA-II and MOEA/D performances.
- Optimize a big circuit as a current conveyor, with both optimizers for the same number of generations and cross-over/mutation parameters to compare NSGA-II and MOEA/D performances.

- Compare performances among both evolutionary techniques and make a conclusion about their behaviors.

1.3 Thesis organization

This work is structured as follows:

In Chapter 2 the basics on multiobjective optimization are described and NSGA-II and MOEA/D algorithms are explained. At the end of this chapter both methods are evaluated with some test functions and a comparison is performed from the metrics of each algorithm.

The Chapter 3 exposes the proposed system and some details about the genetic operators used in this work, the characteristics of the initialization process, how it is possible to use the circuits simulator for measure the electrical parameters and is depicted the flow diagram of both optimizers: NSGA-II optimizer and MOEA/D optimizer.

The Chapter 4 consists of two parts: in the first six different circuits are optimized under three different bias currents and with three different values of length of transistors with the objective to select among all the combinations the best design. In the second stage each one of the best selected designs is optimized over more generations to achieve a better optimization for each one. Both stages works with three objectives (gain, band width and offset) and at the end it is a comparative analysis among both methods.

The Chapter 5 is devoted to perform an exhaustive optimization of current conveyors for four objectives (each one of parasitic resistances) by preserving saturation conditions of the transistors and a closer unity gain in each terminal.

Finally in Chapter 6 are the conclusions of this work.

Chapter 2

MULTIOBJECTIVE OPTIMIZATION

In this chapter are explained the basics on multi-objective optimization and two multi-objective optimization methods are described: Non-Dominated Sorting Genetic Algorithm (NSGA-II) and Multi-Objective Algorithm Based on Decomposition (MOEA/D).

2.1 Multiobjective optimization basics

Lets consider a multiobjective optimization problem (MOP) [25] of the form:

$$\text{minimize } F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x}))^T \tag{2.1}$$

$$\text{subject to } \mathbf{x} \in X$$

where $\mathbf{x} = (x_1, \ldots, x_n)$ is called decision vector, $X \subset \mathbb{R}^n$ is the decision space for the variables, $f_i : \mathbb{R}^n \to \mathbb{R}, i = 1, \ldots, m (m \geq 2)$ are objective functions. $F(\mathbf{x})$ is the objective vector. The attainable objective set is defined as the set $\{F(\mathbf{x}) \mid \mathbf{x} \in X\}$. If $\mathbf{x} \in \mathbb{R}^n$ all objectives are continuous and X is described by:

$$X = \{\mathbf{x} \in \mathbb{R}^n \mid h_j(\mathbf{x}) \leq 0, j = 1, \ldots, m\} \tag{2.2}$$

where h_j are continuous functions and Eq.(2.2) is called a *continuous MOP*. Very often , since the objectives in (2.1) contradict each other, no point in X maximizes all the objectives simultaneously. One has to balance them, the best tradeoffs among the objectives can be defined in terms of Pareto optimality.

The key in multiobjective optimization is the Pareto optimality [6, 7, 26]. A solution is considered as optimal if it can not be improved without deterioration to at least one of its components, then it is probably that there will be more than one Pareto optimal solution and the multiobjective optimization problem finishes when the Pareto optimal set is found.

It is possible to define the Pareto dominance then as $\mathbf{x}_i \prec \mathbf{x}_j$ ($\mathbf{x}_i$ *dominates to* $\mathbf{x}_j$) if all $f_n(\mathbf{x}_i)$ in $F(\mathbf{x}_i)$ are equal or better than all $f_n(\mathbf{x}_j)$ in $F(\mathbf{x}_j)$ and at least one $f_n(\mathbf{x}_i)$ is better than $f_n(\mathbf{x}_j)$, where better means less when the objective is minimize and high when the objective is maximize.

2.1.1 Optimization approaches

There are a lot of optimization methods and it is possible to classify them in different ways, reference [25] proposes the next classification:

- No preference: using a relative simple method and the user opinions are not taken into consideration and at the end it is presented only one solution.

- A posteriori: the idea is to find the Pareto optimal set and after this the solutions are presented to manually select the most preferred solution, examples of these methods are: Weighting Method, ϵ-Constraint, Weighted Metrics and Scalarizing Method.

- A priori: in this kind of methods the user must specify preferences, hopes and opinions, the disadvantage is that generally the user does not know beforehand what is possible to attain in the problem, examples of these methods are: Lexicographic Ordering and Goal Programming.

- Interactive: here are discriminated the solutions which are not useful to the user avoiding overload information, examples of these methods are: Geoffirion-Dyer-Feinberg Method, GUESS Method, Satisfacing Trade-Off Method, Tchebycheff Method, Referent Point Method, Light Beam Search and NIMBUS.

2.2 Non-dominated Sorting Genetic Aslgorithm (NSGA-II)

This algorithm achieve the Pareto front of a multiobjective problem by sorting and ranking all solutions in order to choose the better solutions to make a new offspring, this means, by ranking all the population in different Pareto sub-fronts that it will be possible to know which solutions show better performance. In this algorithm is contemplated a way to choose the best solution between two solutions in the same sub-front preserving diversity, in this form it is possible to select the best part of a population without losing diversity. Then NSGA-II is based on two main procedures : *Fast Nondominated Sort* and *Crowding Distance Assignment.* These two procedures ensure elitism and it is possible to add constraints to ensure that the solutions are feasible.

2.2.1 NSGA-II algorithm

At first it is necessary to randomly initialize the parameters and start by building two populations (P_o and Q_o) each one of size N, from random values into a feasible region. The NSGA-II procedure in each generation consists of rebuilding the current population (R_t) from the two original populations (P_t and Q_t) then the new size of current population will be of g.

Now through a nondominated sorting all solutions in R_t are ranked and cataloged in a family of sub-fronts, this procedure will be explained later. In the next step is necessary to create from the current population R_t (previously ranked and ordered by sub-front number) a new offspring (P_{t+1}), the objective will be to choose from a population of size $2N$, N solutions which belong to the first sub-fronts. In this manner the last sub-front could be greater than it is necessary, then it is used a measure ($i_{distance}$) that allows to identify the better solutions and preserving elitism by selecting the solutions that are far the rest, this is possible simply by modifying a little bit the concept of pareto dominance as follows:

$$i \prec_n j \text{ if } \Big[(i_{rank} < j_{rank}) \text{ or } (i_{rank} = j_{rank})\Big] \text{ and } (i_{rank} > j_{rank})$$

In Fig. 2.1 [6] is shown all the NSGA-II procedure where is easy to understand how nondominated sorting and crowding distances help to generate new offsprings. Then the main procedure could be summarized like:

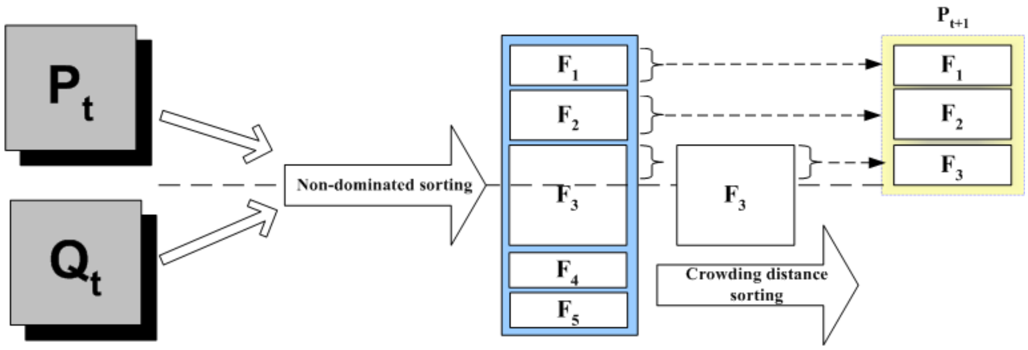


Figure 2.1: NSGA-II procedure

Algorithm 1 NSGA-II Algorithm

P_0=random, Q_0=random //initialize
t=0
$P_{t+1} = \emptyset$ and $i = 1$
repeat
  $R_t = P_t \cup Q_t$ //combine parent and offspring population
  F= fast-non-dominated-sort(R_t) //$F = (F_1, F_2, ...)$, all nondominated fronts of R_t
  crowding-distance-assignment(F_i) //calculate crowding-distance in F_i
  repeat
    $P_{t+1} = P_{t+1} \cup F_i$ //include ith nondominated front in the parent pop
    $i = i + 1$ //include ith nondominated front in the parent pop
  until $|P_{t+1}| + |F_i| \leq N$ //until the parent population is filled
  Sort(F_i,$\prec_n$) //sort in descending order using $\prec_n$
  $P_{t+1} = P_{t+1} \cup F_i[1 : (N - |P_{t+1}|)]$ //choose the first $(N - |P_{t+1}|)$ elements of F_i
  Q_{t+1}=make-new-pop(P_{t+1}) //use genetic operators to create it
until stop criteria

2.2.2 Fast nondominated sort

This procedure is responsible to rank each solution into a sub-front, and starts by selecting the nondominated solutions among the current population (R_t), this first group of solutions will be labeled as the solutions into the first sub-front (F_1) and are separated from R_t. For the remaining solutions in R_t are selected the nondominated solutions again but this time they are labeled into the second sub-front (F_2) and separated from R_t like the solutions in (F_1) were separated before. This procedure continues until all solutions in R_t are ranked into a sub-front.

The procedure uses a counter for each solution, such counter allows to know how many solutions dominate to each solution (n_p where p is the *p-solution*). In the same way there is a set who contains all the solutions dominated for each solution (all solutions in S_p are dominated by *p-solution*). First are taken the solutions with counter equal to zero and to each solution in their set of dominated solutions are diminished their counter in one. In this way the next sub-front is composed by the remaining solutions with counter equal to zero. This continues until all solutions have been ranked. The procedure is performed in Algorithm 2 as follows [6]:

Algorithm 2 Fast Nondominated Sort Algorithm

for each $p \in P$ **do**
    $S_p = \emptyset$
    $n_p = 0$
    for each $q \in P$ **do**
        if $(p \prec q)$ **then**
            $S_p = S_p \cup \{q\}$
        else if $(q \prec p)$ **then**
            $n_p = n_p + 1$
        end if
        if $n_p = 0$ **then**
            $p_{rank} = 1$
            $F_1 = F_1 \cup \{p\}$
        end if
    end for
end for
i=0
while $F_i \neq \emptyset$ **do**
    $Q = \emptyset$
    for each $p \in F_i$ **do**
        for each $q \in S_p$ **do**
            $n_q = n_q - 1$
            if n_q=0 **then**
                $q_{rank} = i + 1$
                $Q = Q \cup \{q\}$
            end if
        end for
    end for
    $i = i + 1$
    $F_i = Q$
end while

2.2.3 Crowding distance asssignment

This is the second procedure to help to select solutions which will generate the offspring, and has sense where is necessary to choose the last members of the population P_{t+1} into a sub-front, because all sub-front members then have other ranking parameter into their sub-front. The main idea is to perform a density estimation named *crowding distance* ($i_{distance}$) by sorting in ascending order the solutions for each one of the objective functions, then for each objective it is first selected the smallest and largest limit found and an infinite value is assigned to their crowding distances.

Next it is calculated for the rest of the functions values their crowding distance, that is as follows: with the sorted objective values is going to take the immediately upper and lower function value of each one of the objective values, and the difference of these two values is normalized (using the smallest and largest of current function value). This procedure continues and the sum of all the normalizations of each objective will be the crowded distance of current solution. This procedure is shown in Algorithm [6]:

Algorithm 3 Crowding Distance Assigment

$l = |T|$ //number of solutions in T
for each i **do**
  set $T[i]_{distance} = 0$ //number of solutions in T
  for each objective m **do**
    T= sort(T, m) //sort using each objective value
    $T[1]_{distance} = T[l]_{distance} = \infty$ //boundaries are always selected for all other points
    for $i = 2$ to (l-1) **do**
      $T[i]_{distance} = T[i]_{distance} + (T[i+1] \cdot m - T[i-1] \cdot m)/(f_m^{max} - f_m^{min})$
    end for
  end for
end for

2.3 Multiobjective Evolutionary Algorithm Based on Decomposition (MOEA/D)

The basic idea of MOEA/D is the decomposition of a multiobjective problem in scalar optimization subproblems by a *weights vector*, this vector associates a weight (λ) for each subproblem where each subproblem is every single individual in the population which are going to try to improve itself and to its nearby (*neighborhoods*) .

After the initialization of the parameters the first step in MOEA/D is related to define the N spread weights vector (to each individual corresponds one λ_i) and one way to determine each spread weight vector can be by a parameter H in a sequence as in 2.3 [26]:

$$\left\{\frac{0}{H}, \frac{1}{H}, \ldots, \frac{H}{H}\right\} \tag{2.3}$$

Therefore, for $m = 2$, $N = H + 1$, but for $m > 2$ the number of such vectors is:

$$N = C_{H+m-1}^{m-1} \tag{2.4}$$

Now for each individual of the population corresponds one λ_i, in this condition it is possible to define a number (T) of neighborhoods for each λ_i. Then it is necessary to calculate the Euclidean distance between each λ_i, finally for each λ_i is going to be (T) neighborhoods nearby and they will be saved in B.

In each generation there is a population of N points $\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^N \in X$ where $\mathbf{x}^i$ is the current solution to the i_{th} subproblem and there are $FV^1, FV^2, \ldots, FV^N$, where FV^i is the F-value of $\mathbf{x}^i$.

In the procedure it is necessary to generate a new individual $\mathbf{y}$ who will be compared with all its neighborhood by applying a decomposition approach ($g[\mathbf{x} \mid \lambda^j]$) such as *Tchebycheff or Weighted Sum Approach* and each neighbor worse than this new individual will be replaced by it.

2.3.1 Decomposition methods

The multiobjective optimization problem tries to find the Pareto optimal front, and sometimes is useful to try the problem like a vector optimization [25], but there exists several ways to do it. This work selects two different performances: *Weighted Sum Approach* and *Tchebycheff Approach*:

- *Weighted Sum Method*

In Weighted Sum Method, the problem is solved by Eq. (2.5):

$$\text{minimize } g^{WS}(\mathbf{x} \mid \lambda) = \sum_{i}^{m} \lambda_i f_i(\mathbf{x}) \tag{2.5}$$

$$\text{subject to } \mathbf{x} \in X$$

Where $\lambda \geq 0$ for all $i = 1, 2, ..., m$ and $\sum_i^m \lambda_i = 1$. X is the set of feasible limits.

- *Tchebycheff Approach*

In this approach, the scalar optimization problem is shown in the next equation:

$$\text{minimize } g^{TE}(\mathbf{x} \mid \lambda, \mathbf{Z}^*) = \max \{\lambda_i \mid f_i(\mathbf{x}) - z_i^* \mid\}_{1 \leq i \leq m} \tag{2.6}$$

$$\text{subject to } \mathbf{x} \in X$$

Where $\mathbf{Z}^* = (z_1^*, z_2^*, \ldots, z_m^*)^T$ is a reference point like the best objective functions found and if the objective is minimize then $z_i* = \min \{f_i(\mathbf{x}) \mid \mathbf{x} \in X\}$ for each $i = 1, \ldots, m$.

Algorithm 4 MOEA/D Algorithm

build an uniform spread of N weight vectors //$\lambda_1, \lambda_2, \ldots, \lambda^N$
$B(i) = \{i_1, i_2, \ldots, i_T\}$ // neighboring each weighted vector
$t = 1$, POP=random() , set $E = \emptyset$, T //initialize
repeat
  for $i = 1, 2, \ldots, N$ **do**
    randomly select parents from $B(i)$
    generate new individual y //using genetic operators
    for each $j \in B(i)$ **do**
      if $g(\mathbf{Y} \mid \lambda^j) \leq g(\mathbf{x} \mid \lambda^j)$ **then**
        $\mathbf{x}_i = \mathbf{Y}$ //if $\mathbf{Y}$ is better than $\mathbf{x}^i$ replace the new solution
        $FV^j = F(\mathbf{Y})$
      end if
    end for
  end for
  remove from EP all vectors dominated by $F(\mathbf{Y})$
until stop criteria

2.3.2 MOEA/D algorithm

In Algorithm 4 is shown the MOEA/D algorithm:

In this manner MOEA/D provides a simple way of performance decomposition into a multiobjective evolutionary algorithm preserving diversity due to decomposition in N scalar subproblems keeping lower computational complexity [27].

2.4 Constraints and Metrics

2.4.1 Constraints

The constraint-handling method used in this work is based in tournament selection proposed in [6] and consists in picking up from the population and the better is chosen. In the presence of constraints, each solution can be either feasible or infeasible. Thus, there may be at most three situations: both solutions are feasible, one is feasible and other is not or both are infeasible. Then to choose the better solution, the definition of *domination* between two solutions i and j

is:

> A solution i is said to be constrained-dominated in a solution j, if any of the following conditions is true:
>
> 1. Solution i is feasible and solution j is no.
> 2. Solutions i and j are both infeasible, but solution i has a smaller overall constraint violation.
> 3. Solutions i and j are feasible and solution i dominates solution j.

This new concept of constrained-domination could be applied for NSGA-II and for MOEAD.

2.4.2 Metrics

In multiobjective optimization there are two goals : 1) convergence to the Pareto-optimal set and 2) maintenance of diversity in solutions of the Pareto-optimal set. These two tasks cannot be measured adequately with one performance metric [6].

- Set Convergence metric (C-metric)

Let A and B be two approximations to the PF of a MOP. $C(A, B)$ is defined as the percentage of the solutions in B that are dominated by at least one solution in A [7]:

$$C(A, B) = \frac{\mid \{u \in B \mid \exists v \in A : u \text{ dominates } v\} \mid}{\mid B \mid} \tag{2.7}$$

$C(A, B)$ is not necessarily equal to 1-$C(B, A)$. $C(A, B) = 1$ means that all solutions in B are dominated by some solutions in A.

- Distance form Representatives in the PF (D-metric)

Let P^* be a set of uniformly distributed points along the true Pareto Front. Let A be an approximation to the PF, the average distance from P^* to A is defined as [7]:

$$D(A, P^*) = \frac{\sum_{v \in P^*} d(v, A)}{\mid P^* \mid} \tag{2.8}$$

Where $d(v, A)$ is the minimum Euclidean distance between v and the points in A. The problem here is that it is necessary to know the true Pareto Front then in this work it will be used only for the Test Functions.

2.5 Test problems

Zitzler *et al.* [28], provided a comparison of various evolutionary approaches to multiobjective optimization using six carefully chosen test functions, each one involves a particular feature that is known to cause difficulty in the evolutionary optimization process, mainly in converging to the Pareto-optimal front, then in Table 2.1 are shown the test functions used in this work, where $ZDT1, ZDT2, ZDT3, ZDT4$ and $ZDT6$ are the name of the selected functions, n is the number of variables used for the function $(x_1, x_2, \ldots, x_n)$ and bounds are the limit of maximum and minimum allowed for each variable. For the five selected functions the objective is to be minimized.

The test function $ZDT1$ has a convex Pareto-optimal front, $ZDT2$ is the nonconvex contrapart to $ZDT1$. $ZDT3$ represents the discreteness feature, its Pareto-optimal front consist of several noncontiguous convex parts. $ZDT4$ contains 21^9 local Pareto-optimal fronts then its ability to deal with multimodality. $ZDT5$ was not selected because represents a binary string. Finally $ZDT6$ includes two difficulties caused by the nonuniformity of the search space: the Pareto-optimal solutions are nonuniformly distributed along the Pareto front and the density of the solutions is slowest near the Pareto-optimal front and highest away from the front.

For constrained problems have been studied three test functions $CONSTR$, SRN and TNK. The first one ($CONSTR$) a part of the unconstrainded Pareto-optimal region is not feasible. The second problem (SRN), the coinstrained Pareto-optimal set is a subset of the unconstrained Pareto-optimal set. The third problem (TNK) has a discontinuous Pareto-optimal region. This is shown in Table 2.2.

2.6 Test function results

To check the two methods (NSGAII and MOEA/D) they will be tested and compared with the test functions explained before, all functions will be tested with N=100 and 250 generations, the cross-over probability is 0.9 and mutation probability is 0.05. In Figs. 2.2 and 2.3 are shown the results of optimization for each method where in the left side are displayed the NSGA-II results and in the right side are displayed the MOEA/D results. In Table 2.3 are shown the

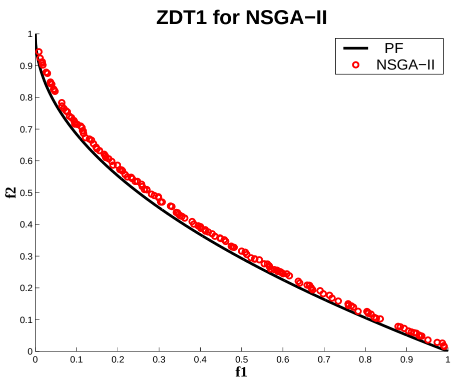
Table 2.1: Test Functions

Function	n	Bounds	Functions	Optimal Sol.
$ZDT1$	10	$x_i \in [0,1]$	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = g(\mathbf{x})[1 - \sqrt{\frac{x_1}{g(\mathbf{x})}}\ ]$ $g(\mathbf{x}) = \frac{9}{n-1}\ \sum_{i=2}^{n} x_i +\ 1$	$x_1 \in [0,1]$ $x_i = 0,$ $i = 2, \ldots, n$
$ZDT2$	12	$x_i \in [0,1]$	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = g(\mathbf{x})[1 -\ (\frac{x_1}{g(\mathbf{x})})^2\ ]$ $g(\mathbf{x}) = \frac{9}{n-1}\ \sum_{i=2}^{n} x_i +\ 1$	$x_1 \in [0,1]$ $x_i = 0,$ $i = 2, \ldots, n$
$ZDT3$	12	$x_i \in [0,1]$	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = g(\mathbf{x})[1 - \sqrt{\frac{x_1}{g(\mathbf{x})}} - \frac{x_1}{g(\mathbf{x})}\sin(10\pi x)]$ $g(\mathbf{x}) = \frac{9}{n-1}\ \sum_{i=2}^{n} x_i +\ 1$	$x_1 \in [0,1]$ $x_i = 0,$ $i = 2, \ldots, n$
$ZDT4$	10	$x_1 \in [0,1]$ $x_{2,\ldots,n} \in [-5,5]$	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = g(\mathbf{x})[1 - \sqrt{\frac{x_1}{g(\mathbf{x})}}\ ]$ $g(\mathbf{x}) = 1 + 10(n-1) + \sum_{i=2}^{n}[x_i^2 - 10\cos(4\pi x_i)]$	$x_1 \in [0,1]$ $x_i = 0,$ $i = 2, \ldots, n$
$ZDT6$	10	$x_i \in [0,1]$	$f_1(\mathbf{x}) = 1 - \exp(-4x_1)\ \sin^6(6\pi x_1)$ $f_2(\mathbf{x}) = g(\mathbf{x})[1 -\ (\frac{x_1}{g(\mathbf{x})})^2\ ]$ $g(\mathbf{x}) = 9\,[\frac{\sum_{i=2}^{n} x_i}{n-1}]^{0.25} +\ 1$	$x_1 \in [0,1]$ $x_i = 0,$ $i = 2, \ldots, n$

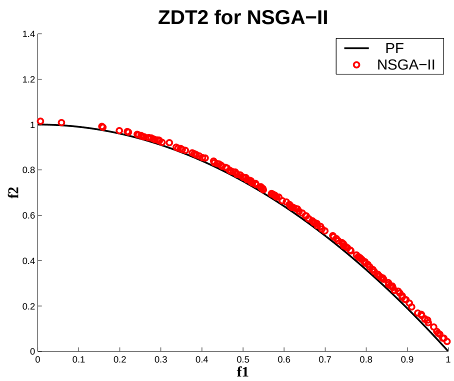
metrics measured for NSGAII and MOEAD for each test function, where A is NSGA-II performance and B is MOEA/D performance:

Table 2.2: Constrained Test Functions

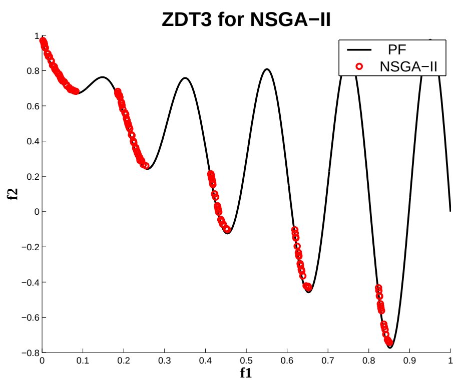
Function	n	Bounds	Functions	Constraints
CONSTR	2	$x_1 \in$ [0.1,1] $x_2 \in$ [0,5]	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = \frac{x_2+1}{x_1}$	$g_1(\mathbf{x}) = 9x_1 + x_2 \geq 6$ $g_2(\mathbf{x}) = 9x_1 - x_2 \geq 1$
SRN	2	$x_i \in$ [-20,20] $i = 1, 2$	$f_1(\mathbf{x}) = (x_1 - 2)^2 + (x_2 - 1)^2 + 2$ $f_2(\mathbf{x}) = 9x_1 - (x_2 - 1)^2$	$g_1(\mathbf{x}) = x_1^2 + x_2^2 \leq 225$ $g_2(\mathbf{x}) = x_1^2 - 3x_2^2 \leq -10$
TNK	2	$x_i \in [0,\pi]$ $i = 1, 2$	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = x_2$	$g_1(\mathbf{x}) = 1 - x_1^2 - x_2^2+$ $0.1 \cdot \cos(16 \arctan(\frac{x_1}{x_2})) \leq 0$ $g_2(\mathbf{x}) = (x_1 - 0.5)^2+$ $(x_2 - 0.5)^2 \leq 5$



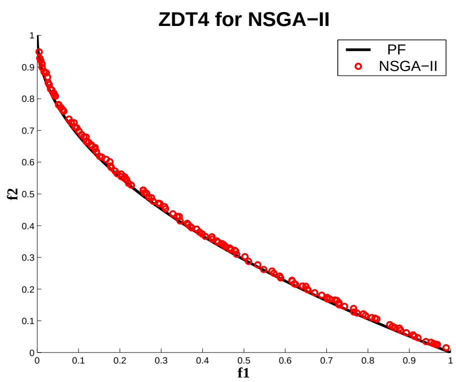
(a) ZDT1 for NSGA-II.



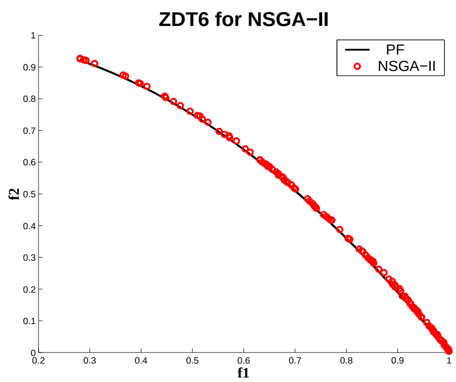
(b) ZDT2 for NSGA-II



(c) ZDT3 for NSGA-II

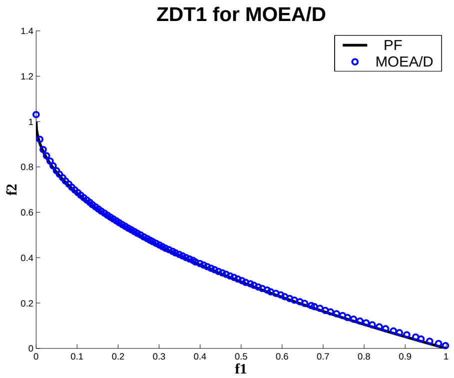


(d) ZDT4 for NSGA-II

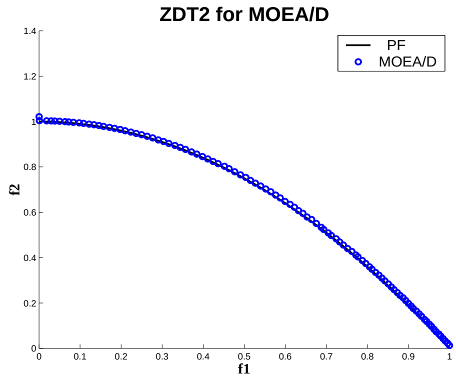


(e) ZDT6 for NSGA-II

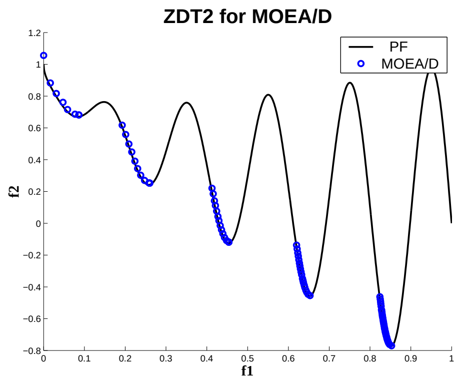
Figure 2.2: NSGA-II performances for ZDT Functions



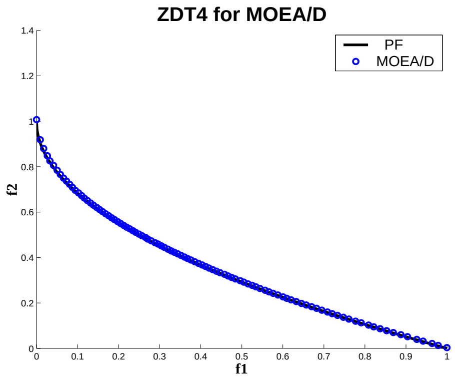
(a) ZDT1 for MOEA/D



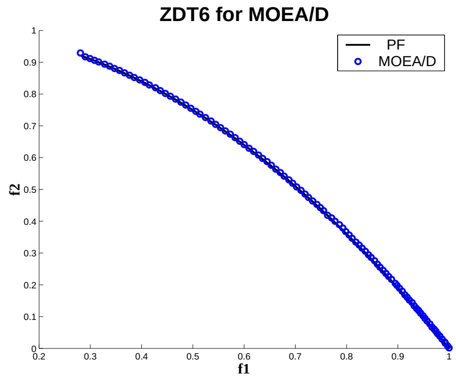
(b) ZDT2 for MOEA/D



(c) ZDT3 for MOEA/D



(d) ZDT4 for MOEA/D



(e) ZDT6 for MOEA/D

Figure 2.3: MOEA/D performances for ZDT Functions

For constraint functions in Fig. 2.4 is shown a comparative results for NSGAII and MOEA/D for three test functions explained before. Here is important to note that NSGAII performance has improved their operation with respect to MOEA/D. In Table 2.4 there are C-Metric and D-Metric for both performances on constraint functions where A is NSGA-II performance and B is MOEA/D performance.

Table 2.3: C-Metric and D-Metric for Test Functions, A is NSGA-II and B is MOEA/D

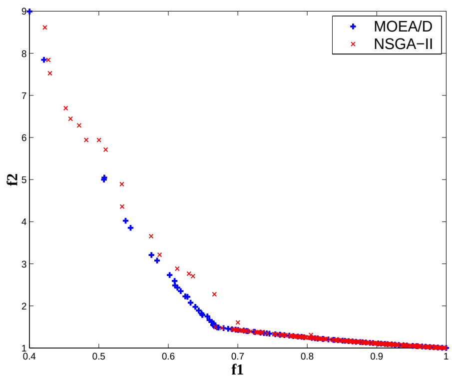
Test Function	C-Metric		D-Metric	
	$C(A,B)$	$C(B,A)$	A	B
$ZDT1$	0.04	0.8421	4.8991×10^{-13}	1.1522×10^{-10}
$ZDT2$	0.01	0.85294	3.9457×10^{-14}	2.057×10^{-13}
$ZDT3$	0.06	0.53913	9.9317×10^{-14}	1.03×10^{-13}
$ZDT4$	0.06	0.42254	7.8346×10^{-21}	3.45×10^{-6}
$ZDT6$	0.07	0.35878	9.201×10^{-14}	3.45×10^{-4}

Table 2.4: C-Metric and D-Metric for Constraint Test Functions, A is NSGA-II and B is MOEA/D

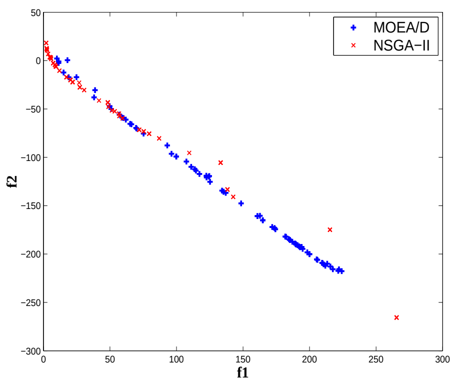
Test Function	C-Metric		D-Metric	
	$C(A,B)$	$C(B,A)$	A	B
$CONSTR$	0.17528	0.052	1.8047×10^{6}	1.379×10^{6}
SRN	0.06097	0.22	5.5983×10^{25}	9.5930×10^{23}
TNK	0.064	0.33929	1.0595×10^{-8}	5.0974×10^{-8}

From Fig. 2.3 it is possible to note that MOEA/D achieve better Pareto-optimal front further for $ZDT1$ and $ZDT2$, indeed from Table 2.3, C-Metric confirm it. However, on the other hand D-Metric tell us how NSGA-II is better because has less distance among the found points.

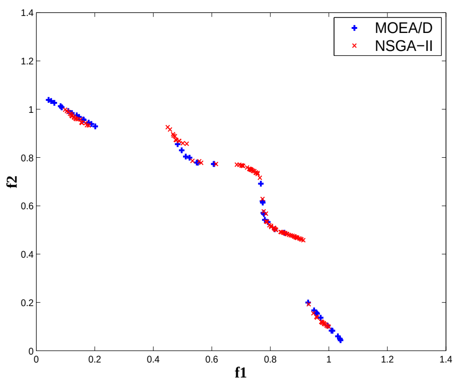
From Fig. 2.4 it is possible to note that NSGA-II improves its performance, achieves better Pareto-optimal front and furthermore for SRN could find points where MOEA/D could not. From Table 2.4, C-Metric tell us how MOEA/D percentage dominance over NSGA-II has diminished and D-Metric is similar for both.



(a) NSGA-II and MOEA/D for CONSTR



(b) NSGA-II and MOEA/D for SRN



(c) NSGA-II and MOEA/D for TNK

Figure 2.4: NSGA-II and MOEA/D performances for Constraints Test Functions

Chapter 3

OPTIMIZER SYSTEM

The proposed optimization system works in MATLAB code and the circuit simulations are made with a HSPICE simulator, which is linked to the MATLAB program. The system is able to optimize with NSGA-II or MOEA/D and it is able to change the way on how to generate offsprings by choosing between two different genetic operators: basic cross-over or differential evolution operators, which will be explained with detail. In this part it will be explained the offspring generation, initialization step, evaluation procedure and finally the complete optimization system.

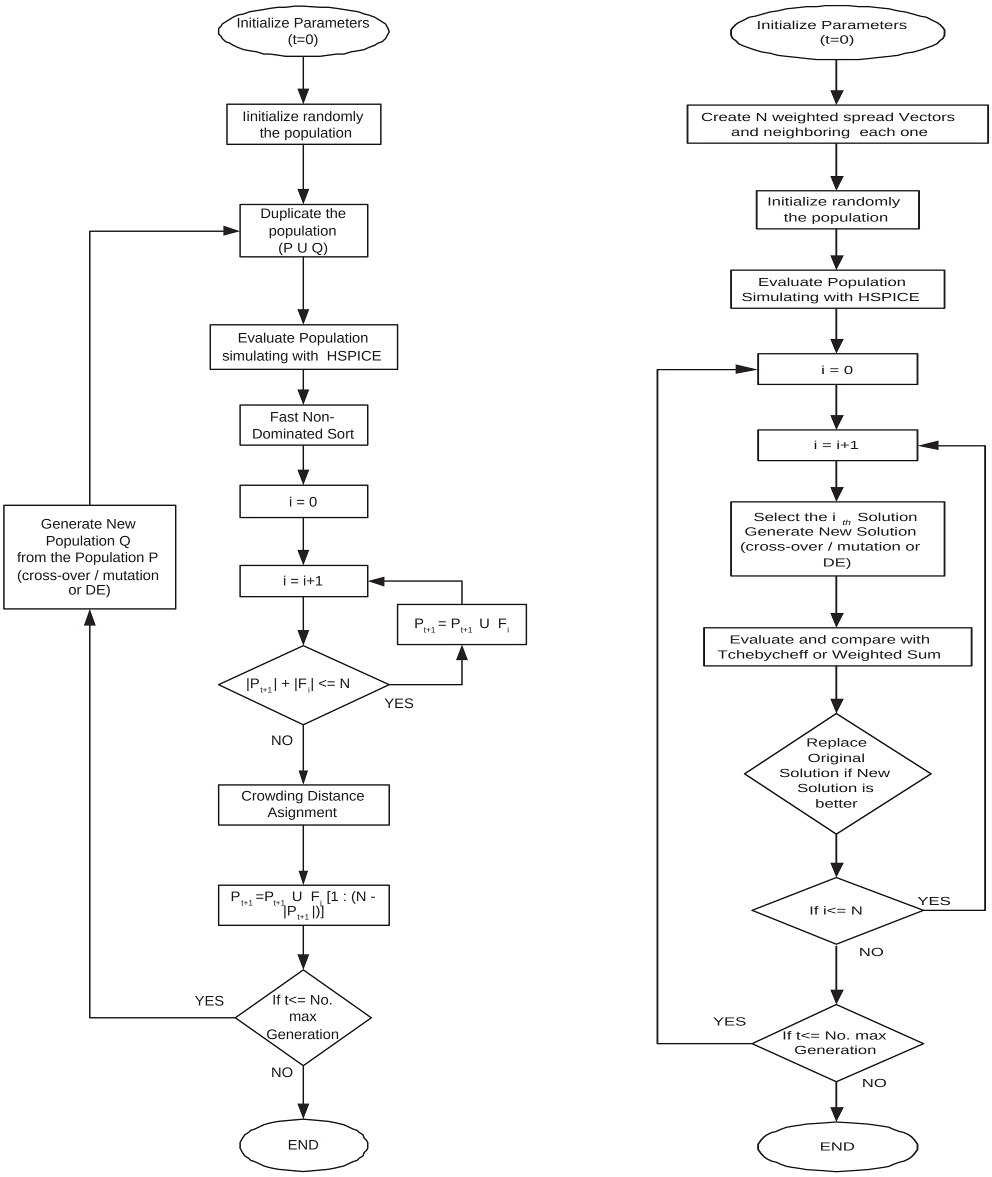
3.1 System description

The system has two general stages: Initialization and Optimization, but its structure is different depending on the optimization engine, then in the Fig. 3.1 are depicted the flow diagrams for both optimization engines.

In Fig. 3.1, t denotes the current number of generations and N the number of solutions in each generation. In case of NSGA-II optimizer i represents the current sub-front and for MOEA/D optimizer i represents the current solution. It is possible to see how in the NSGA-II optimizer the process is performed with the whole the population, and in contrast, in MOEA/D optimizer the process is with each solution.

In the initialization step the parameters (number maximum of generations, number of objectives, solutions and variables, probability of cross-over and mutation, C, R, etc.) exposed in the previous section, it is necessary to initialize the variable t to identify the current generation into the sizing process with this counter.

In Fig. 3.1(a) is possible to appreciate how it is necessary in each generation to evaluate the population by linking HSPICE by modifying each transistor width required and recollecting these results from the output listing , next to make the Fast-nondominated sort, create the new



(a) NSGA-II Optimizer Flow Diagram

(b) MOEA/D Optimizer Flow Diagram

Figure 3.1: NSGA-II and MOEA/D Circuit Optimizer Flow Diagram

population (P_{t+1}) with the firsts sub-fronts, the last sub-front needed to full-fill the new population is assigned with a Crowding Distance and finally is created a new population again (Q_{t+1}) but by applying genetics operators over (P_{t+1}), this process continues until the current generation reach the set maximum number of generations.

In Fig. 3.1(b) the first steps are similar to Fig. 3.1(a) but this time it is necessary to create N weighted spread vectors in the way exposed in the MOEA/D section. Afterwards, it is necessary to find the neighborhood of each weighted vector, then the initial population is created randomly over the search space and evaluate it (as in Fig. 3.1(a)). But the next step is to take one by one of the solutions into the population and create a new solution from this solution, after it is necessary to evaluate with HSPICE and to recollect theirs performances and decide if replace the current solution with this new solution. This process continues until the whole population is regenerated and evaluated and finally the process finishes until the current generation reaches the set maximum number of generations.

In the next chapters will be test the system with different circuits, all them are simulated with a Level 49 CMOS Technology of 0.35 μm and will be demonstrated the flexibility of the system by testing with different number of variables and objectives and preserving constraints such as saturation condition in all transistors.

3.2 Genetic operators

Evolutionary computation is inspired by Darwinian evolutionary theory which is oriented to recreate biological phenomena by using computers and other artificial systems. For instance, genetic algorithms (GAs), which operates on the principle of survival of the fittest, have the capability to generate new design solutions from a population of existing solutions, and discarding the solutions which have an inferior performance or fitness. GAs begin with an initial collection of random solutions called initial population. Each individual in the population is called chromosome and represents a possible solution to the problem. The population evolves through iterations called generations, and in each generation the individuals are evaluated using an aptitude measure. The next population is formed by descendents created by combining two chromosomes of the current generation using the crossover and the mutation operators.

In this work it is possible to choose between two different genetic operators: cross-over/mutation or differential evolution operator (DE) [29]. Lets us define the current population as:

$$X = \mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^n \tag{3.1}$$

$$\mathbf{x}^i = x_1^i, x_2^i, \ldots, x_k^i \tag{3.2}$$

Where X is the current population and $\mathbf{x}^i$ is each one of the solutions in X, so there are n solutions and x_j^i is each variable into $\mathbf{x}^i$ so that each solution has k variables. Each new generation is formed by applying a genetic operator on one or more than one solutions.

3.2.1 Basic cross-over / mutation

The first genetic operator used in this work is basic cross-over/mutation. A random probability is chosen for cross-over and mutation, the probability for cross-over is higher than the mutation probability.

The cross-over operator consists in selecting randomly two solutions among the population to generate a new solution taking randomly each variable of one of the both solutions. The mutation consists in selecting randomly one solution among the population and randomly perturbing each variable by a factor (σ).

3.2.2 Differential Evolution operator (DE)

The second genetic operator used in this work is the Differential Evolution Operator (DE) [7, 27, 29], and consists of randomly choosing three solutions: $\mathbf{x}^a, \mathbf{x}^b$ and $\mathbf{x}^c$ from X. A new solution $\mathbf{x}_{new} = x_1^{new}, x_2^{new}, \ldots, x_k^{new}$ is generated in the following way:

$$\text{For each } i = 1, 2, \ldots, k$$
$$\mathbf{x}_i^{new} = \begin{cases} x_i^a + R \cdot (x_i^b - x_i^c) & \text{if rand} < \text{C} \\ x_i^a & \text{otherwise} \end{cases} \tag{3.3}$$

Where R is a constant factor which controls the amplification of the diferential variation and C is the cross-over probability.

3.3 Initialization procedure

Initialization process is responsible to initialize all the different parameters of the general process, of the genetic operator and the multi-objective optimization engine such as:

- Parameters of the process:
  - Maximum number of generations.
  - Number of variables (number of transistors to size: $\{W_1, W_2, \ldots, W_k\}$).
  - Number of objectives (electrical parameters to improve: gain, band width, offset, etc.).
  - Limit maximum and minimum of each variable ($\{LimitH_1, LimitH_2, \ldots, LimitH_k\}$ and $\{LimitL_1, LimitL_2, \ldots, LimitL_k\}$).
  - Create the first generation randomly inside thes bound limits.
- Parameters of the genetic operators:
  - For cross-over / mutation : probability of cross-over (90%), probability of mutation (10%) and a perturbing factor σ (5%).
  - For DE: R (50%) and C (100%).
- Parameters of multi-objective optimization engine:
  - For NSGA-II: Number of solutions (N).
  - For MOEA/D: Number of solutions (H, exposed in section 2.3).

The proposed system is based on sizing widths of transistors so that the first step consits to build a SPICE netlist from a previous selected circuit where each transistor width is a variable named W_i, for $i = 1, 2, \ldots, k$, and k is the number of transistors to sized. Furthermore it is possible to optimize all transistors in the analog circuit under design. Next is an example of the netlist as an input to the proposed optimization system:

```
* TEST CIRCUIT
VD VDD 0 DC 1.5
VS VSS 0 DC -1.5
Iref 0 6 DC 50u
M1 1 1 2 VDD MODP L=1u W=W1
M3 2 1 X VDD MODP L=1u W=W2
M5 3 3 VDD VDD MODP L=1u W=W3
. . .
. . .
. . .
.END
```

From this example is important to note how the variables needs to be named in a strict sequence order but does not matter which transistor has which variable.

3.4 Performances measurements

To measure the different performances (objectives) it is necessary to link to the system a circuit simulator such as HSPICE, which is able to make measurements of electrical parameters. According to the circuit topology, the measurement process to compute electrical performances makes a few changes to the circuit, by adding some elements. In this case, for the unity gain cells, unity gain is the main performance parameter and the bandwidth is hoped to be increased but the gain must be conserved close to unity. To compute these performances an AC source is added at the input port and the magnitude is measured at the output port. The other performances that must be considered are input and output resistances, because they contribute to parasitic effects producing losses. The measurement of these parameters is done by applying two-port networks. Using HSPICE, an AC analysis is performed with the statement .MEASURE that defines results on successive simulations [30]. Measure instruction prints user-defined electrical specifications of a circuit, and the results could be manipulated in a post-processing step. The .MEASURE basic syntaxes is [31]:

$$.MEASURE < DC|AC|TRAN > resultname$$
$$+ TRIG...TARG...$$

where $resultname$ is the name chosen to save results, $TRIG$ identifies the beginning of trigger specifications and $TARG$ identifies the beginning of target specifications. The syntax could be different depending of the type of measurement, and it is possible to use the next syntaxes too:

$$.MEASURE < DC|AC|TRAN > resultname$$
$$+ func \quad out_{variable} < FROM = val >< TO = val >$$

where $func$ is a specific function such as MIN, MAX or AVG, $out_{variable}$ is name of any output variable whose function ($func$) is measured through simulation. Finally, it is possible to create a library which contains specific measurements for AC or DC analysis, next is shown an example of such library:

```
.LIB MEASLIB                                                          * Library name
    .AC dec 100 100 1G                                                * Execute an AC Analysis
    .TF V(X) VIN                                                      * Calculates DC parameters
    .NET V(X) VIN                                                     * Calculates AC parameters
    .MEAS AC AV MAX Vdb(X) FROM=100 TO=100                            *Calculating Gain in db
    .MEAS AC bwy TRIG vdb(X) at=100 TARG Vdb(X) VAL='av-3' cross=1    *Calculating f_{-3db}
    .MEAS AC AVLin MAX V(X) FROM=100 TO=100                           *Calculating Linear Gain
    .MEAS AC zin MAX zin(mag) FROM='bwvf / 10' TO='bwvf / 10'         *Calculating Z_{in} in f_{-3db}/10
    .MEAS AC Zi2 MAX zin(mAG) FROM=100 TO=100                         *Calculating Z_{in} in 100 Hz
    .MEAS AC ZOut MAX zOUT(mAG) FROM=100 TO=100                       *Calculating Z_{out} in 100 Hz

    .DC VIN 1.5 -1.5 .01                                              *Execute a DC Analysis
    .MEAS DC OFFSET TRIG V(x) at=0 targ v(x) val=0 cross=1            *Calculating offset
.ENDL MEASLIB
```

Chapter 4

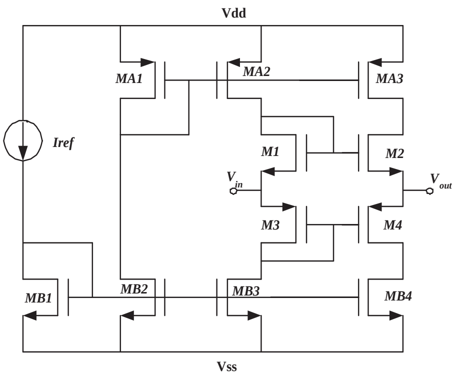
CIRCUIT OPTIMIZATION

It were selected six circuits already synthesized in [2] to be optimized: three voltage followers (VF_A, VF_B and VF_C) and three voltage mirrors (VM_A, VM_B and VM_C) [8]. In Fig. 4.1 are shown these selected circuits which are going to be optimized by NSGA-II and MOEA/D. Finally, the results will be presented in different forms to understand the different interactions among the objective functions.

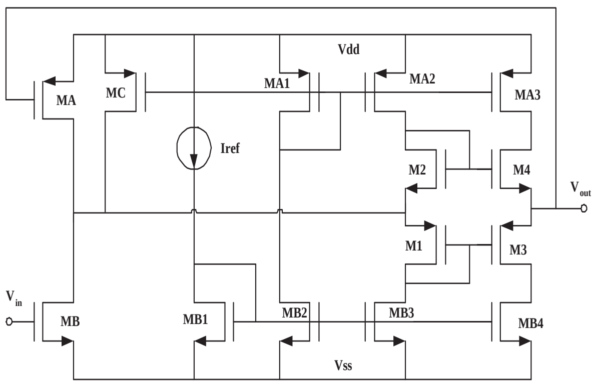
There are two optimization stages: in the first one each circuit is biased with three different currents I_{ref} (10μA, 50μA and 100μA) and for two different transistor lengths for all transistors (0.7μm and 1μm). In the second stage are selected five circuits with the best performances and are tested under the same bias conditions ($I_{ref} = 50\mu A$) and the same transistor length (1μm). For each electrical measurements there is a load capacitor of 1pF.

4.1 First Stage

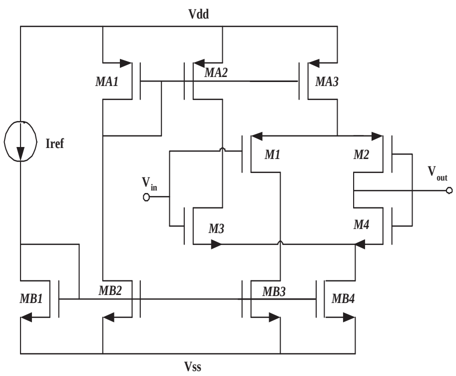
In this stage the six circuits shown in Fig. 4.1 are under three different current bias (10μA, 50μA and 100μA) and each one with two different transistor lengths (0.7μm and 1μm) therefore we have for each circuit six different combinations. The variables for the optimization problem will be widths of the transistors and for each circuit is proposed a different number of variables. In this manner, it is possible to cover different conditions for the optimizer and to see in each case its behavior. The parameters for both methods are the same.



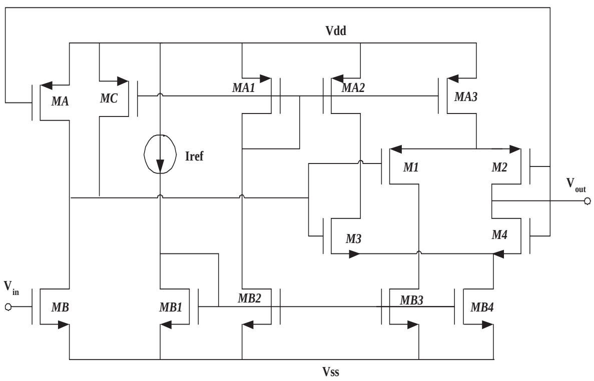
(a) VF_A



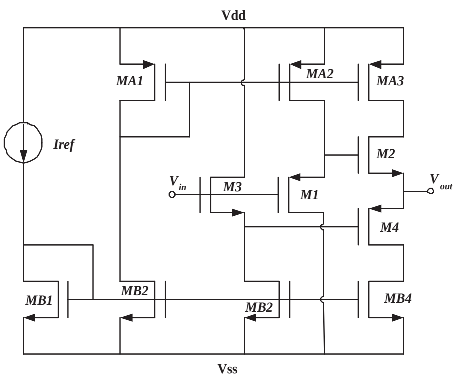
(b) VM_A



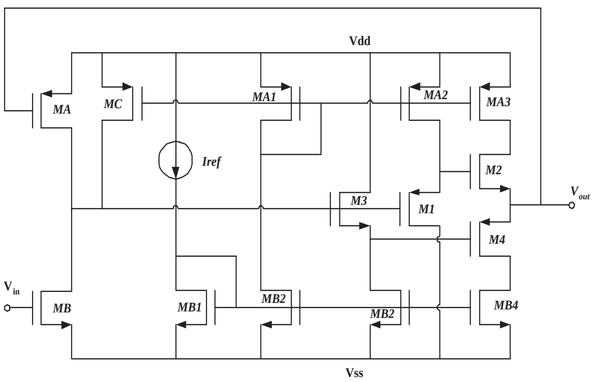
(c) VF_B



(d) VM_B



(e) VF_C



(f) VM_C

Figure 4.1: Selected Circuits for optimization

- N=50
- Number maximum of generations=20
- DE [29] like genetic operator.
- Lower limit of variables $0.35\mu m$ and Upper Limit of variables $300\mu m$
- Generation of initial population randomly between Lower and Upper Limit.
- Optimize three objectives: gain, band width and offset
- Constraint: every transistor in saturation condition.

In Fig 4.1(a) is depicted the schematic of a voltage follower (VF_A)and for this circuit all NMOS transistors have the same width and the PMOS have the same width too then there are only two variables in the optimization problem. In this manner Table 4.1 shows the transistors who are affected for which variable.

Table 4.1: Variables for VF_A in optimization process

Variable Name	Transistors
W_1	MA_1, MA_2, MA_3, M3 ,M4
W_2	MB_1, MB_2, MB_3, MB_4, M1 ,M2

In Fig 4.1(b) is depicted the schematic of a voltage mirror (VM_A) and there are five variables for its optimization process. Table 4.2 shows the transistors who are affected for which variable.

Table 4.2: Variables for VM_A in optimization process

Variable Name	Transistors
W_1	MA_1, MA_2, MA_3, M3 ,M4
W_2	MB_1, MB_2, MB_3, MB_4, M1 ,M2
W_3	MA
W_4	MB
W_5	MC

After in Fig 4.1(c) is depicted the schematic of other voltage follower (VF_B) which has for its optimization process. Table 4.3 shows the transistors who are affected for which variable.

Table 4.3: Variables for VF_B in optimization process

Variable Name	Transistors
W_1	MA_1, MA_2, MA_3 , M3 ,M4
W_2	MB_1, MB_2, MB_3, MB_4, M1 ,M2

Fig 4.1(d) shows a voltage mirror (VM_B) which has seven variables how is shown in Table 4.4.

Table 4.4: Variables for VM_B in optimization process

Variable Name	Transistors
W_1	MA_1, MA_3, MA_3 , M3 ,M4
W_2	MB_1, MB_2, MB_3, MB_4, M1 ,M2
W_3	MA
W_4	MB
W_5	MC

In in Fig 4.1(e) is depicted the schematic of a voltage follower (VF_C) which has six variables for its optimization process how is shown in Table 4.5.

Table 4.5: Variables for VF_C in optimization process

Variable Name	Transistors
W_1	MA_1, MA_2, MA_3
W_2	MB_1, MB_2, MB_3, MB_4
W_3	$M1$
W_4	$M2$
W_5	$M3$
W_6	$M4$

In in Fig 4.1(f) is depicted the last schematic of a voltage mirror (VM_C) which has six variables for its optimization process how is shown in Table 4.6.

After the optimization process, in Tables 4.7-4.18 are selected, for each circuit and for both optimization methods the three best found solutions. In Figs. 4.2- 4.13 are depicted the Pareto

Table 4.6: Variables for VM_C in optimization process

Variable Name	Transistors
W_1	MA_1, MA_2, MA_3
W_2	MB_1, MB_2, MB_3, MB_4
W_3	$M1$
W_4	$M2$
W_5	$M3$
W_6	$M4$
W_7	MA
W_8	MB
W_9	MC

Front achieved for MOEA/D and NSGA-II for each circuit and for their six combinations which are labeled as follows:

a) Iref=10μA and $L = 0.7\mu m$

b) Iref=50μA and $L = 0.7\mu m$

c) Iref=100μA and $L = 0.7\mu m$

d) Iref=10μA and $L = 1\mu m$

e) Iref=50μA and $L = 1\mu m$

f) Iref=100μA and $L = 1\mu m$

Table 4.7: Best points for VF_A with MOEA/D

L	I_{ref}	**W1** (m)	**W2**(m)	$\frac{V}{V}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	4.05E-05	3.71E-05	9.90E-01	3.73E-03	5.15E+07	3.50E+05	3.36E+03
0.7 μm	10 μA	2.71E-05	2.97E-05	9.89E-01	3.48E-03	5.23E+07	3.58E+05	3.65E+03
0.7 μm	10 μA	1.46E-06	7.81E-05	9.81E-01	-1.94E-05	2.95E+07	3.42E+05	6.40E+03
0.7 μm	50 μA	2.03E-04	2.39E-04	9.90E-01	3.73E-03	9.33E+07	6.92E+04	6.74E+02
0.7 μm	50 μA	7.12E-05	5.62E-05	9.88E-01	3.09E-03	1.48E+08	7.34E+04	8.44E+02
0.7 μm	50 μA	7.42E-06	2.76E-04	9.81E-01	2.61E-05	6.93E+07	6.84E+04	1.28E+03
0.7 μm	100 μA	2.99E-04	2.90E-04	9.89E-01	3.55E-03	1.32E+08	3.58E+04	3.56E+02
0.7 μm	100 μA	9.90E-05	1.05E-04	9.87E-01	2.82E-03	2.06E+08	3.65E+04	4.67E+02
0.7 μm	100 μA	1.25E-05	1.70E-04	9.77E-01	5.56E-06	1.42E+08	2.91E+04	6.74E+02
1 μm	10 μA	5.63E-05	1.34E-04	9.95E-01	2.31E-03	3.41E+07	6.15E+05	2.96E+03
1 μm	10 μA	3.38E-05	2.52E-05	9.94E-01	2.17E-03	4.62E+07	5.93E+05	3.40E+03
1 μm	10 μA	1.06E-06	8.85E-05	9.86E-01	1.98E-06	2.35E+07	3.72E+05	5.30E+03
1 μm	50 μA	2.86E-04	2.99E-04	9.95E-01	2.32E-03	6.57E+07	1.22E+05	6.01E+02
1 μm	50 μA	4.34E-05	5.22E-05	9.90E-01	1.72E-03	1.25E+08	9.66E+04	9.73E+02
1 μm	50 μA	5.60E-06	2.57E-04	9.85E-01	9.88E-05	5.56E+07	7.26E+04	1.12E+03
1 μm	100 μA	2.99E-04	2.89E-04	9.94E-01	2.13E-03	1.06E+08	6.01E+04	3.46E+02
1 μm	100 μA	6.30E-05	5.23E-05	9.82E-01	1.90E-03	1.83E+08	2.75E+04	5.27E+02
1 μm	100 μA	9.80E-06	2.78E-04	9.76E-01	4.08E-05	8.76E+07	2.39E+04	5.82E+02

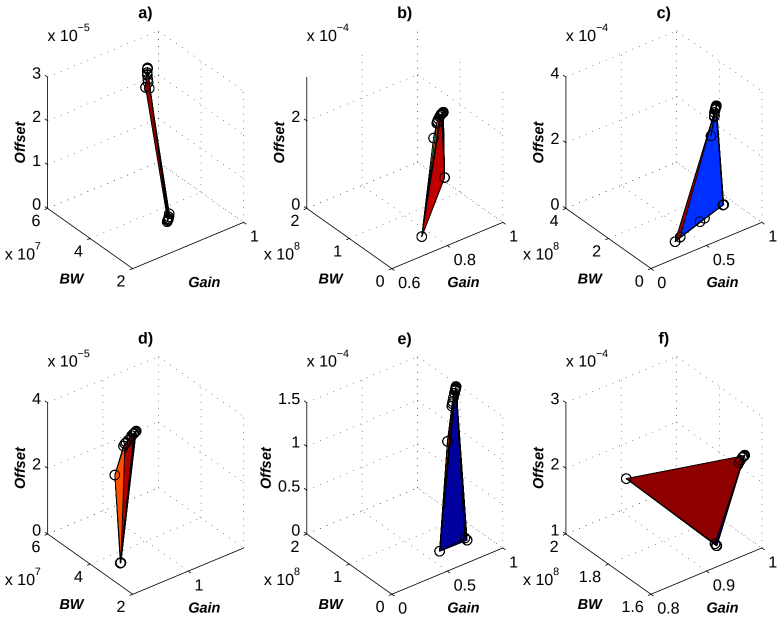


Figure 4.2: Pareto Front for six combinations of VF_A optimized by MOEA/D

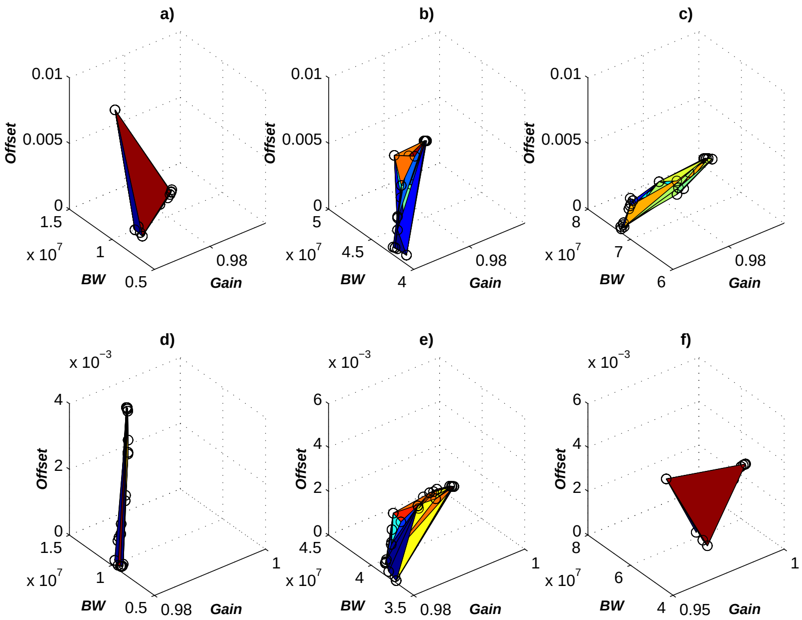


Figure 4.3: Pareto Front for six combinations of VF_B optimized by MOEA/D

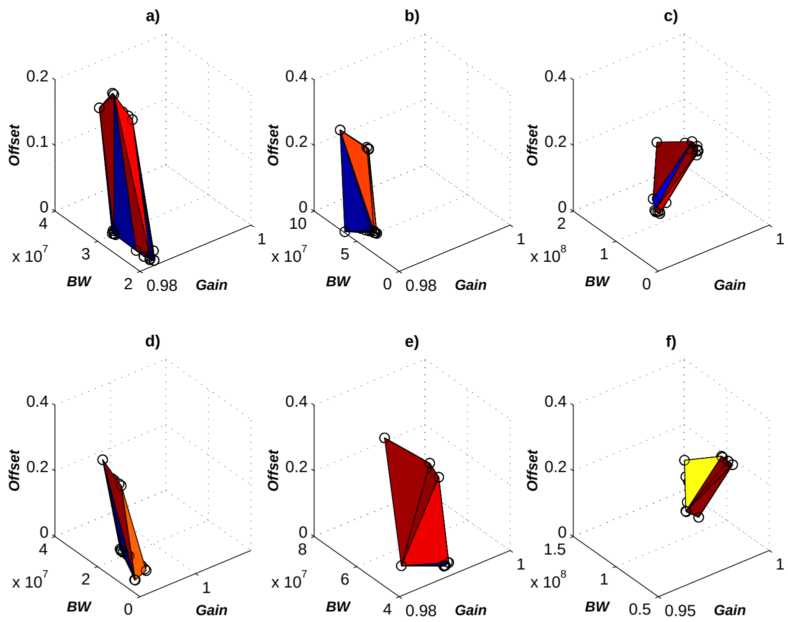


Figure 4.4: Pareto Front for six combinations of VF_C optimized by MOEA/D

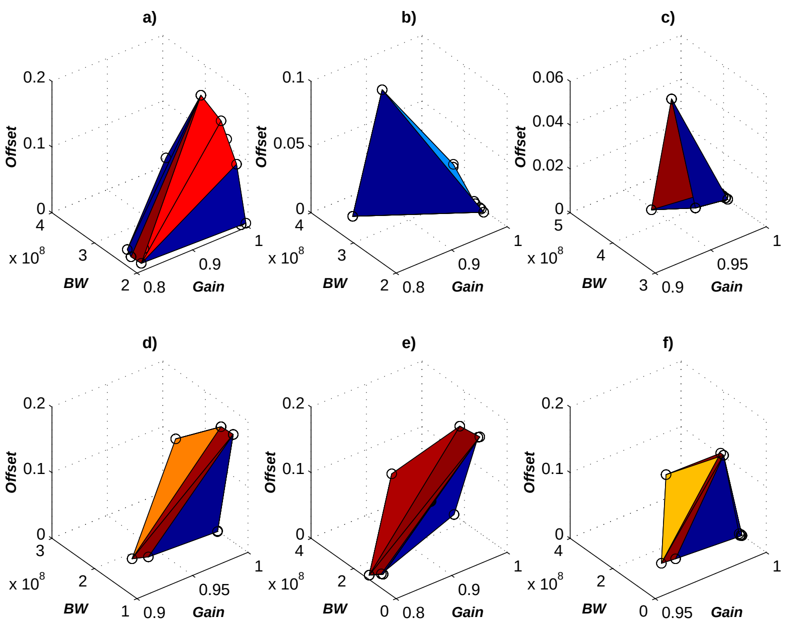


Figure 4.5: Pareto Front for six combinations of VM_A optimized by MOEA/D

Table 4.8: Best points for VF_B with MOEA/D

L	I_{ref}	**W1** (m)	**W2**(m)	$\frac{V}{V}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	6.11E-06	2.55E-06	9.76E-01	-3.69E-03	7.27E+06	2.31E+09	2.09E+04
0.7 μm	10 μA	3.14E-05	1.85E-05	9.76E-01	7.32E-03	1.33E+07	2.68E+08	1.49E+04
0.7 μm	10 μA	1.29E-05	4.04E-06	9.74E-01	-1.88E-06	1.00E+07	7.81E+08	2.00E+04
0.7 μm	50 μA	7.08E-05	9.28E-05	9.79E-01	6.25E-03	4.44E+07	1.06E+08	2.88E+03
0.7 μm	50 μA	4.44E-05	5.45E-05	9.76E-01	4.60E-03	4.64E+07	1.34E+08	3.43E+03
0.7 μm	50 μA	1.54E-05	4.15E-05	9.71E-01	-2.04E-05	4.32E+07	1.98E+08	4.12E+03
0.7 μm	100 μA	1.18E-04	1.86E-04	9.79E-01	5.98E-03	6.37E+07	6.26E+07	1.44E+03
0.7 μm	100 μA	4.79E-05	5.40E-05	9.73E-01	1.61E-03	7.42E+07	1.32E+08	2.10E+03
0.7 μm	100 μA	3.32E-05	5.84E-05	9.71E-01	3.03E-05	7.36E+07	1.41E+08	2.15E+03
1 μm	10 μA	2.29E-05	2.75E-05	9.86E-01	3.87E-03	1.24E+07	1.44E+08	1.54E+04
1 μm	10 μA	2.35E-05	2.74E-05	9.86E-01	3.90E-03	1.24E+07	1.44E+08	1.54E+04
1 μm	10 μA	9.39E-06	6.04E-06	9.81E-01	-1.86E-06	9.45E+06	3.92E+08	2.39E+04
1 μm	50 μA	9.49E-05	1.36E-04	9.88E-01	3.93E-03	3.57E+07	5.26E+07	2.78E+03
1 μm	50 μA	4.01E-05	5.13E-05	9.85E-01	1.76E-03	4.05E+07	8.34E+07	3.78E+03
1 μm	50 μA	2.27E-05	5.52E-05	9.82E-01	-5.89E-07	3.96E+07	8.79E+07	4.00E+03
1 μm	100 μA	1.93E-04	2.73E-04	9.88E-01	3.96E-03	4.60E+07	2.73E+07	1.38E+03
1 μm	100 μA	2.92E-05	8.02E-05	9.69E-01	-2.89E-03	6.33E+07	5.69E+07	2.19E+03
1 μm	100 μA	4.32E-05	1.07E-04	9.82E-01	2.16E-06	6.23E+07	4.96E+07	1.98E+03

Table 4.9: Best points for VF_C with MOEA/D

L	I_{ref}	**W1** (m)	**W2** (m)	**W3** (m)	**W4** (m)	**W5** (m)	**WM4** (m)	$\left\lvert\frac{V}{V}\right\rvert$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	6.05E-06	2.24E-05	2.50E-05	8.78E-05	9.60E-05	9.08E-05	9.87E-01	1.59E-01	3.06E+07	9.18E+09	2.88E+03
0.7 μm	10 μA	2.74E-05	1.64E-05	1.57E-05	5.20E-05	1.06E-04	6.47E-05	9.86E-01	1.45E-01	3.78E+07	9.36E+09	3.22E+03
0.7 μm	10 μA	3.80E-05	2.30E-06	2.57E-06	5.89E-05	1.02E-04	8.32E-05	9.83E-01	-3.39E-05	2.75E+07	1.00E+10	4.01E+03
0.7 μm	50 μA	5.37E-05	1.05E-04	1.57E-04	1.31E-04	2.84E-04	2.90E-04	9.87E-01	1.87E-01	8.15E+07	2.38E+09	6.26E+02
0.7 μm	50 μA	2.16E-04	1.01E-04	1.20E-04	1.45E-04	1.30E-04	1.39E-04	9.84E-01	2.25E-01	9.62E+07	3.91E+09	7.18E+02
0.7 μm	50 μA	1.38E-04	3.15E-06	1.08E-05	2.43E-04	3.42E-05	2.58E-04	9.80E-01	-2.44E-06	6.71E+07	1.00E+10	1.05E+03
0.7 μm	100 μA	2.16E-05	2.39E-04	1.71E-04	4.60E-05	2.92E-04	2.24E-04	9.86E-01	2.21E-01	9.03E+07	2.23E+09	3.50E+02
0.7 μm	100 μA	1.71E-04	6.67E-05	4.40E-05	8.63E-05	2.15E-04	1.37E-04	9.83E-01	1.68E-01	1.56E+08	4.09E+09	4.82E+02
0.7 μm	100 μA	1.07E-04	1.48E-05	1.04E-05	1.01E-04	2.26E-04	1.32E-04	9.79E-01	5.89E-03	1.20E+08	4.67E+09	1.01E+03
1 μm	10 μA	3.06E-05	3.15E-05	3.75E-05	6.79E-05	1.43E-04	1.12E-04	9.93E-01	1.73E-01	3.04E+07	6.31E+09	3.01E+03
1 μm	10 μA	5.16E-05	4.05E-05	2.81E-05	6.42E-05	8.07E-05	3.80E-05	9.93E-01	2.17E-01	3.41E+07	1.00E+10	3.32E+03
1 μm	10 μA	7.14E-06	1.81E-06	2.63E-06	1.40E-05	3.65E-05	7.07E-05	9.90E-01	-5.88E-05	2.15E+07	1.00E+10	4.49E+03
1 μm	50 μA	3.44E-05	1.35E-04	1.23E-04	2.86E-04	2.99E-04	2.31E-04	9.93E-01	2.05E-01	6.08E+07	2.55E+09	6.22E+02
1 μm	50 μA	1.72E-04	1.03E-04	1.22E-04	1.66E-04	1.81E-04	1.64E-04	9.92E-01	2.23E-01	7.83E+07	3.37E+09	6.76E+02
1 μm	50 μA	2.15E-04	1.10E-05	1.05E-05	1.22E-04	2.37E-04	2.53E-04	9.88E-01	-1.26E-06	5.44E+07	4.65E+09	1.94E+03
1 μm	100 μA	4.88E-05	2.50E-04	2.89E-04	7.77E-05	2.86E-04	2.99E-04	9.92E-01	2.43E-01	8.37E+07	1.73E+09	3.33E+02
1 μm	100 μA	2.94E-04	3.47E-05	5.62E-05	1.22E-04	1.05E-04	1.72E-04	9.88E-01	1.81E-01	1.19E+08	6.00E+09	6.40E+02
1 μm	100 μA	2.75E-04	9.71E-06	1.39E-05	2.80E-04	5.17E-05	7.85E-05	9.83E-01	7.98E-02	8.81E+07	1.00E+10	8.44E+02

Table 4.10: Best points for VM_A with MOEA/D

L	I_{ref}	**W1** (m)	**W2** (m)	**M3** (m)	**M4** (m)	**M5** (m)	$\frac{V}{V}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	3.58E-05	1.35E-05	1.46E-04	4.94E-05	1.59E-04	1.00E+00	-1.11E-01	2.50E+08	1.72E+08	1.41E+02
0.7 μm	10 μA	2.62E-05	8.14E-06	1.70E-04	5.96E-05	1.09E-05	1.00E+00	-1.50E-01	3.09E+08	1.45E+08	1.40E+02
0.7 μm	10 μA	2.34E-05	8.07E-06	8.29E-05	2.56E-05	1.88E-04	1.00E+00	4.80E-05	2.16E+08	3.20E+08	1.84E+02
0.7 μm	50 μA	1.61E-04	3.80E-05	1.18E-04	3.63E-05	2.58E-04	1.00E+00	-1.52E-03	2.78E+08	2.33E+08	3.61E+01
0.7 μm	50 μA	1.75E-04	3.84E-05	2.88E-04	8.74E-05	7.47E-05	1.08E+00	-7.44E-02	3.91E+08	8.62E+07	3.06E+01
0.7 μm	50 μA	1.65E-04	3.76E-05	1.30E-04	3.97E-05	2.70E-04	1.01E+00	-1.36E-06	2.84E+08	2.09E+08	3.58E+01
0.7 μm	100 μA	1.59E-04	4.35E-05	1.96E-04	5.99E-05	2.57E-04	1.00E+00	1.00E-03	4.15E+08	1.43E+08	2.14E+01
0.7 μm	100 μA	5.66E-05	5.94E-05	1.49E-04	4.84E-05	2.31E-04	9.76E-01	-4.13E-02	4.60E+08	1.91E+08	2.74E+01
0.7 μm	100 μA	6.54E-05	7.66E-05	1.81E-04	5.26E-05	2.94E-04	1.05E+00	2.50E-05	4.35E+08	1.50E+08	2.74E+01
1 μm	10 μA	3.23E-05	7.91E-05	1.32E-04	4.64E-05	1.40E-04	1.00E+00	-1.62E-01	1.63E+08	1.28E+08	8.84E+01
1 μm	10 μA	1.20E-05	9.13E-06	1.91E-04	6.58E-05	1.45E-04	9.83E-01	-1.27E-01	2.26E+08	9.17E+07	1.18E+02
1 μm	10 μA	3.47E-05	5.47E-06	1.16E-04	3.49E-05	1.47E-04	1.01E+00	-5.21E-05	1.86E+08	1.55E+08	1.20E+02
1 μm	50 μA	1.57E-04	1.41E-04	1.06E-04	3.53E-05	2.50E-04	1.00E+00	-1.12E-01	1.65E+08	1.76E+08	2.13E+01
1 μm	50 μA	2.93E-05	3.25E-05	2.97E-04	8.84E-05	1.72E-04	1.06E+00	-5.71E-02	3.73E+08	6.07E+07	3.47E+01
1 μm	50 μA	6.10E-05	3.81E-05	1.26E-04	3.82E-05	2.49E-04	1.00E+00	-4.08E-04	2.73E+08	1.51E+08	3.16E+01
1 μm	100 μA	2.19E-04	1.60E-04	1.15E-04	3.72E-05	2.43E-04	1.00E+00	-7.77E-02	1.73E+08	1.74E+08	1.32E+01
1 μm	100 μA	3.81E-05	8.52E-05	1.10E-04	3.54E-05	1.43E-04	9.84E-01	-7.14E-02	3.03E+08	2.08E+08	2.63E+01
1 μm	100 μA	1.69E-04	1.34E-04	8.31E-05	2.46E-05	2.45E-04	1.02E+00	-1.14E-06	1.55E+08	2.47E+08	1.62E+01

Table 4.11: Best points for VM_B with MOEA/D

L	I_{ref}	**W1** (m)	**W2** (m)	**M3** (m)	**M4** (m)	**M5** (m)	$\frac{V}{V}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	4.39E-05	1.42E-05	1.51E-04	6.26E-05	2.23E-06	1.00E+00	-2.35E-01	1.60E+08	3.25E+08	1.01E+03
0.7 μm	10 μA	1.92E-05	1.15E-05	7.95E-05	2.89E-05	1.97E-05	9.30E-01	-1.62E-01	1.87E+08	6.99E+08	9.64E+02
0.7 μm	10 μA	6.90E-06	8.40E-06	5.21E-05	1.79E-05	2.70E-05	9.11E-01	-1.15E-01	1.80E+08	1.13E+09	8.99E+02
0.7 μm	50 μA	2.03E-05	1.75E-05	1.47E-04	5.92E-05	6.25E-05	1.00E+00	-1.94E-01	3.41E+08	3.43E+08	2.56E+02
0.7 μm	50 μA	2.29E-05	3.28E-05	1.62E-04	6.62E-05	1.96E-05	9.97E-01	-2.21E-01	3.72E+08	3.07E+08	2.07E+02
0.7 μm	50 μA	4.02E-05	2.04E-05	4.17E-05	1.42E-05	1.69E-04	9.94E-01	2.28E-04	2.00E+08	1.50E+09	2.26E+02
0.7 μm	100 μA	2.26E-05	6.36E-05	1.73E-04	5.60E-05	2.36E-04	1.00E+00	5.01E-02	3.27E+08	3.76E+08	1.14E+02
0.7 μm	100 μA	8.68E-05	3.77E-05	2.21E-04	8.26E-05	1.92E-04	9.60E-01	-1.55E-01	3.76E+08	2.46E+08	1.09E+02
0.7 μm	100 μA	2.61E-05	5.55E-05	1.99E-04	6.78E-05	2.72E-04	1.00E+00	2.75E-05	3.35E+08	3.05E+08	1.14E+02
1 μm	10 μA	4.78E-05	1.69E-05	1.64E-04	6.47E-05	1.83E-05	1.00E+00	-2.24E-01	1.42E+08	2.23E+08	7.32E+02
1 μm	10 μA	2.78E-05	1.12E-05	1.71E-04	5.62E-05	1.28E-06	9.01E-01	-1.29E-01	1.54E+08	2.53E+08	7.29E+02
1 μm	10 μA	3.83E-06	1.53E-05	1.16E-04	3.48E-05	1.03E-04	9.26E-01	1.27E-03	1.10E+08	4.09E+08	1.94E+03
1 μm	50 μA	2.77E-05	1.34E-04	2.97E-04	1.11E-04	1.94E-04	1.00E+00	-1.61E-01	2.18E+08	1.28E+08	1.11E+02
1 μm	50 μA	4.25E-05	5.28E-05	2.89E-04	1.06E-04	9.64E-05	9.66E-01	-1.72E-01	2.57E+08	1.35E+08	1.26E+02
1 μm	50 μA	1.54E-05	1.17E-04	1.00E-04	3.23E-05	1.08E-04	9.90E-01	-2.94E-04	1.95E+08	4.41E+08	1.30E+02
1 μm	100 μA	1.23E-04	8.82E-05	2.71E-04	1.05E-04	1.26E-04	1.00E+00	-2.06E-01	2.91E+08	1.37E+08	7.49E+01
1 μm	100 μA	1.23E-04	8.82E-05	2.71E-04	1.05E-04	1.26E-04	1.00E+00	-2.06E-01	2.91E+08	1.37E+08	7.49E+01
1 μm	100 μA	3.07E-05	1.55E-04	2.14E-04	6.55E-05	1.84E-04	9.43E-01	-7.56E-05	2.55E+08	2.19E+08	6.31E+01

Table 4.12: Best points for VM_C with MOEA/D

L	I_{ref}	**W1**	**W2**	**W3**	**W4**	**W5**	**W6**	**W7**	**W8**	**W9**	$\frac{V}{V}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	3.22E-05	2.46E-05	2.21E-05	4.08E-05	4.89E-05	5.39E-05	1.10E-04	4.67E-05	2.11E-04	1.00E+00	-2.25E-01	2.02E+08	2.29E+04	3.01E+02
0.7 μm	10 μA	3.93E-05	1.79E-05	1.16E-05	6.36E-05	5.44E-05	6.67E-05	1.26E-04	4.67E-05	1.28E-04	9.28E-01	-1.62E-01	2.27E+08	1.96E+04	3.46E+02
0.7 μm	10 μA	3.76E-05	2.27E-05	2.35E-05	1.64E-05	4.74E-05	5.87E-05	3.27E-05	1.12E-05	2.10E-04	9.66E-01	-8.91E-03	1.28E+08	1.46E+05	2.43E+02
0.7 μm	10 μA	4.30E-05	1.86E-05	2.14E-05	1.24E-05	5.13E-05	6.47E-05	4.75E-05	1.63E-05	2.30E-04	9.72E-01	-1.30E-02	1.41E+08	1.03E+05	2.14E+02
0.7 μm	50 μA	1.97E-04	1.39E-04	1.50E-04	7.62E-06	1.62E-04	2.39E-04	1.77E-04	6.15E-05	1.36E-04	1.00E+00	1.26E-03	3.29E+08	1.19E+04	6.23E+01
0.7 μm	50 μA	1.81E-04	1.69E-04	1.13E-04	4.46E-05	2.02E-04	2.21E-04	2.83E-04	1.05E-04	5.06E-05	9.28E-01	-1.62E-01	4.25E+08	4.74E+03	6.89E+01
0.7 μm	50 μA	1.87E-04	1.86E-04	1.33E-04	5.84E-06	1.94E-04	2.38E-04	1.85E-04	6.49E-05	1.12E-04	1.01E+00	-3.14E-05	3.42E+08	1.05E+04	9.17E+01
0.7 μm	50 μA	1.94E-04	1.47E-04	1.46E-04	7.63E-06	1.81E-04	2.48E-04	1.78E-04	6.20E-05	1.37E-04	1.00E+00	-3.60E-05	3.28E+08	1.18E+04	5.87E+01
0.7 μm	100 μA	2.34E-04	1.08E-04	1.32E-04	1.68E-04	1.81E-04	1.82E-04	2.39E-04	1.02E-04	1.82E-04	1.00E+00	-2.29E-01	4.96E+08	4.29E+03	2.22E+01
0.7 μm	100 μA	7.21E-05	1.49E-04	9.86E-05	9.01E-05	1.93E-04	1.62E-04	2.18E-04	8.53E-05	7.98E-05	9.54E-01	-1.82E-01	5.48E+08	4.74E+03	3.25E+01
0.7 μm	100 μA	2.37E-04	1.63E-04	4.50E-05	8.50E-05	1.53E-04	6.67E-05	8.39E-05	2.89E-05	1.89E-04	9.33E-01	-5.59E-02	3.81E+08	1.72E+04	2.52E+01
0.7 μm	100 μA	2.37E-04	1.63E-04	4.50E-05	8.50E-05	1.53E-04	6.67E-05	8.39E-05	2.89E-05	1.89E-04	9.33E-01	-5.59E-02	3.81E+08	1.72E+04	2.52E+01
1 μm	10 μA	5.40E-05	3.23E-05	3.84E-05	6.79E-05	8.59E-05	1.21E-04	1.41E-04	5.71E-05	1.16E-04	1.00E+00	-2.29E-01	1.95E+08	1.40E+04	2.57E+02
1 μm	10 μA	5.82E-05	3.50E-05	4.27E-05	5.29E-05	7.21E-05	1.19E-04	1.56E-04	6.31E-05	8.06E-05	9.98E-01	-2.29E-01	2.07E+08	1.19E+04	2.65E+02
1 μm	10 μA	6.07E-05	4.39E-05	3.52E-05	2.47E-06	1.20E-04	2.07E-04	2.05E-04	7.21E-05	2.11E-04	1.06E+00	-2.96E-03	1.48E+08	1.72E+04	3.20E+02
1 μm	10 μA	5.57E-05	4.93E-05	3.10E-05	3.77E-06	1.30E-04	2.17E-04	1.98E-04	6.59E-05	2.47E-04	1.00E+00	-6.75E-03	1.50E+08	1.92E+04	2.76E+02
1 μm	50 μA	1.87E-04	2.53E-04	2.73E-04	6.25E-06	1.59E-04	2.07E-04	1.10E-04	3.67E-05	5.05E-05	1.00E+00	-4.17E-03	2.43E+08	2.05E+04	6.43E+01
1 μm	50 μA	1.52E-04	1.38E-04	1.41E-04	1.31E-04	2.06E-04	2.03E-04	2.13E-04	8.28E-05	9.07E-05	9.74E-01	-2.02E-01	3.20E+08	5.81E+03	2.98E+01
1 μm	50 μA	2.13E-04	2.28E-04	2.41E-04	5.75E-06	2.18E-04	2.66E-04	1.14E-04	3.66E-05	4.17E-05	9.66E-01	7.93E-05	2.41E+08	1.97E+04	3.93E+01

Table 4.13: Best points for VF_A with NSGA-II

L	I_{ref}	**W1** (m)	**W2**(m)	$\frac{I}{I}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	4.00E-05	3.31E-05	9.90E-01	3.72E-03	5.18E+07	3.51E+05	3.37E+03
0.7 μm	10 μA	3.04E-05	2.70E-05	9.89E-01	3.56E-03	5.23E+07	3.58E+05	3.55E+03
0.7 μm	10 μA	2.57E-06	9.01E-06	9.79E-01	1.44E-03	2.81E+07	3.04E+05	6.74E+03
0.7 μm	50 μA	9.95E-05	6.73E-05	9.89E-01	3.32E-03	1.40E+08	7.35E+04	7.76E+02
0.7 μm	50 μA	4.45E-05	3.89E-05	9.86E-01	2.76E-03	1.54E+08	7.03E+04	9.59E+02
0.7 μm	50 μA	1.31E-05	2.17E-05	9.70E-01	1.98E-03	1.25E+08	4.18E+04	1.31E+03
0.7 μm	100 μA	9.39E-05	9.42E-05	9.87E-01	2.79E-03	2.10E+08	3.61E+04	4.74E+02
0.7 μm	100 μA	6.06E-05	4.04E-05	9.81E-01	2.62E-03	2.29E+08	2.65E+04	5.26E+02
0.7 μm	100 μA	2.52E-05	4.03E-05	9.67E-01	2.04E-03	2.11E+08	1.92E+04	6.56E+02
1 μm	10 μA	5.42E-05	6.51E-05	9.95E-01	2.30E-03	4.12E+07	6.10E+05	3.03E+03
1 μm	10 μA	2.63E-05	2.47E-05	9.94E-01	2.08E-03	4.63E+07	5.93E+05	3.58E+03
1 μm	10 μA	1.17E-05	5.01E-06	9.85E-01	2.08E-03	3.60E+07	2.85E+05	4.48E+03
1 μm	50 μA	9.75E-05	9.79E-05	9.93E-01	1.99E-03	1.09E+08	1.16E+05	7.73E+02
1 μm	50 μA	4.18E-05	3.34E-05	9.88E-01	1.81E-03	1.27E+08	7.62E+04	9.83E+02
1 μm	50 μA	1.62E-05	3.00E-05	9.74E-01	1.74E-03	1.14E+08	4.52E+04	1.24E+03
1 μm	100 μA	9.84E-05	9.91E-05	9.91E-01	1.78E-03	1.66E+08	4.84E+04	4.70E+02
1 μm	100 μA	5.51E-05	4.15E-05	9.70E-01	2.30E-03	1.86E+08	1.75E+04	5.42E+02
1 μm	100 μA	2.80E-05	4.96E-05	9.59E-01	2.19E-03	1.74E+08	1.51E+04	6.42E+02

Table 4.14: Best points for VF_B with NSGA-II

L	I_{ref}	**W1** (m)	**W2**(m)	$\frac{I}{I}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	1.26E-05	1.81E-05	9.76E-01	5.26E-03	1.25E+07	3.41E+05	1.61E+04
0.7 μm	10 μA	2.53E-05	1.84E-05	9.76E-01	6.87E-03	1.33E+07	2.68E+05	1.51E+04
0.7 μm	10 μA	4.49E-06	1.35E-06	9.72E-01	-5.69E-03	6.03E+06	8.10E+06	2.48E+04
0.7 μm	50 μA	7.26E-05	9.28E-05	9.79E-01	6.29E-03	4.43E+07	3.95E+04	2.87E+03
0.7 μm	50 μA	4.57E-05	5.14E-05	9.76E-01	4.58E-03	4.64E+07	4.37E+04	3.46E+03
0.7 μm	50 μA	1.12E-05	4.36E-06	9.51E-01	-8.59E-03	2.34E+07	7.37E+05	6.48E+03
0.7 μm	100 μA	9.77E-05	9.91E-05	9.77E-01	4.79E-03	7.09E+07	1.90E+04	1.70E+03
0.7 μm	100 μA	5.16E-05	6.53E-05	9.74E-01	2.27E-03	7.46E+07	2.38E+04	2.00E+03
0.7 μm	100 μA	2.33E-05	6.11E-06	9.44E-01	-7.11E-03	4.40E+07	2.20E+05	3.54E+03
1 μm	10 μA	2.60E-05	2.27E-05	9.86E-01	3.94E-03	1.23E+07	1.94E+05	1.58E+04
1 μm	10 μA	3.11E-05	2.67E-05	9.86E-01	4.30E-03	1.24E+07	1.89E+05	1.54E+04
1 μm	10 μA	3.29E-06	9.83E-06	9.29E-01	-1.24E-02	2.45E+06	9.94E+06	5.42E+04
1 μm	50 μA	9.46E-05	9.96E-05	9.88E-01	3.73E-03	3.72E+07	3.53E+04	2.94E+03
1 μm	50 μA	4.33E-05	5.40E-05	9.85E-01	1.98E-03	4.05E+07	3.97E+04	3.71E+03
1 μm	50 μA	3.78E-05	1.02E-05	9.76E-01	6.94E-04	3.33E+07	7.69E+04	5.98E+03
1 μm	100 μA	9.16E-05	9.93E-05	9.85E-01	2.12E-03	6.05E+07	1.87E+04	1.84E+03
1 μm	100 μA	4.38E-05	7.49E-05	9.81E-01	-4.06E-04	6.35E+07	2.17E+04	2.12E+03
1 μm	100 μA	2.83E-05	7.86E-06	9.47E-01	-5.24E-03	4.28E+07	1.38E+05	3.52E+03

Table 4.15: Best points for VF_C with NSGA-II

Lx	**Iref**	**W1**	**WM2**	**WM3**	**W4**	**W5**	**W6**	$\frac{I}{I}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	1.98E-05	3.17E-05	3.60E-05	7.40E-05	6.49E-05	5.63E-05	9.86E-01	1.99E-01	3.73E+07	1.00E+10	2.91E+03
0.7 μm	10 μA	1.40E-05	2.61E-05	3.94E-05	6.17E-05	4.82E-05	5.12E-05	9.86E-01	2.07E-01	3.99E+07	1.00E+10	3.04E+03
0.7 μm	10 μA	2.15E-05	9.93E-07	1.94E-06	4.04E-05	1.52E-05	4.50E-05	9.80E-01	2.66E-03	2.75E+07	1.00E+10	4.96E+03
0.7 μm	50 μA	8.64E-05	9.08E-05	1.02E-04	6.97E-05	2.53E-04	2.35E-04	9.86E-01	1.81E-01	8.77E+07	3.00E+09	6.19E+02
0.7 μm	50 μA	6.06E-05	8.91E-05	3.44E-05	1.48E-04	1.30E-04	4.76E-05	9.84E-01	2.23E-01	1.07E+08	6.36E+09	7.78E+02
0.7 μm	50 μA	1.16E-04	9.61E-06	1.15E-05	2.95E-04	2.68E-04	2.86E-04	9.83E-01	6.85E-03	6.57E+07	4.06E+09	8.53E+02
0.7 μm	100 μA	4.42E-05	1.70E-04	1.70E-04	1.31E-04	2.88E-04	2.63E-04	9.86E-01	2.08E-01	1.16E+08	2.23E+09	3.36E+02
0.7 μm	100 μA	1.18E-04	1.20E-04	1.05E-04	1.45E-04	1.98E-04	1.46E-04	9.84E-01	2.17E-01	1.52E+08	3.33E+09	3.79E+02
0.7 μm	100 μA	2.28E-04	1.36E-05	1.23E-05	3.00E-04	2.24E-04	2.36E-04	9.78E-01	-7.91E-03	9.77E+07	4.57E+09	9.87E+02
1 μm	10 μA	7.67E-06	4.54E-05	3.35E-05	9.79E-05	1.45E-04	9.40E-05	9.93E-01	1.83E-01	2.53E+07	6.42E+09	2.73E+03
1 μm	10 μA	3.11E-05	2.85E-05	2.13E-05	5.39E-05	1.17E-04	5.34E-05	9.93E-01	1.81E-01	3.42E+07	8.35E+09	3.35E+03
1 μm	10 μA	3.14E-05	4.32E-06	3.74E-06	1.14E-04	1.26E-04	1.27E-04	9.87E-01	2.58E-02	1.37E+07	9.29E+09	1.23E+04
1 μm	50 μA	4.02E-05	1.76E-04	2.05E-04	2.94E-04	2.92E-04	2.89E-04	9.93E-01	2.19E-01	5.43E+07	2.10E+09	5.85E+02
1 μm	50 μA	6.98E-05	5.26E-05	3.77E-05	1.38E-04	1.24E-04	7.66E-05	9.91E-01	2.07E-01	9.06E+07	6.51E+09	8.38E+02
1 μm	50 μA	1.34E-04	1.62E-05	1.95E-05	1.78E-04	1.98E-04	1.64E-04	9.91E-01	8.53E-02	7.60E+07	5.17E+09	9.56E+02
1 μm	100 μA	9.88E-05	2.01E-04	2.68E-04	1.78E-04	2.79E-04	2.92E-04	9.92E-01	2.37E-01	9.97E+07	1.82E+09	3.43E+02
1 μm	100 μA	1.21E-04	1.17E-04	1.48E-04	9.20E-05	1.30E-04	1.43E-04	9.90E-01	2.64E-01	1.23E+08	3.43E+09	4.17E+02
1 μm	100 μA	1.02E-04	1.45E-05	1.49E-05	1.34E-04	1.25E-04	1.42E-04	9.86E-01	3.81E-02	9.98E+07	7.40E+09	1.10E+03

Table 4.16: Best points for VM_A with NSGA-II

Lx	**Iref**	**W1**	**W2**	**W3**	**W4**	**W5**	$\frac{I}{I}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	3.18E-05	7.00E-05	3.42E-06	1.16E-06	4.51E-05	9.89E-01	-6.16E-02	3.26E+07	9.68E+09	2.42E+02
0.7 μm	10 μA	2.83E-05	6.64E-05	3.20E-05	1.26E-05	4.18E-05	9.34E-01	-1.96E-01	1.53E+08	8.12E+08	1.39E+02
0.7 μm	10 μA	3.18E-05	7.00E-05	3.42E-06	1.16E-06	4.51E-05	9.89E-01	-6.16E-02	3.26E+07	9.68E+09	2.42E+02
0.7 μm	50 μA	5.23E-05	4.58E-05	2.82E-06	7.29E-07	1.11E-05	9.90E-01	1.67E-01	2.43E+07	1.00E+10	1.84E+02
0.7 μm	50 μA	4.90E-05	3.95E-05	2.82E-06	7.29E-07	7.31E-06	1.02E+00	1.34E-01	2.69E+07	1.00E+10	1.80E+02
0.7 μm	50 μA	5.51E-05	4.10E-05	2.82E-06	7.29E-07	7.31E-06	1.02E+00	1.32E-01	2.47E+07	1.00E+10	1.72E+02
0.7 μm	100 μA	5.16E-05	7.71E-05	1.80E-05	4.43E-06	6.79E-05	1.02E+00	1.91E-01	1.15E+08	2.33E+09	8.11E+01
0.7 μm	100 μA	3.58E-05	5.82E-05	2.59E-05	6.85E-06	9.84E-05	9.20E-01	1.86E-01	2.02E+08	1.74E+09	9.66E+01
0.7 μm	100 μA	4.66E-05	8.45E-05	1.66E-05	4.86E-06	7.67E-05	9.52E-01	1.05E-01	1.15E+08	2.27E+09	5.71E+01
1 μm	10 μA	5.22E-05	7.69E-05	5.75E-05	1.98E-05	5.56E-05	1.01E+00	-1.52E-01	1.26E+08	2.96E+08	8.39E+01
1 μm	10 μA	5.04E-05	4.10E-05	6.87E-05	2.77E-05	8.82E-05	9.19E-01	-2.14E-01	1.58E+08	2.59E+08	7.85E+01
1 μm	10 μA	5.22E-05	7.69E-05	5.75E-05	1.98E-05	5.56E-05	1.01E+00	-1.52E-01	1.26E+08	2.96E+08	8.39E+01
1 μm	50 μA	9.35E-05	8.95E-05	9.68E-06	2.70E-06	9.20E-05	9.70E-01	1.71E-01	3.33E+07	2.75E+09	5.59E+01
1 μm	50 μA	5.00E-05	7.47E-05	1.47E-05	4.17E-06	5.64E-05	1.04E+00	4.64E-02	7.55E+07	1.59E+09	5.15E+01
1 μm	50 μA	9.28E-05	8.51E-05	8.25E-06	2.36E-06	2.83E-05	1.04E+00	3.35E-02	2.97E+07	3.18E+09	5.49E+01
1 μm	100 μA	5.83E-05	8.20E-05	2.53E-05	5.58E-06	6.77E-05	9.99E-01	2.13E-01	1.01E+08	1.44E+09	1.03E+02
1 μm	100 μA	5.07E-05	9.32E-05	2.82E-05	5.79E-06	5.53E-05	1.09E+00	2.09E-01	1.02E+08	1.25E+09	1.01E+02
1 μm	100 μA	6.20E-05	8.46E-05	1.25E-05	3.64E-06	4.69E-05	9.36E-01	9.98E-02	5.77E+07	2.43E+09	5.14E+01

Table 4.17: Best points for VM_B with NSGA-II

Lx	**Iref**	**W1**	**W2**	**W3**	**W4**	**W5**	$\frac{I}{I}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	1.20E-05	2.64E-06	8.90E-06	2.94E-06	8.37E-05	1.06E+00	1.20E-01	6.25E+07	8.16E+09	1.07E+03
0.7 μm	10 μA	1.20E-05	2.64E-06	8.90E-06	2.94E-06	8.37E-05	1.06E+00	1.20E-01	6.25E+07	8.16E+09	1.07E+03
0.7 μm	10 μA	1.20E-05	2.64E-06	8.90E-06	2.94E-06	8.37E-05	1.06E+00	1.20E-01	6.25E+07	8.16E+09	1.07E+03
0.7 μm	50 μA	1.20E-05	2.64E-06	8.90E-06	2.94E-06	8.37E-05	1.06E+00	1.20E-01	6.25E+07	8.16E+09	1.07E+03
0.7 μm	50 μA	1.20E-05	2.64E-06	8.90E-06	2.94E-06	8.37E-05	1.06E+00	1.20E-01	6.25E+07	8.16E+09	1.07E+03
0.7 μm	50 μA	1.20E-05	2.64E-06	8.90E-06	2.94E-06	8.37E-05	1.06E+00	1.20E-01	6.25E+07	8.16E+09	1.07E+03
0.7 μm	100 μA	7.80E-05	6.03E-06	1.41E-05	3.75E-06	9.19E-05	1.01E+00	2.37E-01	1.34E+08	6.63E+09	1.47E+02
0.7 μm	100 μA	4.46E-05	7.53E-05	9.84E-05	3.32E-05	8.80E-05	9.26E-01	-7.70E-02	3.71E+08	6.16E+08	9.22E+01
0.7 μm	100 μA	4.46E-05	7.53E-05	9.84E-05	3.32E-05	8.80E-05	9.26E-01	-7.70E-02	3.71E+08	6.16E+08	9.22E+01
1 μm	10 μA	1.47E-05	6.96E-06	1.13E-05	3.41E-06	7.04E-05	9.81E-01	8.03E-02	6.90E+07	4.45E+09	6.84E+02
1 μm	10 μA	5.16E-05	2.18E-05	7.99E-05	2.81E-05	2.62E-05	9.36E-01	-1.61E-01	1.36E+08	5.09E+08	6.30E+02
1 μm	10 μA	1.47E-05	6.96E-06	1.13E-05	3.41E-06	7.04E-05	9.81E-01	8.03E-02	6.90E+07	4.45E+09	6.84E+02
1 μm	50 μA	7.49E-05	6.25E-05	4.59E-05	1.32E-05	7.16E-05	8.59E-01	-2.70E-02	1.70E+08	1.09E+09	9.82E+01
1 μm	50 μA	7.49E-05	6.25E-05	4.59E-05	1.32E-05	7.16E-05	8.59E-01	-2.70E-02	1.70E+08	1.09E+09	9.82E+01
1 μm	50 μA	7.49E-05	6.25E-05	4.59E-05	1.32E-05	7.16E-05	8.59E-01	-2.70E-02	1.70E+08	1.09E+09	9.82E+01
1 μm	100 μA	8.19E-05	7.48E-05	3.23E-05	1.40E-05	2.55E-05	1.07E+00	-2.49E-01	2.10E+08	1.05E+09	8.10E+01
1 μm	100 μA	8.19E-05	7.48E-05	3.23E-05	1.40E-05	2.55E-05	1.07E+00	-2.49E-01	2.10E+08	1.05E+09	8.10E+01
1 μm	100 μA	8.94E-05	9.15E-05	1.87E-05	5.33E-06	3.16E-05	8.73E-01	8.79E-03	1.40E+08	2.78E+09	5.57E+01

Table 4.18: Best points for VM_C with NSGA-II

Lx	**Iref**	**W1**	**W2**	**WM3**	**W4**	**W5**	**W6**	**W7**	**W8**	**W9**	$\frac{I}{I}$	**OFF** (V)	**BW** (Hz)	R_I (Ω)	R_O (Ω)
0.7 μm	10 μA	9.29E-06	9.71E-06	8.18E-06	1.03E-04	6.02E-05	8.74E-05	2.82E-04	1.19E-04	8.80E-05	1.00E+00	-2.40E-01	2.19E+08	1.75E+08	4.47E+02
0.7 μm	10 μA	1.49E-05	2.50E-05	3.02E-05	8.43E-05	6.04E-05	1.76E-04	2.60E-04	5.83E-05	1.29E-05	6.99E-01	6.09E-02	2.84E+08	3.43E+08	2.37E+02
0.7 μm	10 μA	3.83E-05	2.37E-05	1.37E-05	9.42E-05	1.04E-04	9.91E-05	1.86E-04	4.95E-05	6.82E-05	7.72E-01	-9.95E-03	2.41E+08	4.04E+08	2.55E+02
0.7 μm	50 μA	6.04E-05	1.32E-04	1.49E-04	7.61E-05	2.64E-04	2.36E-04	2.27E-04	9.69E-05	7.18E-05	9.99E-01	-2.36E-01	4.14E+08	2.25E+08	3.51E+01
0.7 μm	50 μA	6.08E-05	1.52E-04	1.18E-04	4.20E-05	2.38E-04	2.81E-04	2.94E-04	1.29E-04	1.57E-05	1.01E+00	-2.57E-01	4.76E+08	1.69E+08	8.50E+01
0.7 μm	50 μA	1.42E-04	1.78E-04	1.76E-04	7.87E-05	2.17E-04	2.51E-04	2.18E-04	5.70E-05	9.23E-06	7.61E-01	-1.06E-03	4.13E+08	3.57E+08	3.43E+01
0.7 μm	100 μA	2.38E-04	2.36E-04	2.28E-04	6.73E-05	1.47E-04	1.75E-04	2.45E-04	1.05E-04	1.72E-04	1.00E+00	-2.13E-01	4.70E+08	2.13E+08	3.94E+01
0.7 μm	100 μA	1.04E-04	1.94E-04	1.96E-04	1.82E-04	1.62E-04	1.51E-04	2.17E-04	8.57E-05	4.08E-06	9.48E-01	-1.95E-01	5.47E+08	2.58E+08	2.08E+01
0.7 μm	100 μA	2.17E-04	2.64E-04	9.67E-05	3.00E-04	2.87E-04	1.63E-04	2.74E-04	7.18E-05	2.05E-05	7.61E-01	-6.93E-04	4.86E+08	2.86E+08	2.16E+01
1 μm	10 μA	6.13E-05	1.68E-05	5.62E-05	9.97E-05	9.32E-05	3.69E-05	1.06E-04	6.03E-06	2.30E-04	6.13E-01	5.13E-01	9.85E+07	5.28E+09	2.53E+02
1 μm	10 μA	6.13E-05	1.68E-05	5.62E-05	9.97E-05	9.32E-05	3.69E-05	1.06E-04	6.03E-06	2.30E-04	6.13E-01	5.13E-01	9.85E+07	5.28E+09	2.53E+02
1 μm	10 μA	4.23E-05	7.20E-05	3.05E-05	1.14E-06	1.04E-04	7.25E-05	3.82E-05	3.21E-06	8.80E-06	3.51E-01	4.48E-01	8.35E+07	2.60E+09	8.07E+01
1 μm	50 μA	1.80E-04	1.12E-04	7.83E-05	8.19E-05	1.54E-04	1.25E-04	1.38E-04	5.45E-05	1.99E-04	1.00E+00	-1.90E-01	2.78E+08	2.83E+08	4.38E+01
1 μm	50 μA	6.95E-05	1.38E-04	1.55E-04	1.77E-04	1.68E-04	2.13E-04	3.00E-04	1.03E-04	5.29E-05	9.10E-01	-1.42E-01	3.45E+08	1.46E+08	5.14E+01
1 μm	50 μA	2.33E-04	2.09E-04	1.29E-04	1.12E-04	2.79E-04	2.72E-04	1.60E-04	4.04E-05	4.80E-05	7.72E-01	2.39E-03	2.69E+08	3.52E+08	3.82E+01
1 μm	100 μA	8.64E-05	2.84E-04	2.27E-04	1.13E-04	2.21E-04	1.73E-04	2.44E-04	9.67E-05	2.83E-04	1.00E+00	-1.86E-01	3.38E+08	1.66E+08	1.51E+01
1 μm	100 μA	1.70E-04	6.91E-05	9.66E-05	9.91E-05	1.87E-04	2.36E-04	2.77E-04	1.19E-04	2.30E-05	1.03E+00	-2.61E-01	4.60E+08	1.35E+08	1.92E+01
1 μm	100 μA	2.17E-04	1.11E-04	1.28E-04	1.40E-04	1.18E-04	1.33E-04	2.39E-04	6.14E-05	6.44E-05	7.79E-01	-1.22E-03	3.96E+08	2.39E+08	1.54E+01

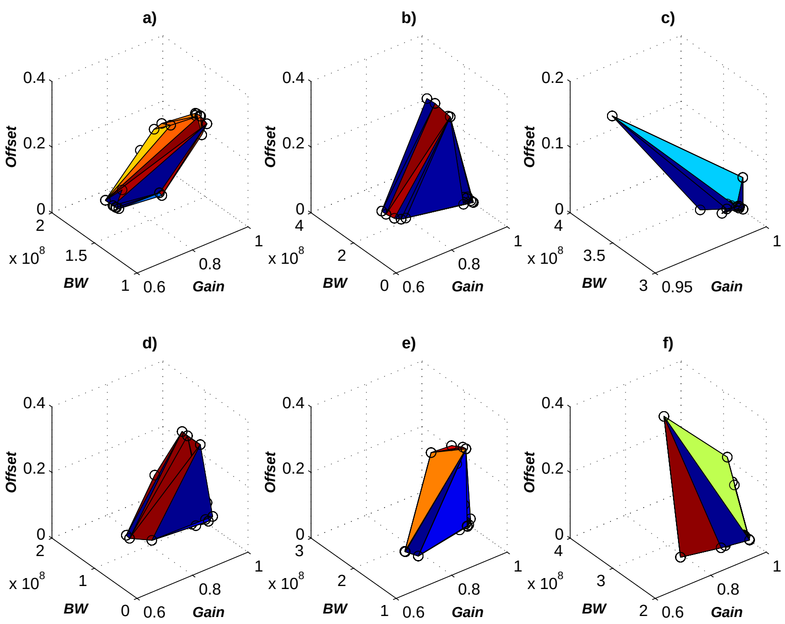


Figure 4.6: Pareto Front for six combinations of VM_B optimized by MOEA/D

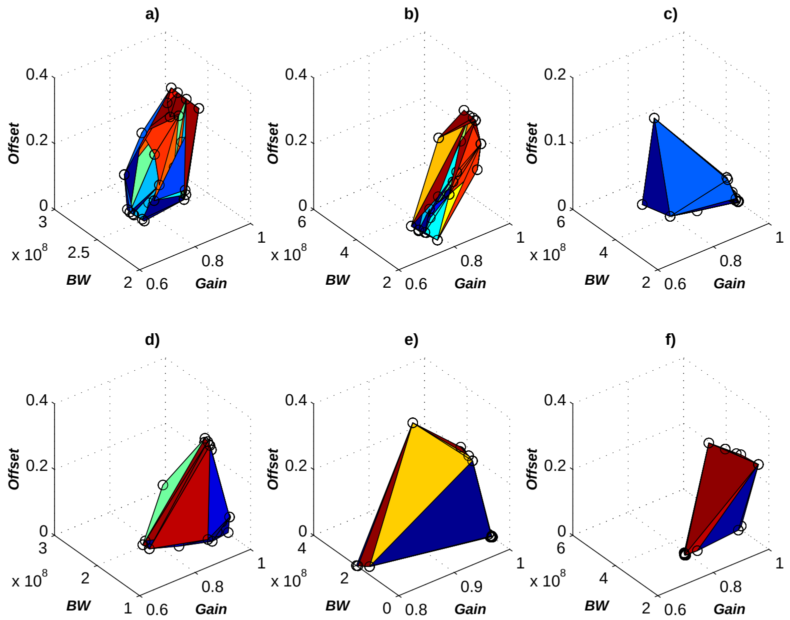


Figure 4.7: Pareto Front for six combinations of VM_C optimized by MOEA/D

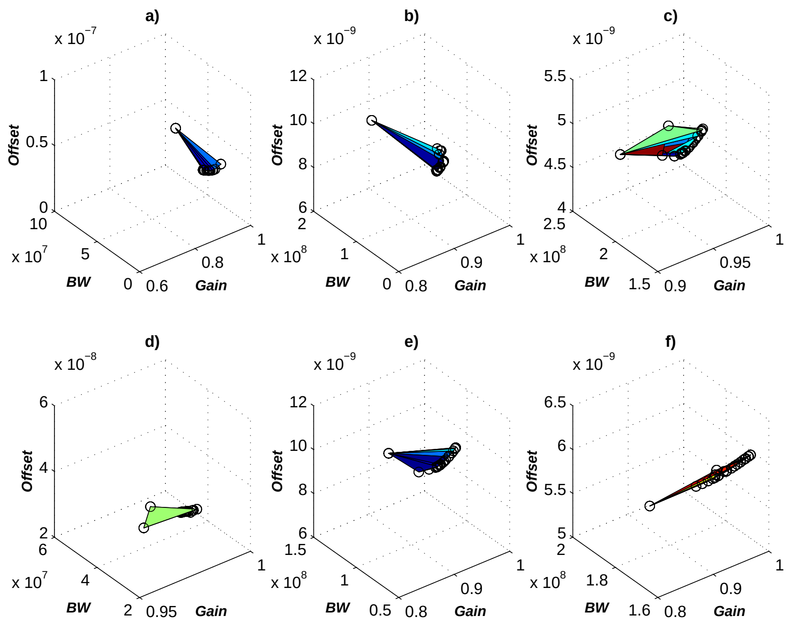


Figure 4.8: Pareto Front for six combinations of VF_A optimized by NSGA-II

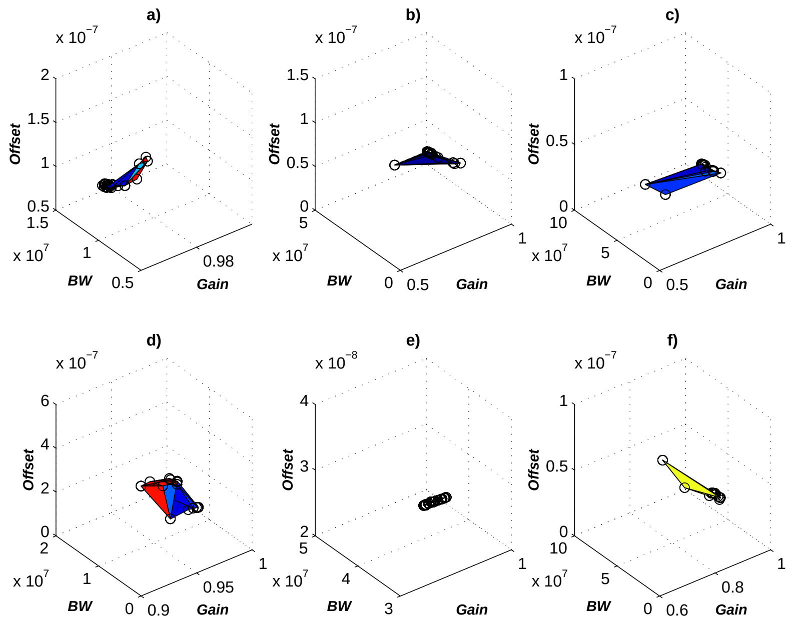


Figure 4.9: Pareto Front for six combinations of VF_B optimized by NSGA-II

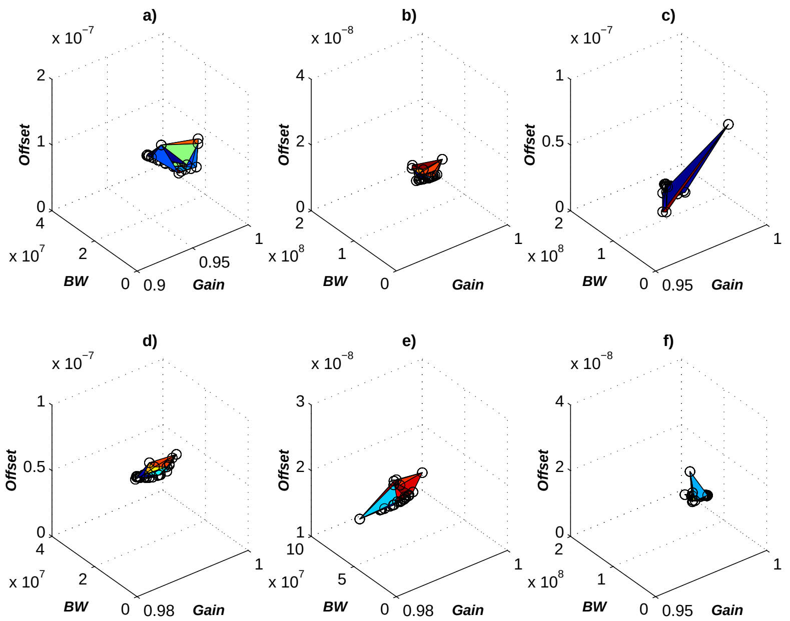


Figure 4.10: Pareto Front for six combinations of VF_C optimized by NSGA-II

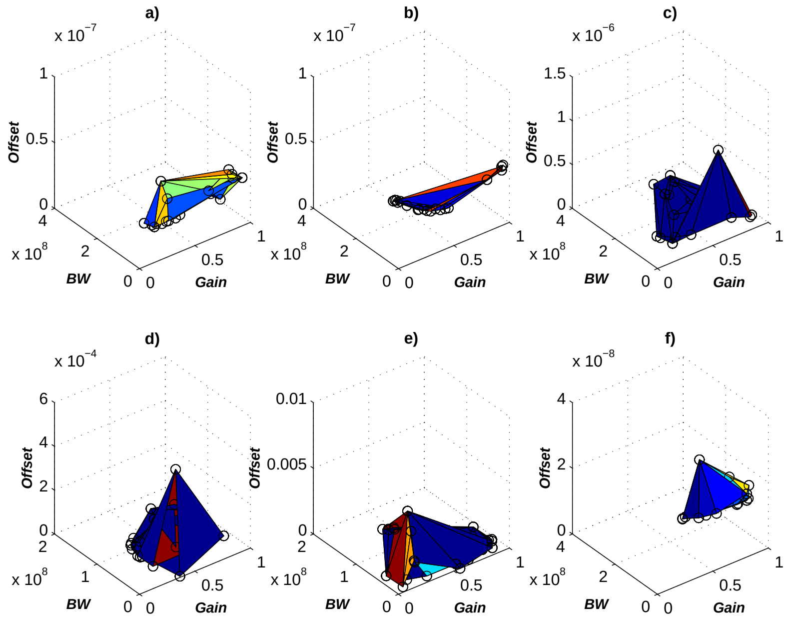


Figure 4.11: Pareto Front for six combinations of VM_A optimized by NSGA-II

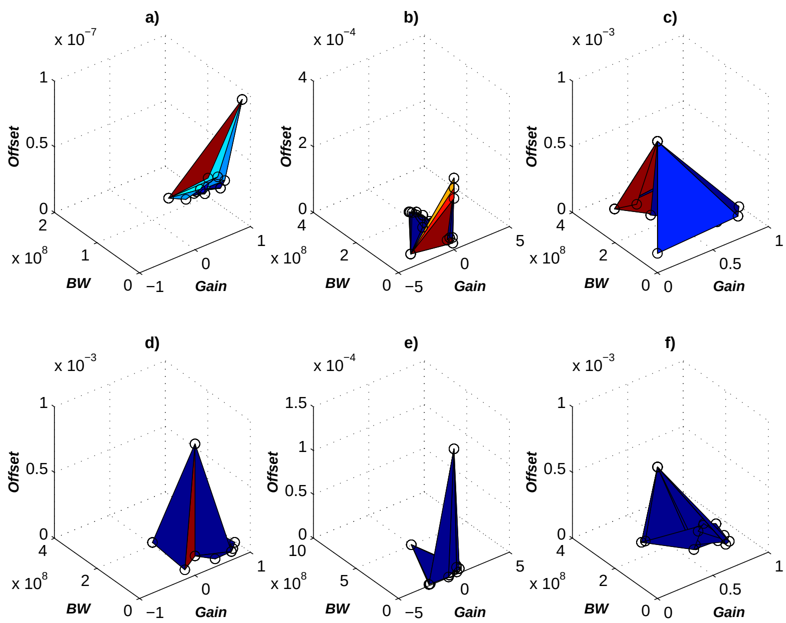


Figure 4.12: Pareto Front for six combinations of VM_B optimized by NSGA-II

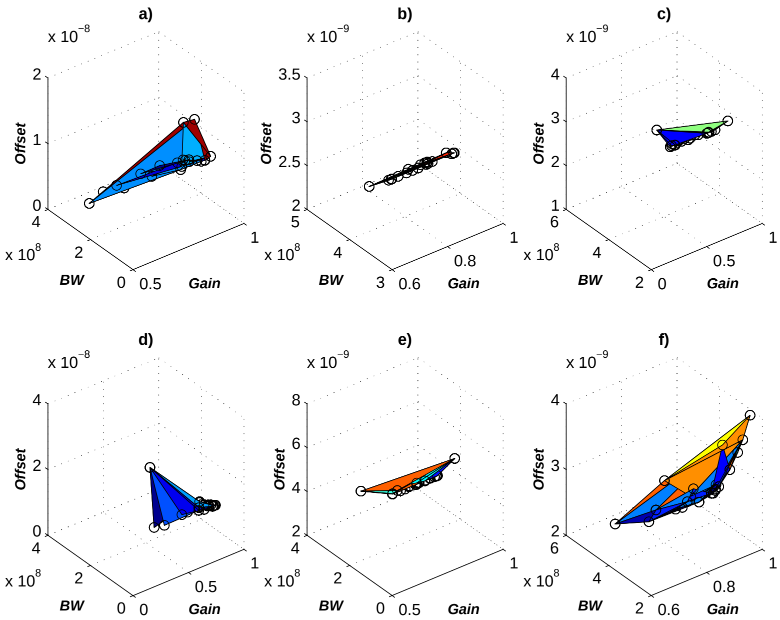


Figure 4.13: Pareto Front for six combinations of VM_C optimized by NSGA-II

4.2 Second Stage

From First Stage it were selected five circuits to be optimized: three voltage followers (VF_A, VF_B and VF_C) and two voltage mirrors (VM_A, and VM_B) in different bias conditions and with different transistor lengths, in this manner scoping the results is possible to choose the bests designs . In Fig. 4.1 are shown these selected circuits which are going to be optimized by NSGA-II and MOEA/D, each of the five circuits will be under the same bias conditions ($I_{ref} = 50\mu A$) and the same transistor length ($1\mu m$).

4.2.1 Analysis types

In this part it were selected the five circuits only biased with $Iref = 50\mu A$ and $L = 1\mu m$ then over 50 generations NSGA-II and MOEA/D are going to optimize them. Both methods are going to be used under the same parameters (number of population, number of generations and genetic operators) but it will be made changes to both methods to scout theirs behaviors under different algorithm conditions like genetics operators and in case of MOEA/D using Weighted Sum Approach and Tchebycheff Approach. Each optimization will be ran 5 times and by comparing both methods the objective is to know under which conditions works better each one, then this task will be done as follows:

I.- MOEA/D (with Tchebycheff Approach) and NSGA-II with DE

II.- MOEA/D (with Tchebycheff Approach) and NSGA-II with basic cross-over/mutation

III.- MOEA/D (with Weighted Sum Approach) with DE and MOEA/D (with Weighted Sum Approach) with basic cross-over/mutation

4.2.2 MOEA/D (with Tchebycheff approach) and NSGA-II with DE

Here, the optimization will be made by generating offsprings with DE genetic operator, the parameters for both methods are the same:

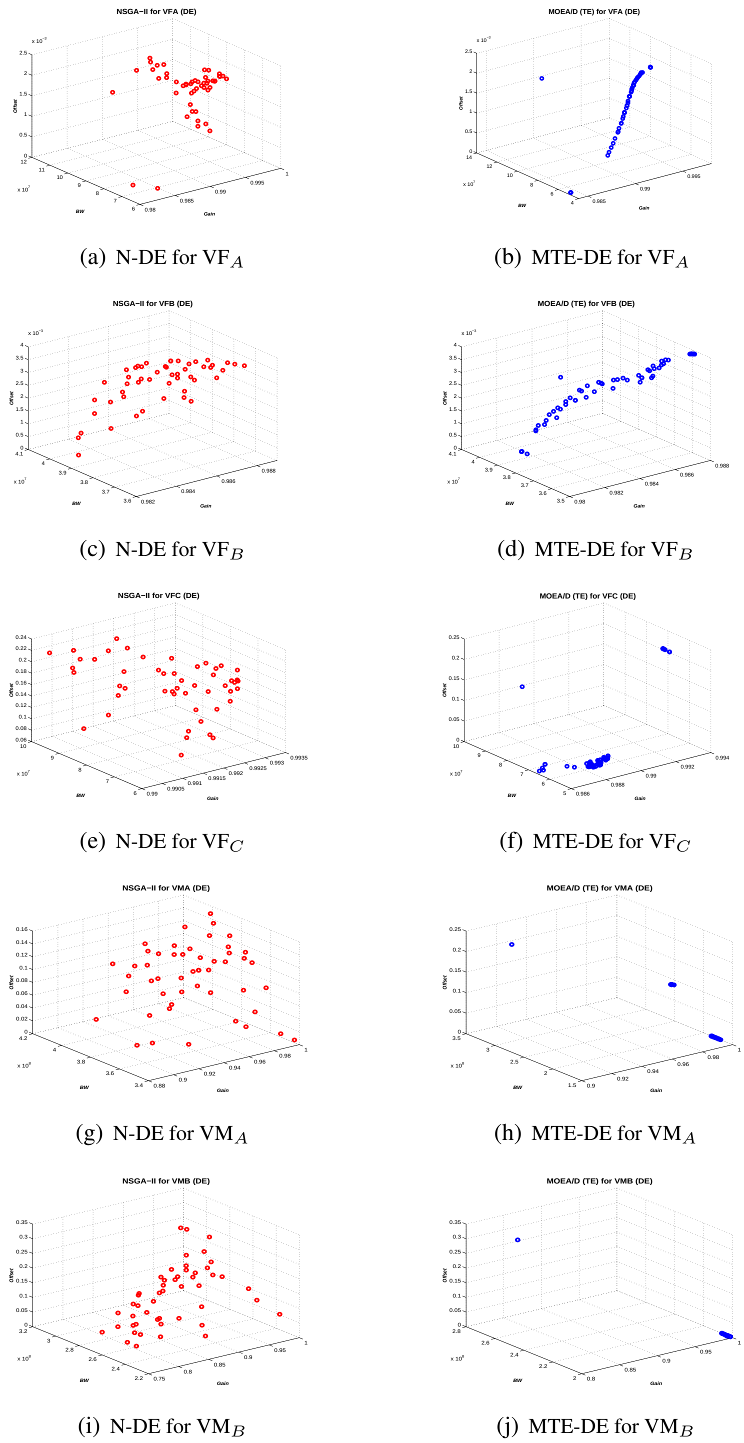
- N=50.
- Number maximum of loops=50.
- DE [29] like genetic operator.
- Lower limit of variables $0.35\mu m$ and Upper Limit of variables $300\mu m$.
- Generation of initial population randomly between Lower and Upper Limit.
- Optimize three objectives: gain, band width and offset.
- Constraint: every transistor in saturation condition.
- Tchebycheff Approach (TE) in MOEA/D.

In Fig. 4.14 are the comparative results for each circuit where it was used DE like genetic operator for 50 generations :

4.2.3 MOEA/D (with Tchebycheff approach) and NSGA-II with basic cross-over/mutation

Here, the optimizatin will be made by generating offsprings with basic cross-over/mutation genetics operator, the parameters for both methods are the same:

- N=50.
- Number maximum of loops=50.
- Basic cross-over/mutation.
- Lower limit of variables $0.35\mu m$ and Upper Limit of variables $300\mu m$.
- Generation of initial population randomly between Lower and Upper Limit.
- Optimize three objectives: gain, band width and offset.
- Constraint: every transistor in saturation condition.



(a) N-DE for VF$_A$

(b) MTE-DE for VF$_A$

(c) N-DE for VF$_B$

(d) MTE-DE for VF$_B$

(e) N-DE for VF$_C$

(f) MTE-DE for VF$_C$

(g) N-DE for VM$_A$

(h) MTE-DE for VM$_A$

(i) N-DE for VM$_B$

(j) MTE-DE for VM$_B$

Figure 4.14: NSGA-II and MOEA/D over 50 generations using DE

- Tchebycheff Approach (TE) in MOEA/D.

In Fig. 4.15 are the comparative results for each circuit where it was used cross-over/mutation like genetic operator for 50 generations :

4.2.4 MOEA/D (weighted sum approach) with DE and MOEA/D (Weighted Sum Approach) with basic cross-over/mutation

In this part now is going to be analyzed MOEA/D but this time using Weighted Sum Approach with DE and with basic cross-over/mutation, and the parameters are:

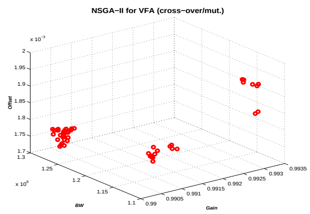
- N=50.
- Number maximum of loops=50.
- Lower limit of variables $0.35\mu m$ and Upper Limit of variables $300\mu m$.
- Generation of initial population randomly between Lower and Upper Limit.
- Optimize three objectives: gain, band width and offset.
- Constraint: every transistor in saturation condition.
- Weighted Sum Approach (WS) in MOEA/D.

In Fig. 4.16 are the comparative results for each circuit where it was used cross-over/mutation like genetic operator for 50 generations :

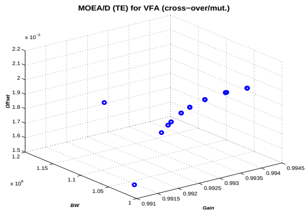
4.2.5 Comparative between performances

In this part it has been measured C-Metrics for all performances and their variations used in the last optimization were optimized five different circuits: VF_A, VF_B, VF_C, VM_A, VM_B. Then in each case is possible to know the behavior of each one of the optimizing methods used in this work to compare with each other. In Tables 4.19 to 4.23 are shown the C-Metrics for all cases for each circuit where *N-DE*, *MTE-DE* , *N-CR*, *MTE-CR* , *MWS-DE* and *MWS-TE* represent each performance used:

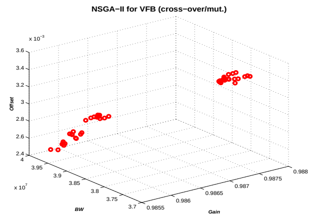
- *N-DE* represents NSGA-II performance with DE as genetic operator.
- *MTE-DE* represents MOEA/D performance with Tchebycheff (TE) approach and DE as genetic operator.
- *N-CR* represents NSGA-II performance with basic cross-over and mutation as genetics operators.



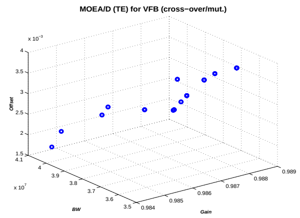
(a) N-CR for VF_A



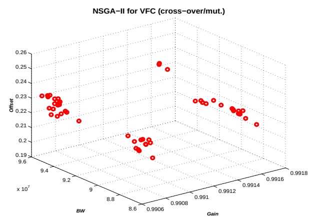
(b) MTE-CR for VF_A



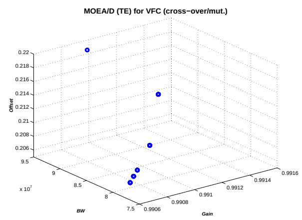
(c) N-CR for VF_B



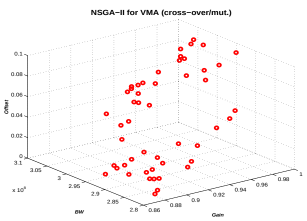
(d) MTE-CR for VF_B



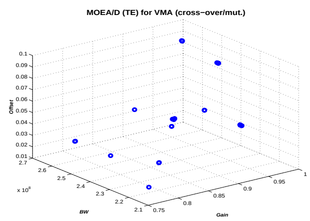
(e) N-CR for VF_C



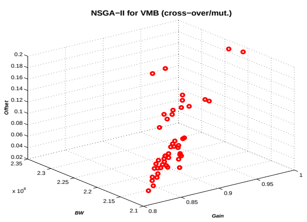
(f) MTE-CR for VF_C



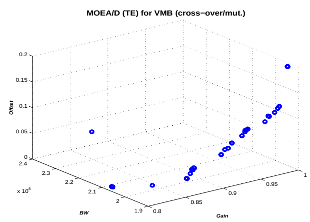
(g) N-CR for VM_A



(h) MTE-CR for VM_A

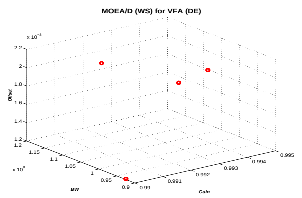


(i) N-CR for VM_B

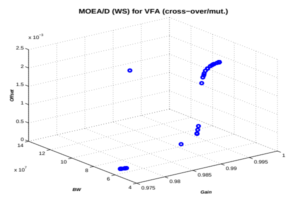


(j) MTE-CR for VM_B

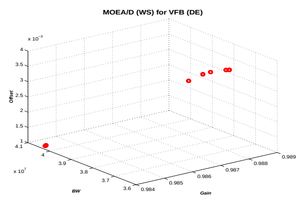
Figure 4.15: NSGA-II and MOEA/D over 50 generations using cross-over/mutation



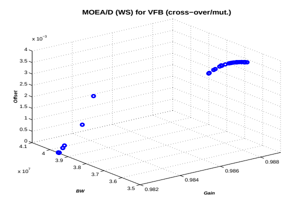
(a) MWS-DE for VF_A



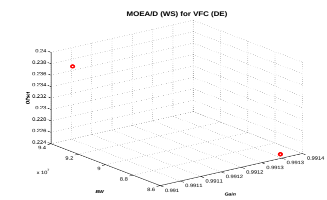
(b) MWS-CR for VF_A



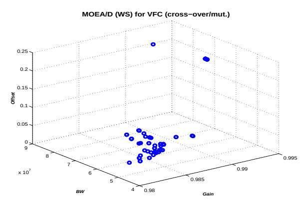
(c) MWS-DE for VF_B



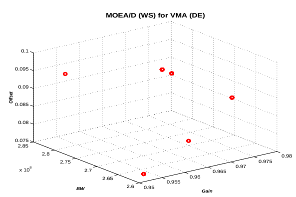
(d) MWS-CR for VF_B



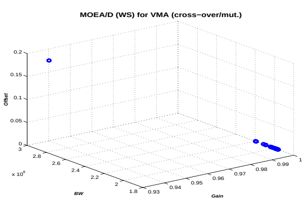
(e) MWS-DE for VF_C



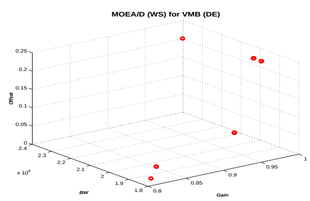
(f) MWS-CR for VF_C



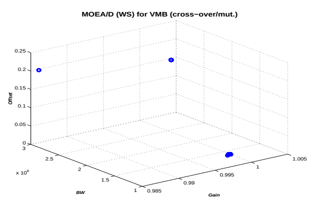
(g) MWS-DE for VM_A



(h) MWS-CR for VM_A



(i) MWS-DE for VM_B



(j) MWS-CR for VM_B

Figure 4.16: MOEA/D with Weigthed Sum over 50 generations using DE and cross-over/mutation

Table 4.19: Comparative Table of C-Metric for each performance for VF_A

	N-DE	***MTE-DE***	***N-CR***	***MTE-CR***	***MWS-DE***	***MWS-CR***
N-DE	0	0.863	1	1	1	0.845
MTE-DE	0.983	0	1	1	1	0.845
N-CR	0.027	0.004	0	0.012	0.024	0
MTE-CR	0.300	0.020	1	0	1	0.017
MWS-DE	0.190	0.020	1	0.964	0	0
MWS-CR	0.975	0.827	1	1	1	0
AVERAGE	0.413	0.289	0.833	0.663	0.671	0.284

Table 4.20: Comparative Table of C-Metric for each performance for VF_B

	N-DE	***MTE-DE***	***N-CR***	***MTE-CR***	***MWS-DE***	***MWS-CR***
N-DE	0	0.885	1	0.896	0.858	0.483
MTE-DE	0.829	0	0.994	0.819	0.596	0.297
N-CR	0.432	0.180	0	0.333	0.231	0.136
MTE-CR	0.941	0.880	1	0	0.700	0.525
MWS-DE	0.965	0.880	1	0.896	0	0.602
MWS-CR	0.516	0.369	0.624	0.559	0.596	0
AVERAGE	0.614	0.532	0.770	0.584	0.497	0.340

- *MTE-CR* represents MOEA/D performance with Tchebycheff (TE) approach and crossover and mutation as genetics operators.
- *MWS-DE* represents MOEA/D performance with Weighted Sum (WS) approach and DE as genetics operators.
- *MWS-CR* represents MOEA/D performance with Weighted Sum (WS) approach and crossover and mutation as genetics operators.

It were selected the only one performance for each circuit and have made a 3x3 plot where is shown the interplay between the objective functions (Gain, BW and Offset).

Table 4.21: Comparative Table of C-Metric for each performance for VF_C

	N-DE	***MTE-DE***	***N-CR***	***MTE-CR***	***MWS-DE***	***MWS-CR***
N-DE	0	0.039	0.867	1	1	0.287
MTE-DE	1	0	1	1	1	0.495
N-CR	0.173	0	0	0.310	0.644	0
MTE-CR	0.083	0.006	0.489	0	0.966	0
MWS-DE	0.048	0	0.178	0.103	0	0
MWS-CR	1	0.148	1	1	1	0
AVERAGE	0.384	0.032	0.589	0.569	0.768	0.130

Table 4.22: Comparative Table of C-Metric for each performance for VM_A

	N-DE	***MTE-DE***	***N-CR***	***MTE-CR***	***MWS-DE***	***MWS-CR***
N-DE	0	0	0	0	0	0.364
MTE-DE	0.566	0	0	0	0	0
N-CR	0.977	0.500	0	0.073	0.231	0.091
MTE-CR	0.977	0.750	0.827	0	0.794	0.091
MWS-DE	0.987	0.583	0.962	0.758	0	0.455
MWS-CR	0.107	0.667	0	0	0	0
AVERAGE	0.602	0.417	0.298	0.138	0.171	0.167

Table 4.23: Comparative Table of C-Metric for each performance for VM_B

	N-DE	***MTE-DE***	***N-CR***	***MTE-CR***	***MWS-DE***	***MWS-CR***
N-DE	0	0.174	0.065	0.442	0.168	0.532
MTE-DE	0.591	0	0.006	0.083	0.084	0.532
N-CR	0.969	0.174	0	0.392	0.721	0.532
MTE-CR	0.916	0.261	0.806	0	0.589	0.532
MWS-DE	0.916	0.435	0.871	0.817	0	0.532
MWS-CR	0.819	1	0.815	0.900	0.616	0
AVERAGE	0.702	0.341	0.427	0.439	0.363	0.443

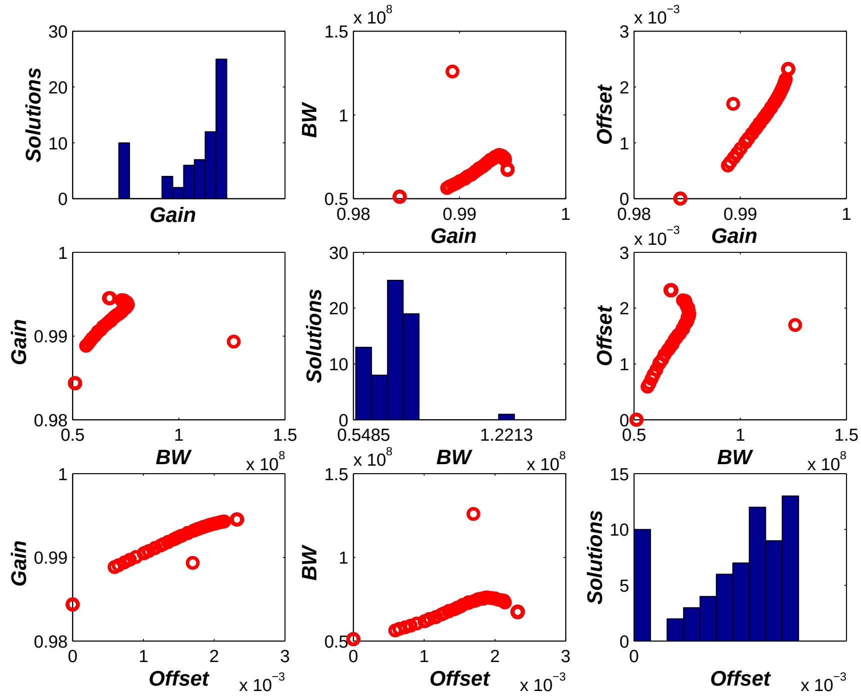


Figure 4.17: MTE-DE for VF_A after 50 generations for one run

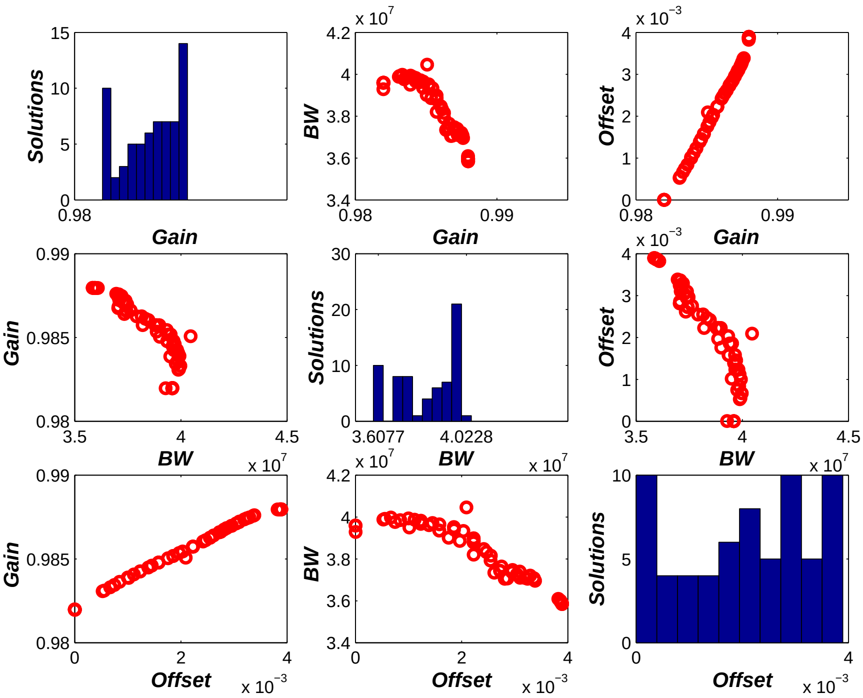


Figure 4.18: MTE-DE for VF$_B$ after 50 generations for one run

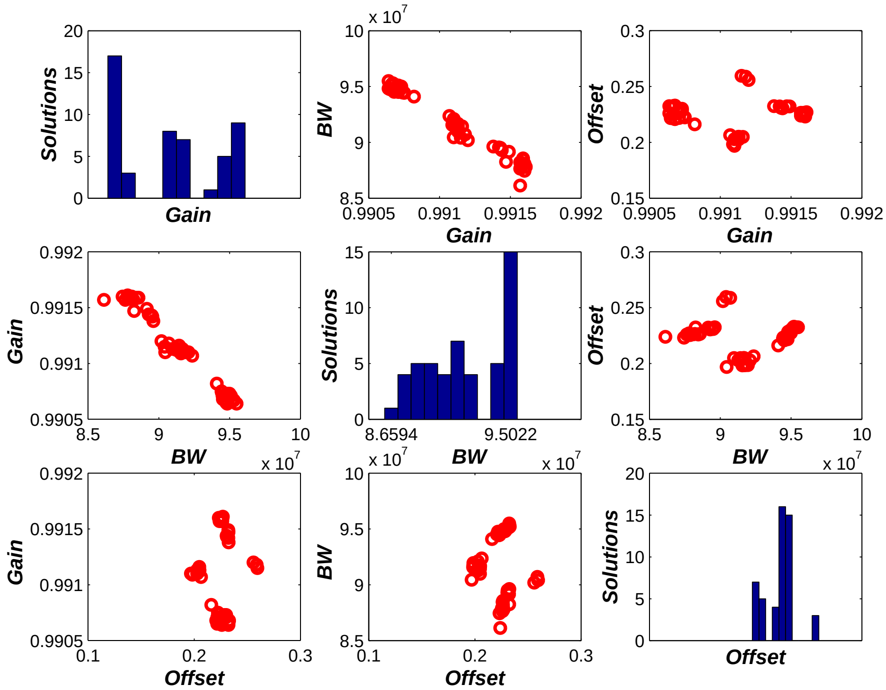


Figure 4.19: N-CR for VF_C after 50 generations for one run

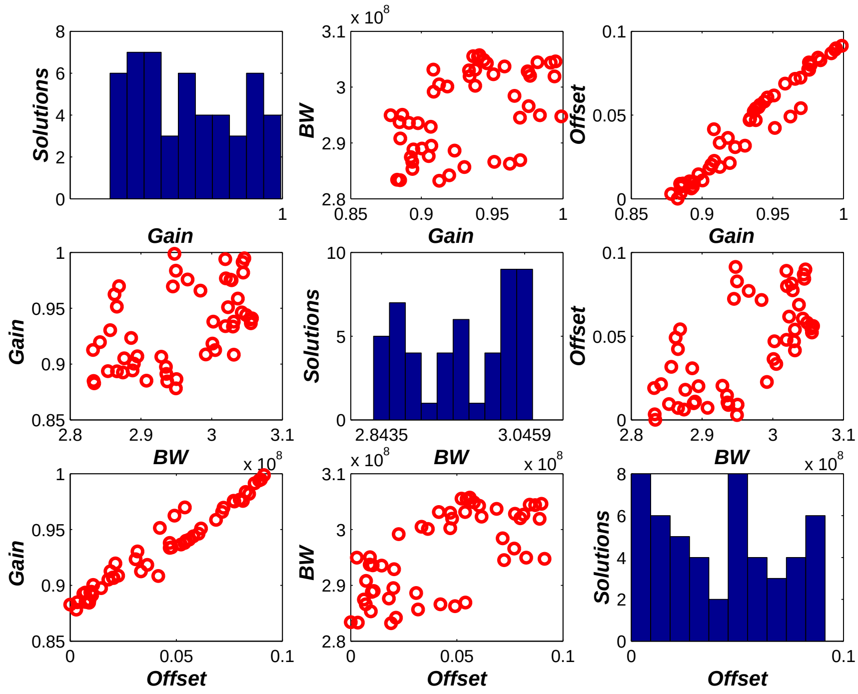


Figure 4.20: N-CR for VM_A after 50 generations for one run

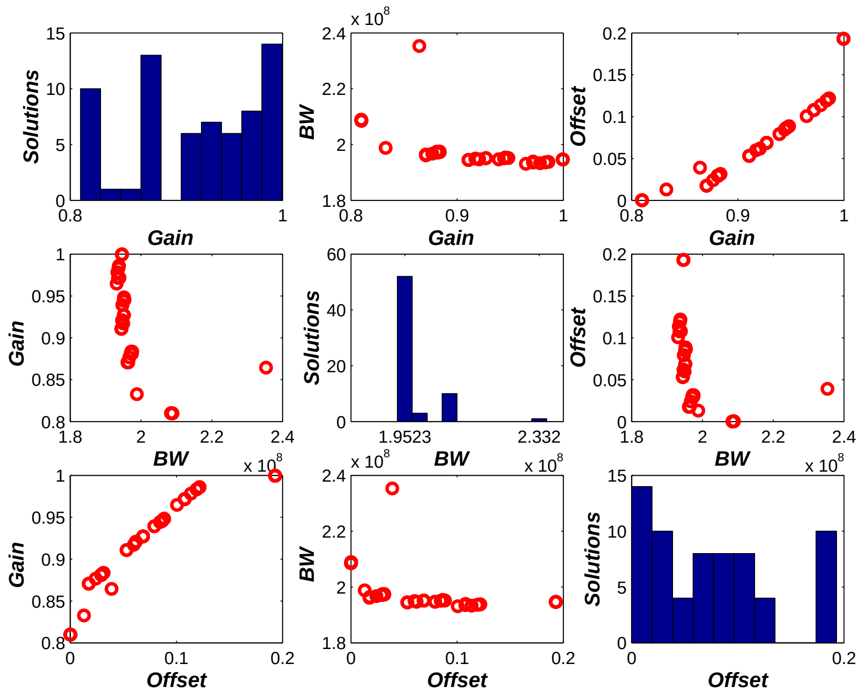


Figure 4.21: MTE-CR for VM_B after 50 generations for one run

Chapter 5

CURRENT CONVEYOR OPTIMIZATION

In this chapter the proposed optimization system will be applied on three different current conveyors: the first one is a first generation direct current conveyor (DOCCI), the second one is a second generation direct current conveyor (DOCCII) and the other one is a third generation inverse current conveyor (DOICCIII). The selected circuits has four ports, then there is a resistance in each terminal so this time the objective will be to improve these four resistances (r_x, r_y, r_{z+} and r_{z-}) but ensuring saturation conditions in all transistors and a closer unity gain in each terminal too (AV_x, AI_y, AI_{z+} and AI_{z-}). The way to include these gains by including them as constraints in the optimization process but only for DCCII one gain is included as objective.

Due to the complexity of the circuits this time all the optimization process will be made with DE operator for NSGA-II and MOEA/D engine, conserving all the rest of parameters like was exposed in 3.3. In all circuits the length for all transistors is $1\mu m$, I_{ref} is $50\mu A$ and the number of solutions are eighty four over 50 generations.

5.1 DOCCI optimization

This circuit is a double output direct first generation current conveyor and it is made by joining a voltage follower and a double current mirror like is shown in Fig. 5.1. The behavior of this circuit is described by the next equations:

- $V_X = V_Y$

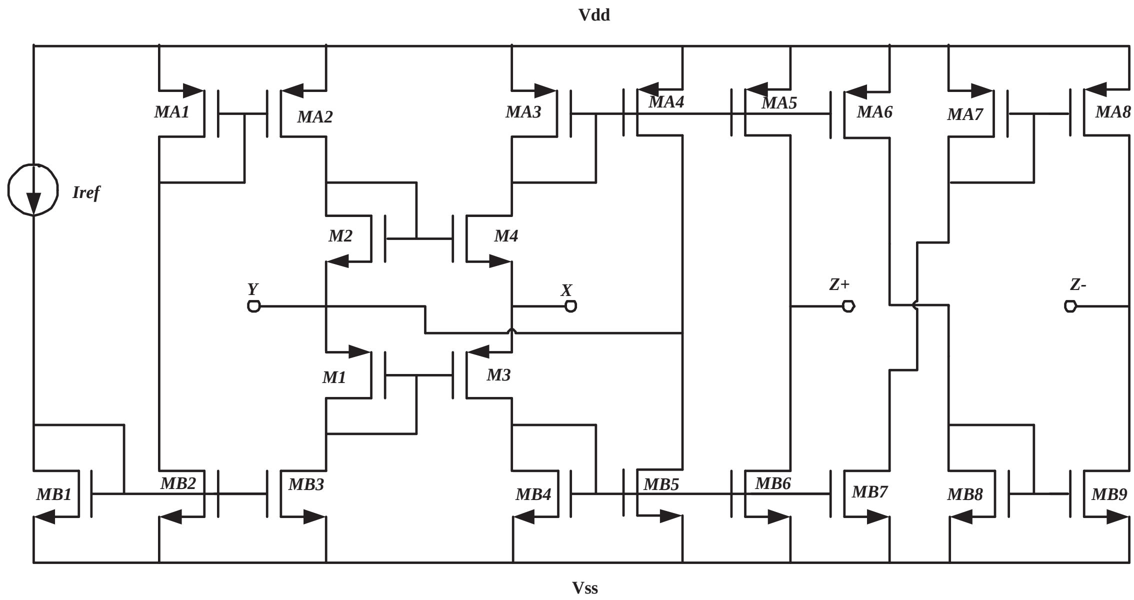


Figure 5.1: Double Output Direct First Generation Current Conveyor (DOCCI)

- $I_Y = I_X$
- $I_{Z+} = I_X$
- $I_{Z-} = -I_X$

For this circuit all NMOS transistors have the same width and the PMOS have the same width too. In this manner Table 5.1 shows the transistors who are affected for which variable.

Table 5.1: Variables for DOCCI in optimization process

Variable Name	Transistors
W_1	$MA_1, MA_2, ..., MA_8$, M1 ,M3
W_2	$MB_1, MB_2, ..., MB_9$, M2 ,M4

The optimization process has the task to improve the terminal resistances in X, Y, Z+ and Z- conserving saturation conditions in all transistors and a closer unity gain in each terminal. After the optimization process is possible to see the results in Figs. 5.2, 5.3 and 5.4 after 50 generations, where *b), c)* and *d)* items are the relationship between all objective functions with the first objective function (R_X), *e), g)* and *h)* items are the relationship between all objective functions with the second objective function (R_Y), *i), j)* and *l)* items are the relationship be-

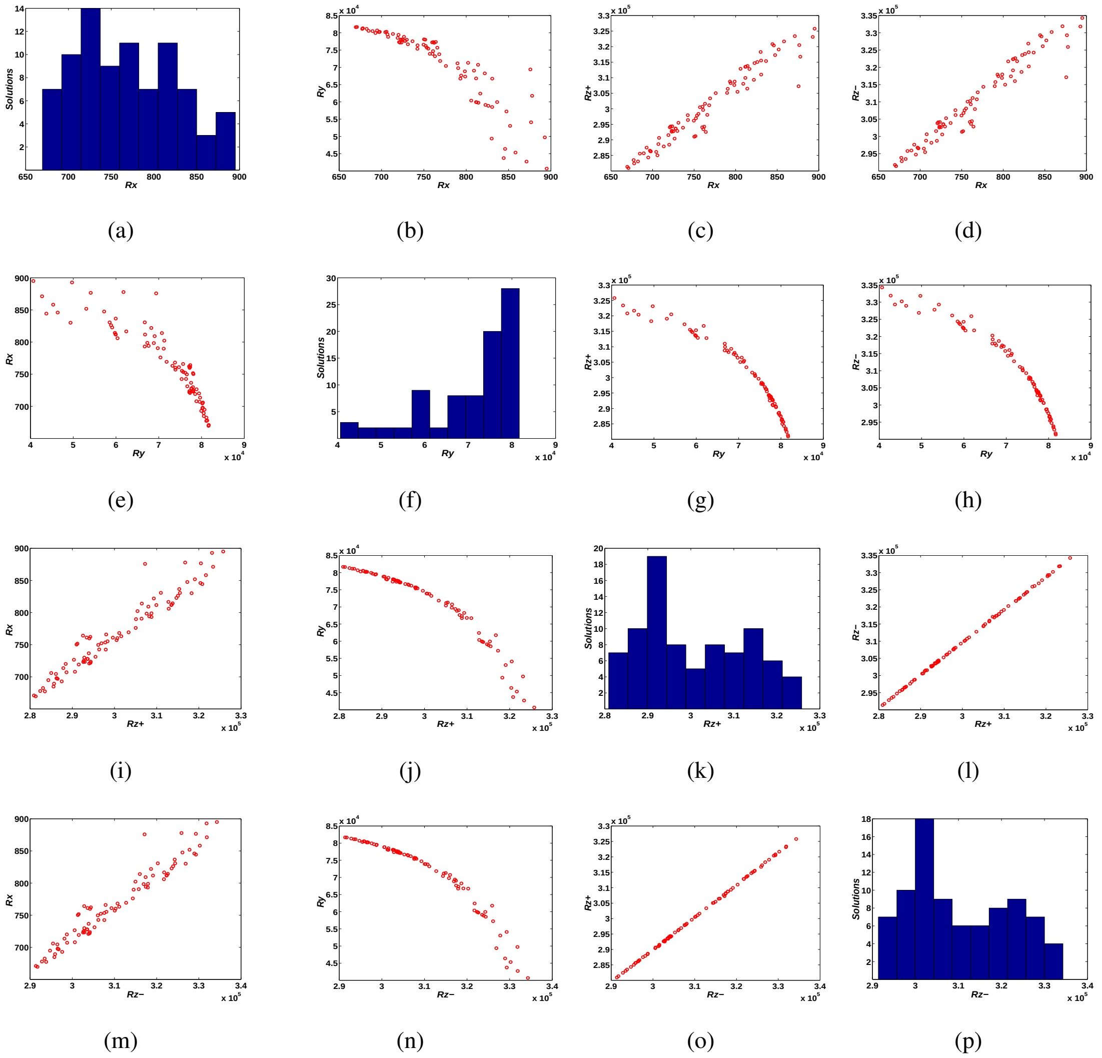


Figure 5.2: N-DE Approach for DOCCI

tween all objective functions with the third objective function (R_{Z+}), *m), n)* and *o)*items are the relationship between all objective functions with the fourth objective function (R_{Z-}). In items *a), f), k)* and *p)* are histograms of the population over the solution space.

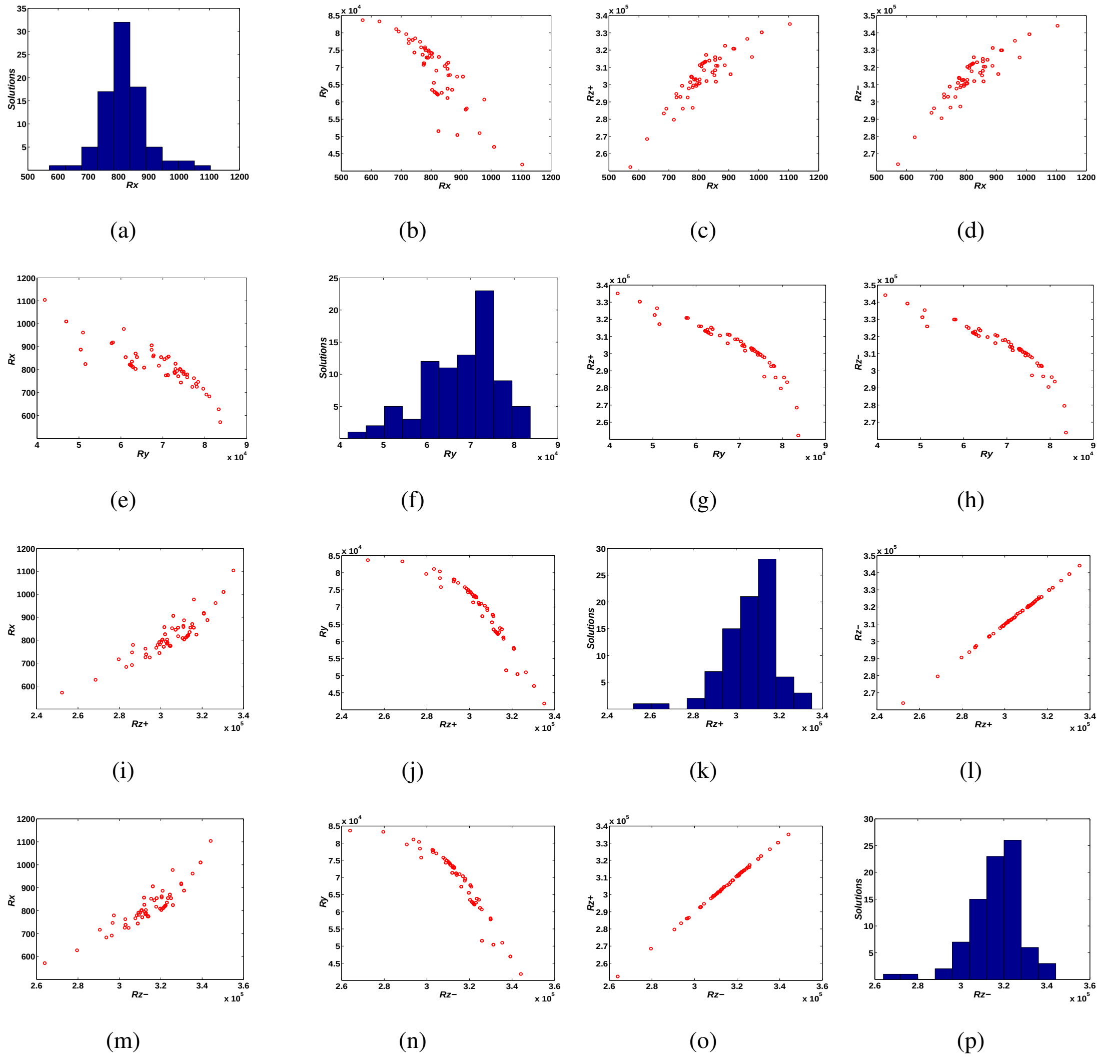


Figure 5.3: MTE-DE Approach for DOCCI

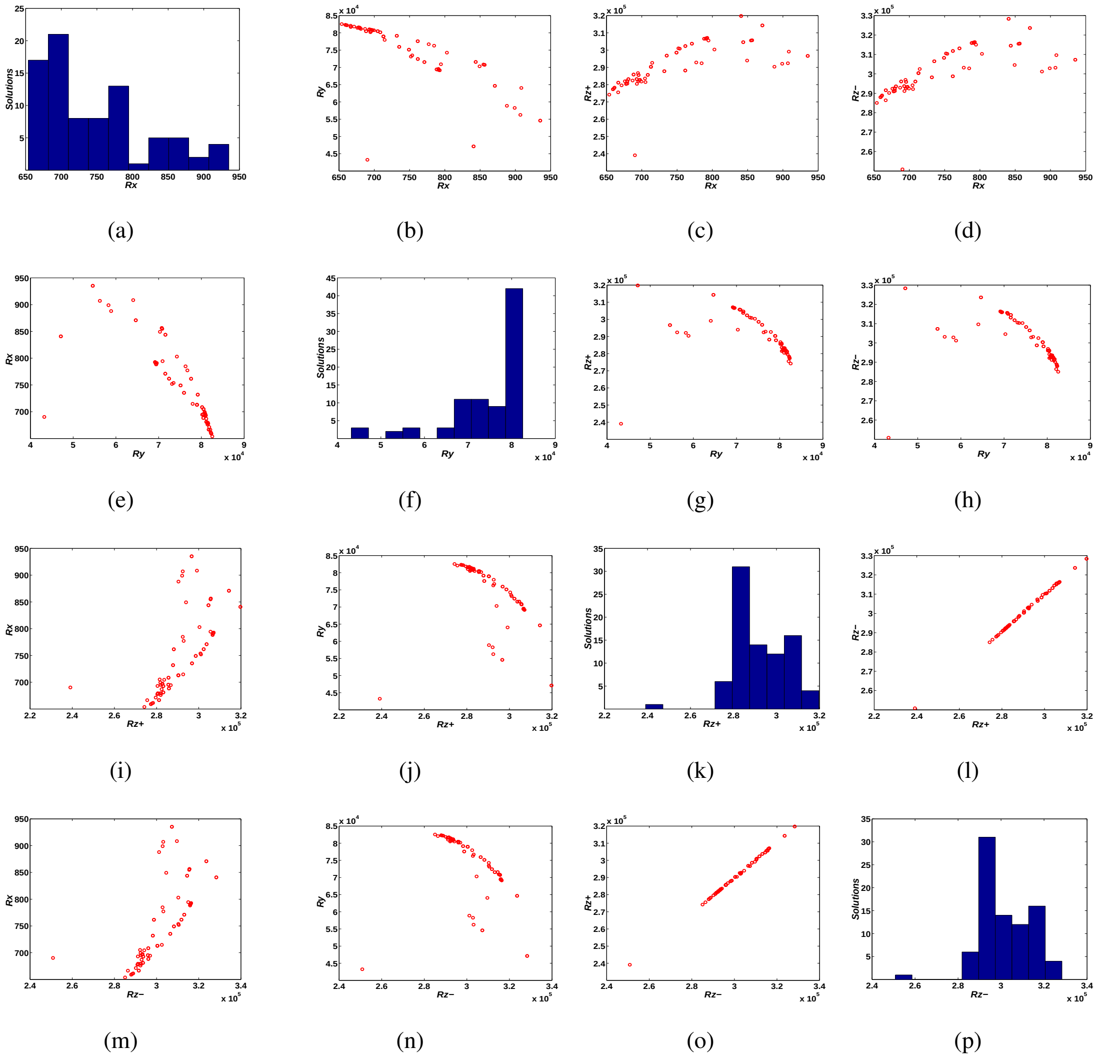


Figure 5.4: MWS-DE Approach for DOCCI

It is possible to select the best solution for each case: NSGA-II applying DE (N-DE), MOEA/D with Tchebycheff Approach applying DE (MTE-DE) and MOEA/D with Weighted Sum Approach applying DE (MWS-DE) and compare the results for the four objectives (r_x, r_y, r_{z+} and r_{z-}) , the constraints ((AV_x, AI_y, AI_{z+} and AI_{z-})) and some others parameters such as Band Width (BW) and Offset. Then in Table 5.2 is possible to see all these results:

Table 5.2: Best Performances for each method for DOCCI

Method	W1	W2	Port	Resistance	Gain	BW	Offset
N-DE	286.68 μm	160.66 μm	X	669.6161Ω	0.99 $(\frac{V}{V})$	64.929 MHz	-0.88872 mV
			Y	81.6413 $K\Omega$	1.0196 $(\frac{I}{I})$	59.226 MHz	-1.9518 μA
			Z+	291.7468 $K\Omega$	1.0110 $(\frac{I}{I})$	54.970 MHz	-1.0326 μA
			Z-	281.3263 $K\Omega$	-1.0544 $(\frac{I}{I})$	39.480 MHz	-0.89686 μA
MTE-DE	242.6 μm	42.091 μm	X	571.3755Ω	0.99247 $(\frac{V}{V})$	81.291 MHz	-1.1018 mV
			Y	83.6698 $K\Omega$	1.0250 $(\frac{I}{I})$	57.264 MHz	-2.3029 μA
			Z+	263.9356 $K\Omega$	1.0217 $(\frac{I}{I})$	55.034 MHz	-0.60497 μA
			Z-	281.3263 $K\Omega$	-1.0679 $(\frac{I}{I})$	41.0751 MHz	-0.82650 μA
MWS-DE	226.65 μm	22.23 μm	X	653.872Ω	0.99115 $(\frac{V}{V})$	77.448 MHz	-0.98917 mV
			Y	82.5407 $K\Omega$	1.0213 $(\frac{I}{I})$	65.376 MHz	-2.015 μA
			Z+	285.04 $K\Omega$	1.0155 $(\frac{I}{I})$	61.442 MHz	-0.84464 μA
			Z-	274.2025 $K\Omega$	-1.0577 $(\frac{I}{I})$	45.061 MHz	-0.87884 μA

5.2 DOCCII optimization

This circuit is a double output direct second generation current conveyor and it is made by joining a voltage follower and a current mirror like is shown in Fig. 5.5. The behavior of this circuit is described by the next equations:

- $V_X = V_Y$
- $I_Y = 0$
- $I_{Z+} = I_X$
- $I_{Z-} = -I_X$

For this circuit all NMOS transistors have the same width and the PMOS have the same width too. In this manner Table 5.3 shows the transistors who are affected for which variable.

Table 5.3: Variables for DOCCII in optimization process

Variable Name	Transistors
W_1	$MA_1, MA_2, \ldots, MA_8$, M1 ,M3
W_2	$MB_1, MB_2, \ldots, MB_9$, M2 ,M4

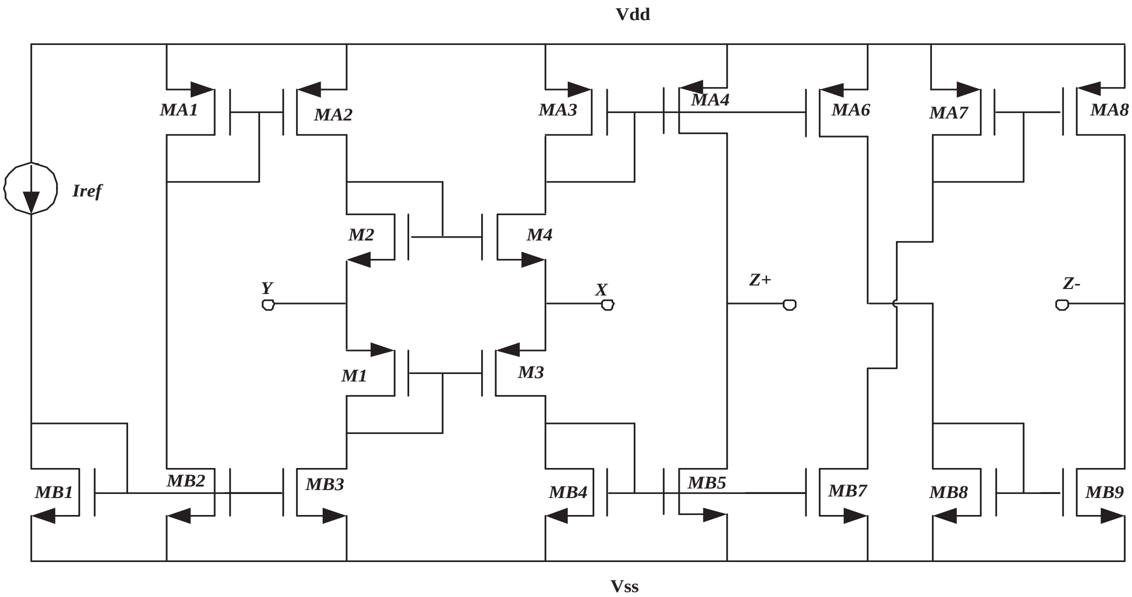


Figure 5.5: Double Output Direct Second Generation Current Conveyor (DOCCII)

The optimization process has the task to improve the terminal resistances in X, Z+, Z- and as a fourth objective is included the gain in terminal X (A_{VX}) conserving saturation conditions in all transistors and a closer unity gain in the rest of terminals. After the optimization process is possible to see the results in Figs. 5.6, 5.7 and 5.8 after 50 generations, where *b), c)* and *d)* items are the relationship between all objective functions with the first objective function (R_X), *e), g)* and *h)* items are the relationship between all objective functions with the second objective function (A_{VX}), *i), j)* and *l)* items are the relationship between all objective functions with the third objective function (R_{Z+}), *m), n)* and *o)*items are the relationship between all objective functions with the fourth objective function (R_{Z-}). In items *a), f), k)* and *p)* are histograms of the population over the solution space.

It is possible to select the best solution for each case: NSGA-II applying DE (N-DE), MOEA/D with Tchebycheff Approach applying DE (MTE-DE) and MOEA/D with Weighted

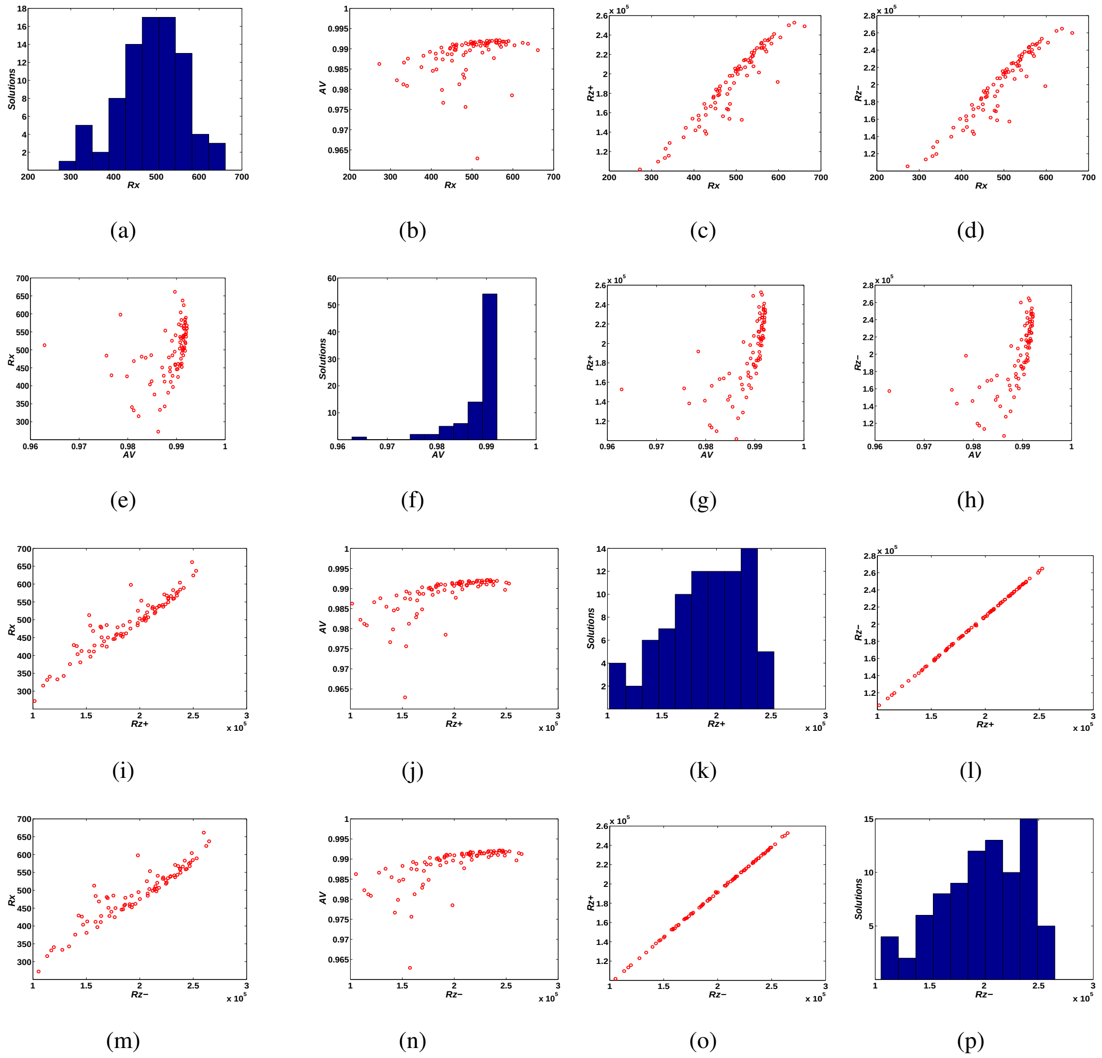


Figure 5.6: N-DE Approach for DOCCII

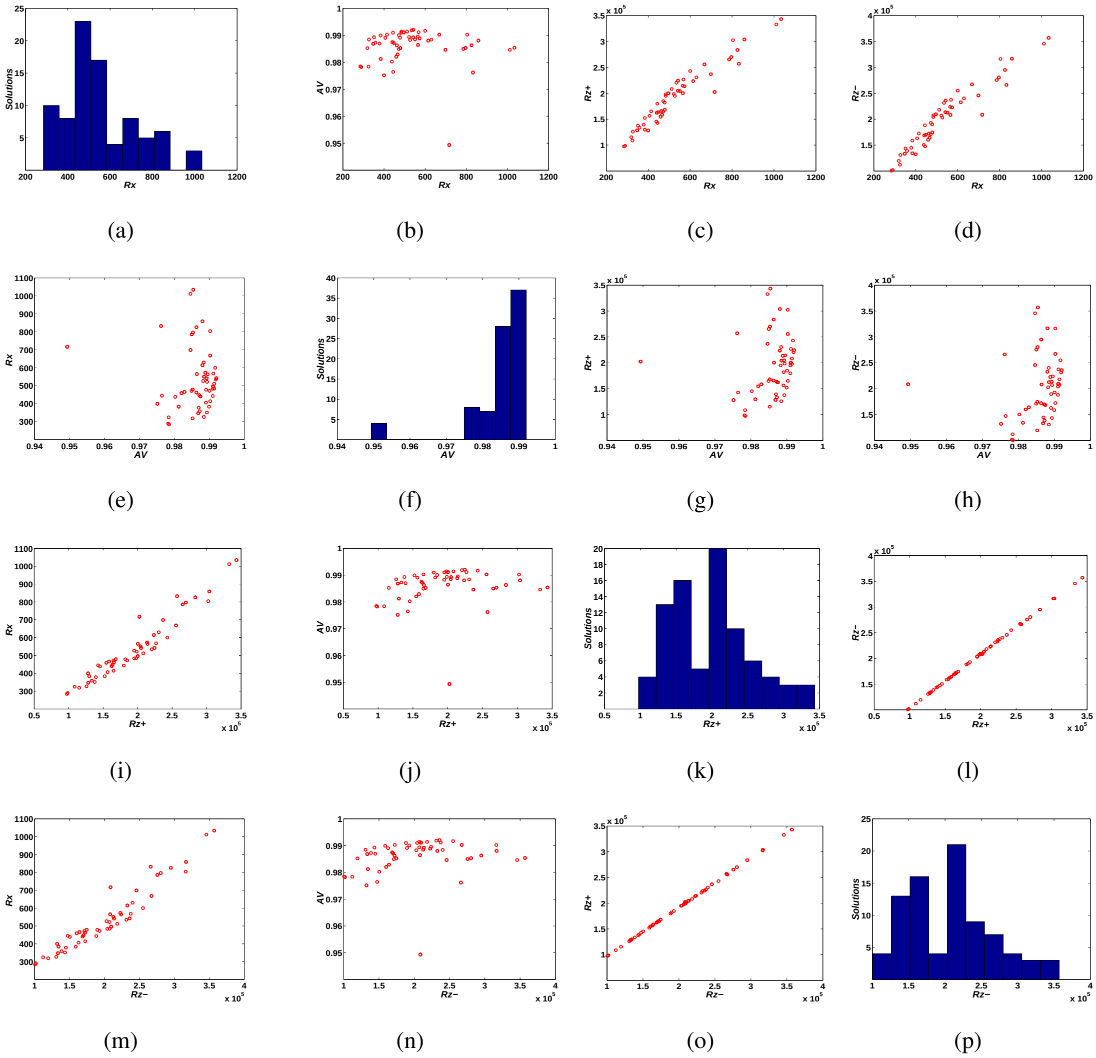


Figure 5.7: MTE-DE Approach for DOCCII

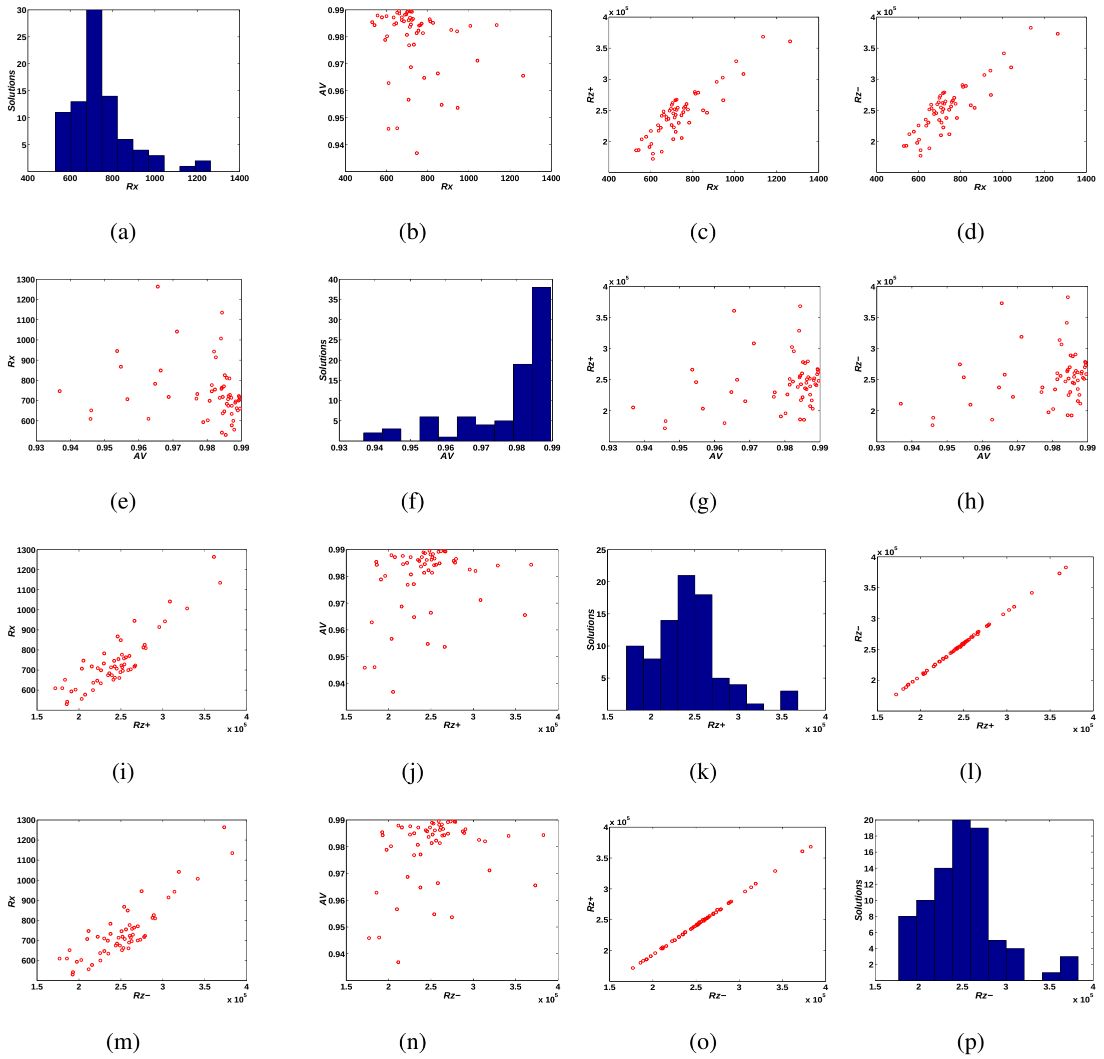


Figure 5.8: MWS-DE Approach for DOCCII

Sum Approach applying DE (MWS-DE) and compare the results for the four objectives (r_x, A_V, r_{z+} and r_{z-}) , the constraints (AI_y, AI_{z+} and AI_{z-})) and some others parameters such as Band Width (BW) and Offset. Then in Table 5.4 is possible to see all these results:

Table 5.4: Best Performances for each method for DOCCII

Method	W1	W2	Port	Resistance	Gain	BW	Offset
N-DE	74.776 μm	72.945 μm	X	637.1473Ω	0.99125 ($\frac{V}{V}$)	117.59 MHz	-2.2060 mV
			Z+	$264.8563\ K\Omega$	1.0240 ($\frac{I}{I}$)	90.763 MHz	2.09 nA
			Z-	$252.7134\ K\Omega$	-1.0702 ($\frac{I}{I}$)	63.965 MHz	-697.47 nA
MTE-DE	60.742 μm	61.722 μm	X	668.4148Ω	0.99025 ($\frac{V}{V}$)	123.14 MHz	-0.90431 mV
			Z+	$267.2798\ K\Omega$	1.0223 ($\frac{I}{I}$)	102.06 MHz	8.825 nA
			Z-	$255.6926\ K\Omega$	-1.0658 ($\frac{I}{I}$)	72.026 MHz	-747.65 nA
MWS-DE	40.288 μm	46.768 μm	X	714.0247Ω	0.987 ($\frac{V}{V}$)	132.47 MHz	4.2685 mV
			Z+	$261.7766\ K\Omega$	1.0186 ($\frac{I}{I}$)	127.55 MHz	22.93 nA
			Z-	$251.6469\ K\Omega$	-1.0569 ($\frac{I}{I}$)	90.203 MHz	-887.07 nA

5.3 DOICCIII optimization

This circuit is a double output inverse third generation current conveyor and it is made by joining a voltage mirror, a current mirror and a current follower like is shown in Fig. 5.9. The behavior of this circuit is described by the next equations:

- $V_X = -V_Y$
- $I_Y = -I_X$
- $I_{Z+} = I_X$
- $I_{Z-} = -I_X$

In this circuit is going to test six variables so Table 5.5 shows the transistors who are affected for which variable.

Table 5.5: Variables for DOICCIII in optimization process

Variable Name	Transistors
W_1	MA_1, MA_2, ..., MA_8 , M1 ,M3
W_2	MB_1, MB_2, ..., MB_9 , M2 ,M4
W_3	MC
W_4	MD
W_5	MA
W_6	MB

The optimization process has the task to improve the terminal resistances in X, Y, Z+ and Z- conserving saturation conditions in all transistors and a closer unity gain. After the optimization process is possible to see the results in Figs. 5.10, 5.11 and 5.12 after 50 generations, where *b), c)* and *d)* items are the relationship between all objective functions with the first objective function (R_X), *e), g)* and *h)* items are the relationship between all objective functions with the second objective function (R_Y), *i), j)* and *l)* items are the relationship between all objective functions with the third objective function (R_{Z+}), *m), n)* and *o)*items are the relationship between all objective functions with the fourth objective function (R_{Z-}). In items *a), f), k)* and *p)* are histograms of the population over the solution space.

It is possible to select the best solution for each case: NSGA-II applying DE (N-DE), MOEA/D with Tchebycheff Approach applying DE (MTE-DE) and MOEA/D with Weighted

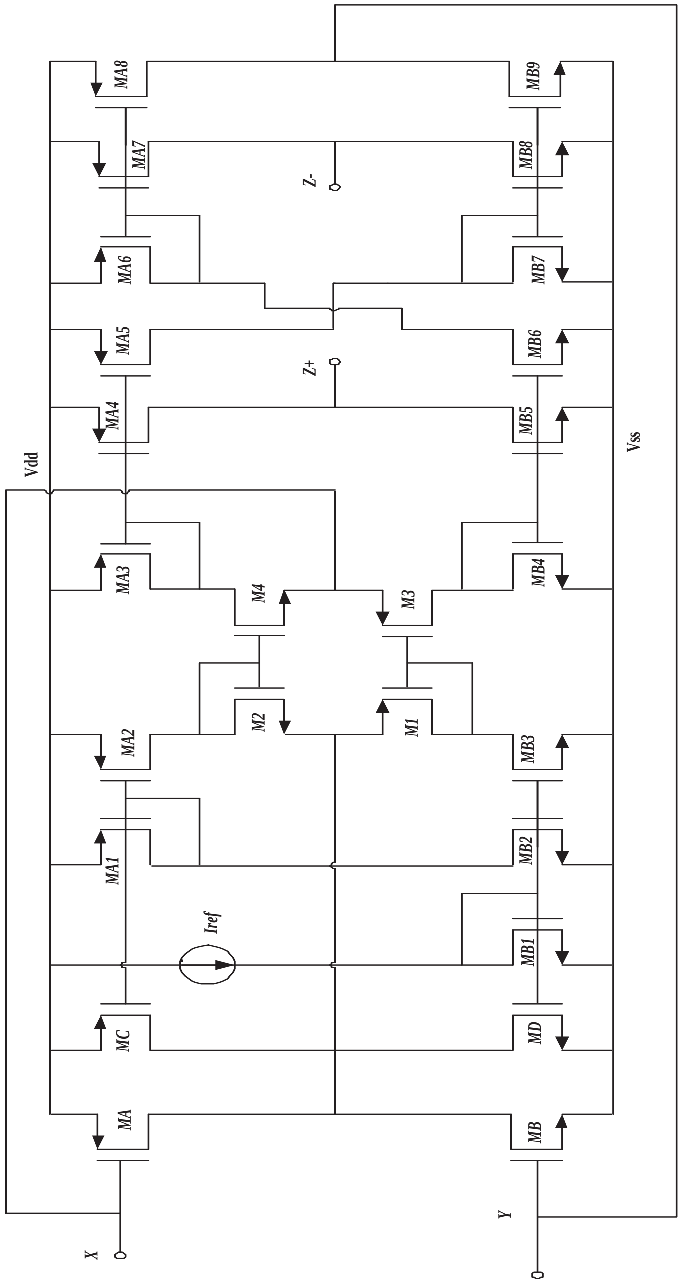
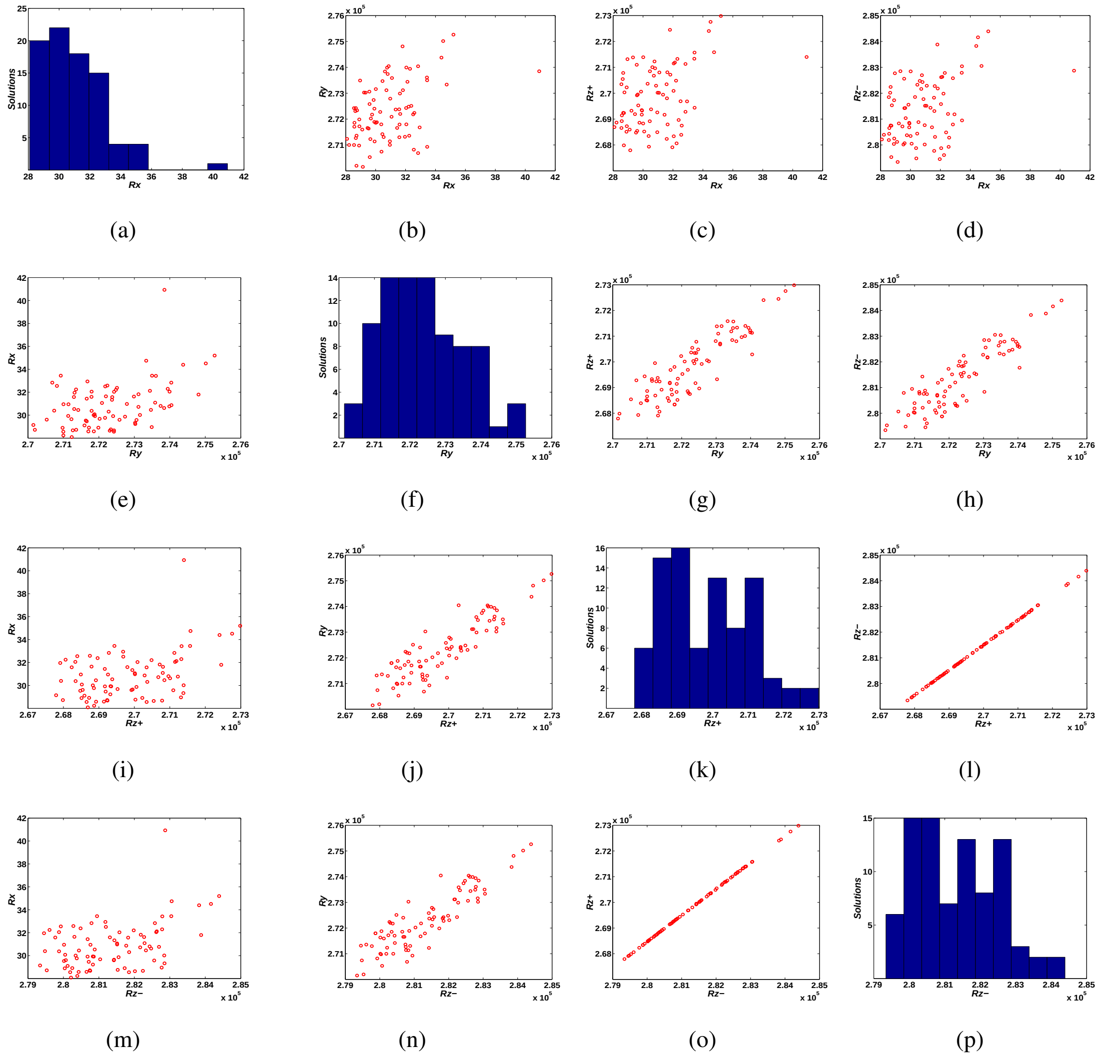


Figure 5.9: Double Output Inverse Third Generation Current Conveyor (DOICCII)

Sum Approach applying DE (MWS-DE) and compare the results for the four objectives (r_x, r_y, r_{z+} and r_{z-}) , the constraints ((AV_x, AI_y, AI_{z+} and AI_{z-})) and some others parameters such as Band Width (BW) and Offset. Then in Table 5.6 is possible to see all these results:



(a) (b) (c) (d)

(e) (f) (g) (h)

(i) (j) (k) (l)

(m) (n) (o) (p)

Figure 5.10: N-DE for DOICCIII

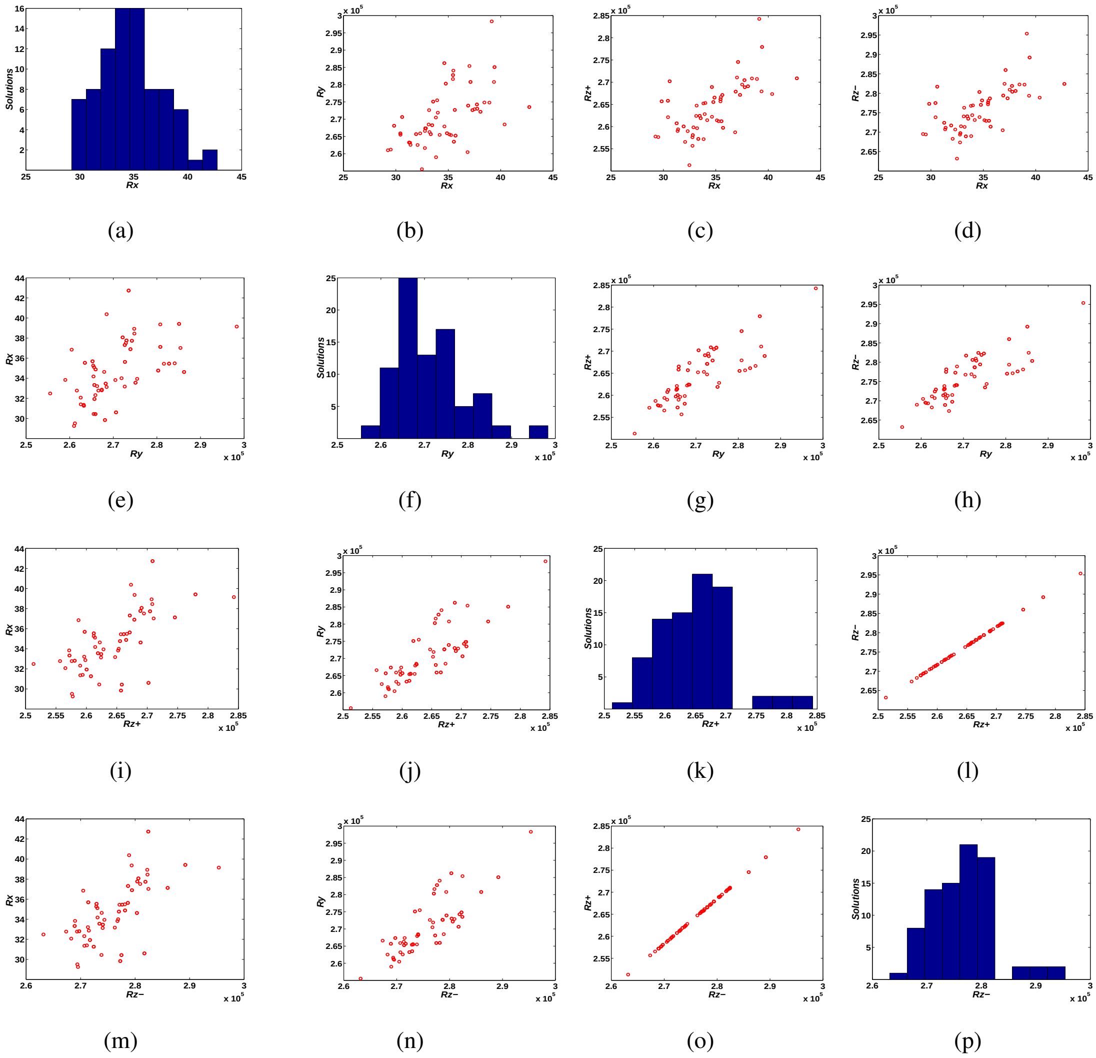


Figure 5.11: MTE-DE Approach for DOICCIII

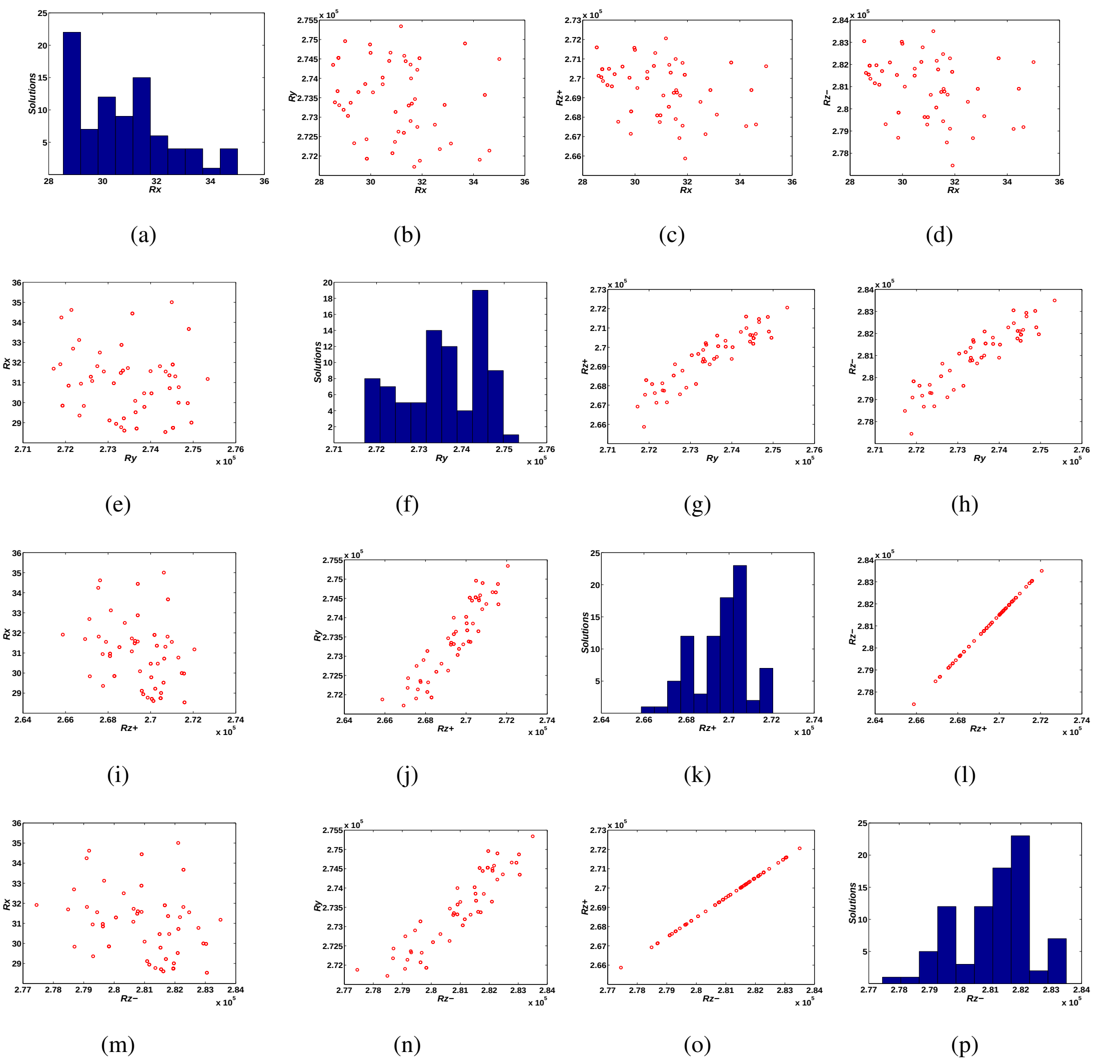


Figure 5.12: MWS-DE Approach for DOICCIII

Table 5.6: Best Performances for each method for DOICCIII

Method	W1 (μm)	W2 (μm)	W3 (μm)	W4 (μm)	W5 (μm)	W6 (μm)	Port	Resistance	Gain	BW	Offset
N-DE	60.619	47.516	261.35	67.622	37.239	12.261	X	33.2370Ω	-1.001 $(\frac{V}{V})$	134.64 MHz	-7.8456 mV
							Y	270.4477 $K\Omega$	-1.0629 $(\frac{I}{I})$	64.1558 MHz	-0.7688 μA
							Z+	280.1098 $K\Omega$	1.0049 $(\frac{I}{I})$	59.172 MHz	-9.1005 nA
							Z-	268.5585 $K\Omega$	-0.9915 $(\frac{I}{I})$	50.005 MHz	-9.8523 nA
MTE-DE	75.437	41.992	223.15	13.821	34.297	11.249	X	30.6025Ω	-0.99748 $(\frac{V}{V})$	133.01 MHz	-5.9372 mV
							Y	270.6297 $K\Omega$	-1.063 $(\frac{I}{I})$	61.012 MHz	-0.7823 μA
							Z+	281.7049 $K\Omega$	0.99398 $(\frac{I}{I})$	58.705 MHz	-9.5671 nA
							Z-	270.2105 $K\Omega$	-0.98878 $(\frac{I}{I})$	48.635 MHz	-9.8548 nA
MTE-DE	47.536	50.731	253.69	67.327	53.474	17.683	X	31.7234Ω	-1.002 $(\frac{V}{V})$	175.75 MHz	-2.0210 mV
							Y	273.4651 $K\Omega$	-1.0618 $(\frac{I}{I})$	67.521 MHz	-0.76746 μA
							Z+	280.6408 $K\Omega$	1.0136 $(\frac{I}{I})$	61.635 MHz	5.8964 nA
							Z-	269.116 $K\Omega$	-0.9926 $(\frac{I}{I})$	53.677 MHz	-9.8519 nA

Chapter 6

CONCLUSIONS

In this work was applied two evolutionary algorithms (NSGA-II and MOEA/D) to perform multi-objective optimization of analog circuits including constraints like saturation conditions in the transistors, gain and/or offset. The proposed optimization system works on a MATLAB code and is able to link SPICE to measure performances.

The first step was to develope two multi-objective engines: NSGA-II and MOEA/D and test them on test functions, so that it was possible to have a priori idea of their advantages and disadvantages. In Section 2.6 it is possible to see the results of both techniques and how MOEA/D has a better convergence to the Pareto front, however NSGA-II has less distance among the found points and sometimes, for constraints functions NSGA-II can achieve points where MOEA/D could not.

Then a MATLAB-based system was developed for circuit optimization, adding to the initializing process the possibility to set the maximum and minimum limit of each variable, where the variables are the transistors widths, starting on test it on small circuits as voltage followers and voltage mirrors with NSGA-II and MOEA/D methods for three objective functions: gain, band width and offset. Afterwards, in Section 4.1 the system was tested on six different circuits biased in different ways with the objective to know which biasing is the best. In Tables 4.7-4.18 are selected, for each circuit and for both optimization methods the three best found solutions to have a better idea about the optimization process, because there are only three objective functions but the remain parameters where included as constraints, then it is possible to see how the process improve their objectives without neglecting other parameters like input and output resistances and the transistors widths are under feasible limits. So this process showed how a current bias of $50\mu A$ and a transistor length of $1\mu m$ presented better performance than the others. It is important to say that in Tables 4.7-4.18 were selected the best solutions in each one of the objective functions, then it could be possible that the others parameters are not the best because the objective of listing that values is to select which bias could be improved in a later section.

Once that was selected the best biasing in Section 4.2 it is made other optimization process but this time deeper because the number of generations are 50 now and for five runs, and using two different genetic operators (DE and basic corss-over/mutation) to create offspring and same like its previous section the number of variables are from two to nine and with the same three objective functions (gain, band width and offset). In Figs. 4.14-4.16 are depicted each one of the performances in scatter plots where it is possible to see their behavior and it is possible to see how the best results become from the DE operator for NSGA-II and MOEA/D with Tchebycheff approach and between these two methods MOEA/D seems the best to built Pareto fronts however, again like was said before NSGA-II achieve solutions where MOEA/D not. It is important to say that in Tables 4.19-4.23 was collected all results from the 5 runs and the metric was measured with all these solutions, here , despite MWS-CR does not build a spread Pareto front can reach to N-DE and MTE-DE performances because their dominate percentage is similar.

Finally, the optimization system was tested on current conveyors which include voltage followers, voltage mirrors, current followers and/or current mirrors and the number of variables is from two to six but this time for four objective functions (resistances in each port and gain for DOCCII). In this manner it is possible to have an idea about the behavior of the system with a circuit with more than one port (three and four ports). For the optimization was selected only three methods (N-DE, MTE-DE and MWS-DE) over 50 generations. Being four objective functions is a difficult task to explore the results, but it is possible to see the interaction among the objectives.

For DOCCI, Fig. 5.2 shows how N-DE has a better Pareto front than the others two methods, but in Figures 5.3 and 5.4 is depicted how a lot number of solutions are around the best values of each objective. For DOCII the solutions are spread in three methods but N-DE (Fig.5.6) build a better uniform Pareto front than the others (Fig.5.7 - 5.8). DOICCIII is the circuit which has the largest number of variables in that section but this time MWS-DE seems to be the best solution for the Pareto front and not only this, in Table 5.6 it is possible to find the best parameters if are compared with the other two methods. In Tables 5.2-5.6 is possible to see how objectives where optimized however as constraints as variables were preserved into acceptable limits.

In Tables 5.2 , 5.4 and 5.6 were selected by hand some of the best solutions because the system find a lot of solutions closer to the Pareto front, then to select a solution is a hard task because there are a lot of solutions which offer alternatively other good solutions so that this selection is a future work.

In this manner, it have been tested the proposed system and the results were into the set limits and the objective functions were improved, with the possibility to choose among a lot of solutions which meet the set requirements, it is clear that it is possible to find satisfactory results with NSGA-II or MOEA/D and in the case of MOEA/D, it is possible that works better Tchebycheff or weighted sum approaches how was said before. The time consumption between NSGA-II and MOEA/D is completely different as it was widely exposed in literature [7, 27, 26]. Other advantage of the system for both methods is that it is not necessary to set initial values, the first generation is made randomly over the space defined for the limits previously specified.

In the future work, there are a lot of options, even with this system is possible varying the parameters such: number of solutions, limits of variables, parameters of genetics operators, and surely the results could be similar or even could be improved. In the other hand, exist a lot of other options for multi-objective optimizations which could be explored to compare them and may be mixed to improve the optimization performance of more complex analog circuits.

Published papers

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2) E. Tlelo-Cuautle, M.A. Duarte-Villaseñor, I. Guerra-Gómez, Automatic synthesis of VFs and VMs by applying genetic algorithms, Circuits, Systems and Signal Processing, vol. 27, no. 3, pp. 391-403, June 2008.

3) I. Guerra-Gómez, E. Tlelo-Cuautle, Peng Li, Georges Gielen, Simulation-Based Optimization of UGCs Performances, IEEE ICCDCS, ISBN 978-1-4244-1957-9, Cancun, México, April 28-30, 2008.

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