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Numerical Simulation of Chua's Circuit Oriented to Circuit Synthesis

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Abstract

The application of numerical methods based on polynomial approximation and multistep algorithms is presented to simulate the behavior of a chaotic oscillator, e.g. Chua's circuit, which consist of five circuit elements: one linear resistor, one inductor, two capacitors and one nonlinear resistor known as Chua's diode. The last element is modeled by an I-V piecewise-linear function to formulate a system of linear dynamic equations. This initial value problem is solved by applying numerical simulation to compute a sequence of chaotic phenomena by solving the state-variables equations describing Chua's circuit. It is highlighted the usefulness of numerical methods to speed-up time simulation to estimate the values of the circuit elements, so that the synthesis of Chua's diode can be realized directly using novel active devices. For instance, it is described the synthesis of Chua's circuit using SPICE and standard CMOS integrated-circuit technology of 0.35μm, to show the control of the I-V slopes and the generation of chaotic phenomena which are in good agreement with that computed from numerical simulation.

Keywords: Numerical simulation, state variables approach, chaotic oscillator, Chua's circuit, electronic design automation, circuit simulation.

1. Introduction

The electronic design automation (EDA) industry is enhancing its design tools for the digital domain where it covers the modeling, simulation and synthesis of electronic systems. However, automated synthesis of analog systems from a description of its desired behavior has not progressed to the point where it is practical except in a few very restricted cases [1]. In this manner, this work is focused to introduce a synthesis approach to design chaotic oscillators, e.g. Chua's circuit [2]-[4], by beginning with the application of numerical simulation based on polynomial approximation and multistep algorithms [5], and ending with the use of SPICE and standard CMOS integrated-circuit (IC) technology of 0.35μm [3].

It is worthy to mention that SPICE provides a limited set of built-in models, those needed to model the components commonly available on ICs, and provide relatively limited ability to add new models at the electronic system level (ESL) [4]. In contrast, an EDA tool is based on behavioral relationships, it provides a very wide variety of features and can be used to efficiently describe a broad range of models at the ESL. Models which are based on sets of mathematical equations to describe the behavior of analog circuits are frequently called analog behavioral models (ABMs) [1]. An ABM is similar to a sub-circuit in that it constitutes a box which is connected to a circuit through electrical ports [5]. Henceforth, from the EDA point of view, analog circuit synthesis can be performed by adopting a top-down approach, where the first cut of design can be modeled and simulated using ABMs, and afterwards each individual block can be synthesized by transistor-circuits designed individually. This process is performed herein to synthesize Chua's diode.

Bifurcation and chaotic phenomena can potentially appear in many nonlinear systems, and most practical electronic circuits are nonlinear to some degree. In electronics, chaotic systems find applications in control, secure communications, dry turbulence, visual sensing, neural networks, nonlinear waves and music [6]-[12]. Besides, Chua's circuit is the only chaotic system which can be easily built, simulated, and tractable mathematically [2]. It consist of five circuit elements: one linear resistor (R), one inductor, two capacitors, and one nonlinear resistor known as Chua's diode. For instance, in section 2 the last element is modeled by an I-V piecewise-linear characteristic in order to allow the formulation of a system of linear state-variable equations [4]. In section 3 is shown the numerical simulation of Chua's circuit by applying polynomial approximation. In section 4 is shown the numerical simulation of Chua's circuit by applying multistep algorithms. In section 5 is described the synthesis of Chua's diode, which is designed with CMOS compatible current-feedback operational amplifiers (CFOAs) [3], using SPICE and standard CMOS IC technology of 0.35µm. It is shown how to control its negative slopes to adjust the breaking points [2]. Finally, the conclusions are summarized in section 6, where it is stressed the advantage of numerical simulation as a good solution to speed-up time simulation of nonlinear systems, and its appropriateness to perform circuit synthesis.

2. Analog Behavioral Modeling of Chua's Circuit

Chua's circuit can be modeled by applying the state variables approach [5]. In this manner, the ABM from Fig. 1 consist of three equations, i.e. one for each state variable, as shown by (1)-(3). In (1), i_{NR} describes the current through Chua's diode, which is a nonlinear resistor. The main idea behind is to transform the problem of solving a nonlinear system of differential equations into a sequence of linear and purely algebraic problem which can be solved straightforward by applying numerical simulation. Furthermore, Chua's diode can be modeled by an I-V piecewise-linear characteristic, as shown by Fig. 2 [2]. As one sees, for voltage signals less than the breakpoint (BP_1) in absolute value the characteristic has a linear segment with negative slope g_1. For absolute voltages larger than BP_1 the characteristic has two linear segments of negative slope g_2. The negative piecewise-linear behavior is valid in the nominal range ($-BP_2$, BP_2), in which the diode is normally operated [2]. For voltages outside this range the slope of the I-V characteristic increases monotonically ultimately becoming positive g_3. In this manner, i_{NR} has the general form given by (4), where m (g_1, g_2, g_3) and Ix should be updated according to (5).

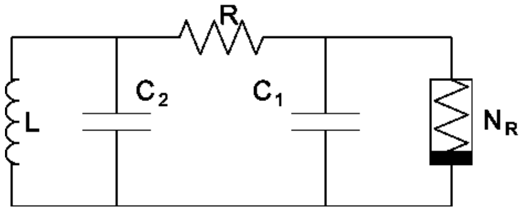


Fig. 1 Chua's circuit.

$$\frac{dV_{C1}}{dt} = -\frac{V_{C1}}{RC_1} + \frac{V_{C2}}{RC_1} - \frac{i_{NR}}{C_1} \qquad (1)$$

$$\frac{dV_{C2}}{dt} = \frac{V_{C1}}{RC_2} - \frac{V_{C2}}{RC_2} - \frac{i_L}{C_2} \qquad (2)$$

$$\frac{dI_L}{dt} = -\frac{V_{C2}}{L} \qquad (3)$$

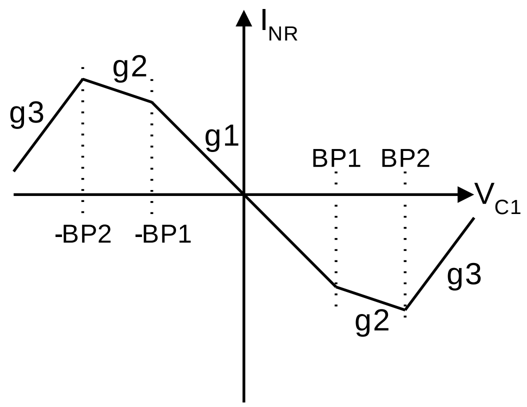


Fig. 2 Driving-point I-V piecewise-linear characteristic of Chua's diode.

$$i_{NR} = mV_{C1} + Ix \tag{4}$$

$$i_{NR} = \begin{cases} -g2V_{C1} + (g1 - g2)BP1 & V_{C1} < -BP1 \\ -g1V_{C1} & -BP1 \le V_{C1} \le BP1 \\ -g2V_{C1} + (g2 - g1)BP1 & V_{C1} > BP1 \end{cases} \tag{5}$$

Equations (1)-(5) leads to three systems of first-order differential equations, as shown by (6)-(8).

$$\begin{bmatrix} \dot{V}_{C1} \\ \dot{V}_{C2} \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} + \frac{g2}{C_1} & \frac{1}{RC_1} & 0 \\ \frac{1}{RC_2} & -\frac{1}{RC_2} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{(g2 - g1)BP1}{C_1} \\ 0 \\ 0 \end{bmatrix}$$

$$V_{C1} < -BP1 \tag{6}$$

$$\begin{bmatrix} \dot{V}_{C1} \\ \dot{V}_{C2} \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} + \frac{g1}{C_1} & \frac{1}{RC_1} & 0 \\ \frac{1}{RC_2} & -\frac{1}{RC_2} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ I_L \end{bmatrix}$$

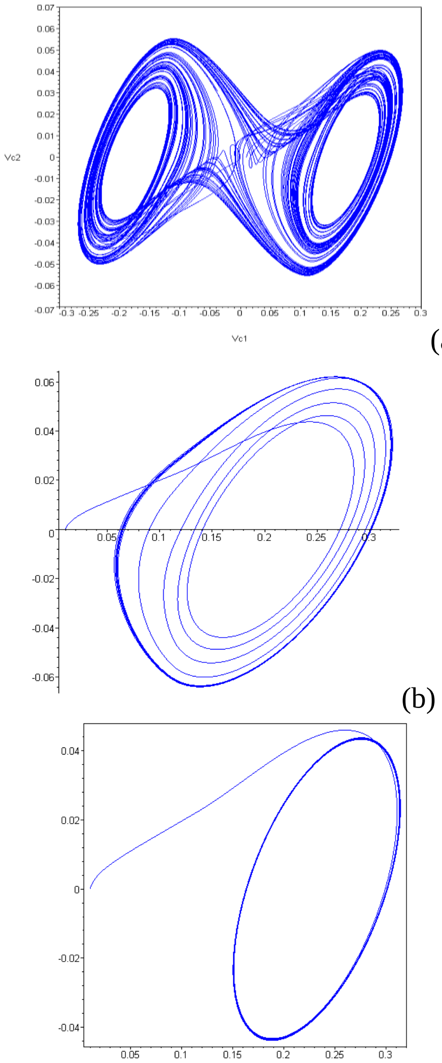
$$-BP1 \le V_{C1} \le BP1 \tag{7}$$

$$\begin{bmatrix} \dot{V}_{C1} \\ \dot{V}_{C2} \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} + \frac{g2}{C_1} & \frac{1}{RC_1} & 0 \\ \frac{1}{RC_2} & -\frac{1}{RC_2} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_{C1} \\ V_{C2} \\ I_L \end{bmatrix} + \begin{bmatrix} \frac{(g1 - g2)BP1}{C_1} \\ 0 \\ 0 \end{bmatrix}$$

$$V_{C1} > BP1 \tag{8}$$

3. Numerical Simulation by Polynomial Approximation

The numerical simulation of (6)-(8) by applying the Forward Euler (FE) approximation given by (9), has been already demonstrated in [4]. Among other polynomial based methods, the explicit fourth-order Runge-Kutta (RK) method is the most widely used for large step-size and greater accuracy. It is given by (10), where the k variables are evaluated by (11), A and b are the state-matrix and I-V characteristic of Chua's diode taken from (6)-(8), and h and x_n are the step-size and state vector, respectively.



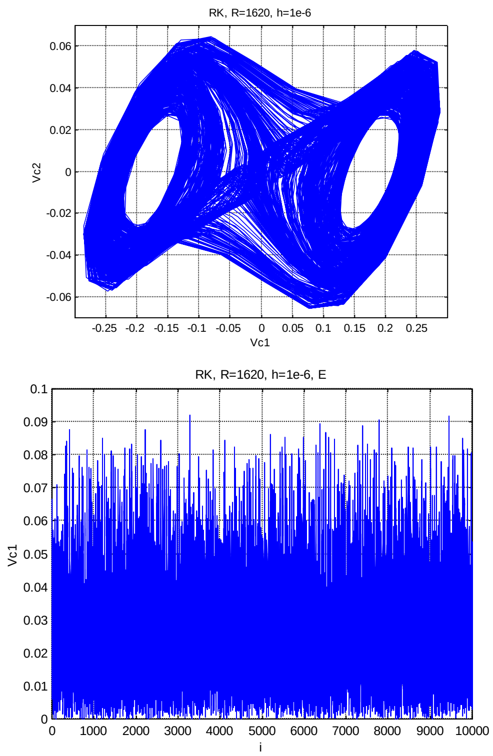
(a) (b) (c)

Fig. 3 Numerical simulations of Chua's circuit by applying the fourth-order RK method with 10,000 iterations: (a) Double-scroll with R=1650, (b) one chaotic attractor with R=1705, and (c) one-period attractor with R=1760.

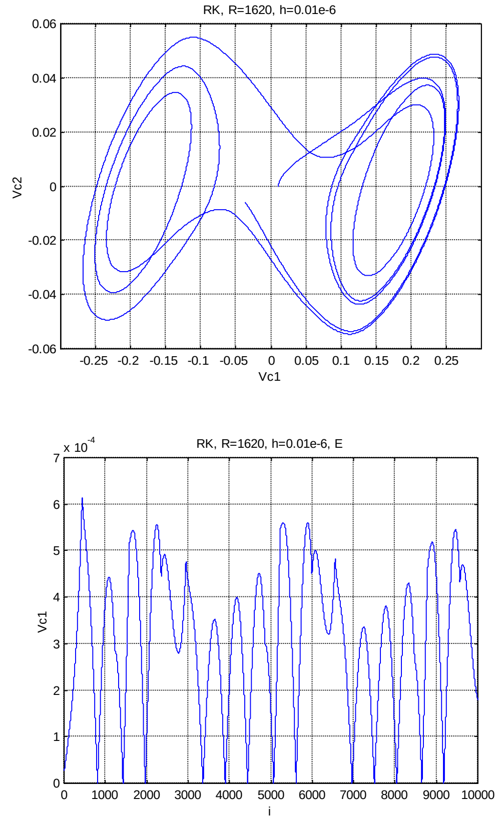
$$X_{n+1} = X_n + hf(t_n, x_n) \tag{9}$$

$$X_{n+1} = X_n + \frac{1}{6}(k1 + 2k2 + 2k3 + k4) \tag{10}$$

$$\begin{aligned} k1 &= h[Ax_n + b] \\ k2 &= h\left[A(x_n + \frac{1}{2}k1) + b\right] \\ k3 &= h\left[A(x_n + \frac{1}{2}k2) + b\right] \\ k4 &= h[A(x_n + k3) + b] \end{aligned} \tag{11}$$



(a)



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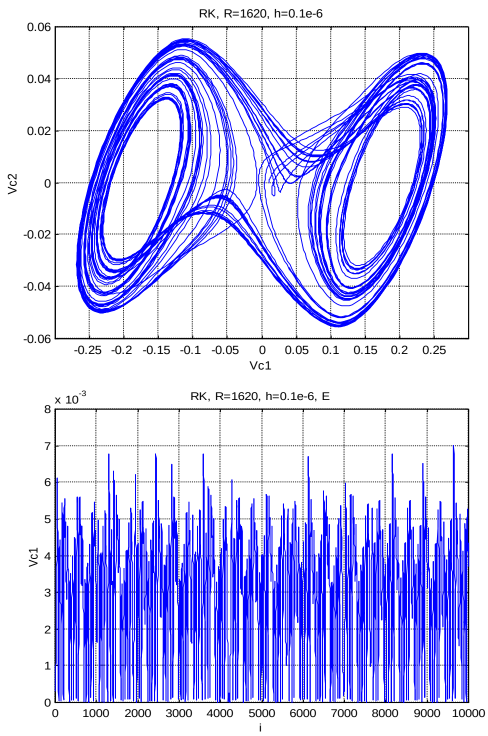
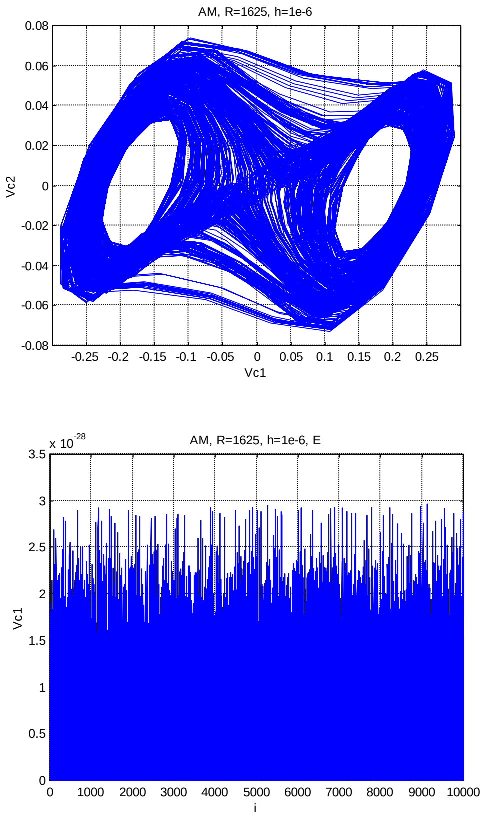


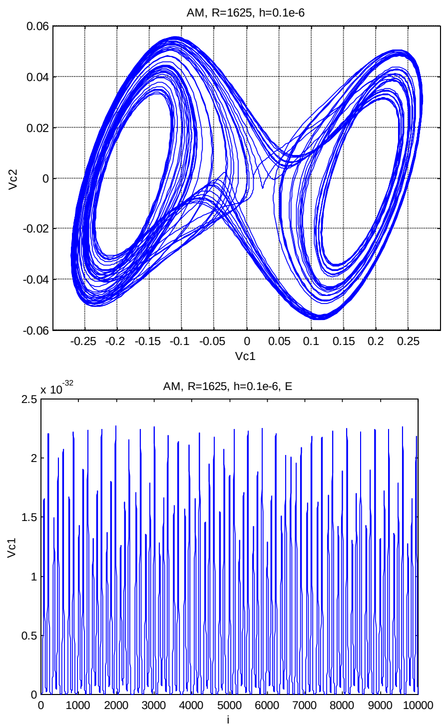
(b)

Fig. 4 Numerical simulations of Chua's circuit by applying the fourth-order RK method: Double-scroll and its arithmetic error with (a) h=1e-6, (b) h=0.1e-6, and (c) h=0.01e-6 with 10,000 iterations.

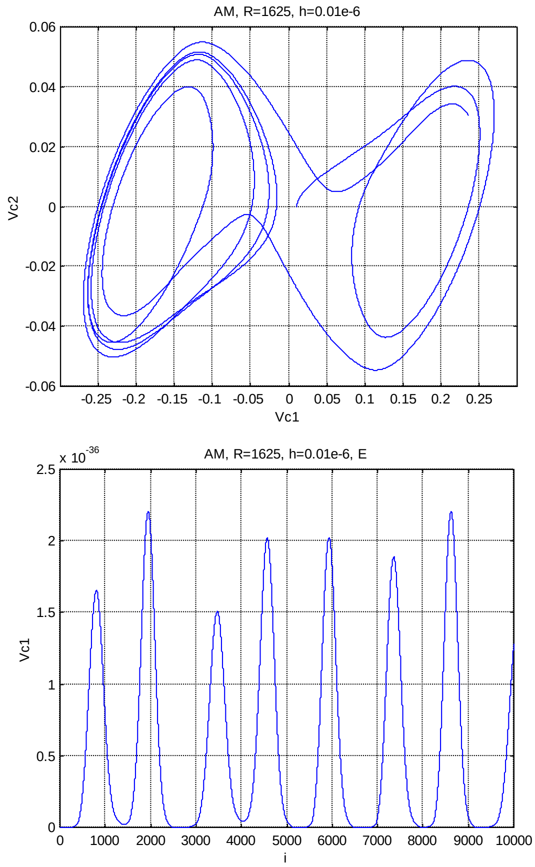
By setting: C_1=450pF, C_2=1.5nF, L=1mH, g_1=1/1358, g_2=1/2464, g_3=1/1600, BP_1=0.114V, BP_2=0.4V, h=0.0000001, $V_{C1}(0)$=0.01V, $V_{C2}(0)$=0V and $I_L(0)$=0A, Fig. 3 shows a sequence of chaotic behaviors and Figure 4 shows the generation of the double-scroll attractor with R=1620, at three different step sizes. Although the error is larger when the step size is large, for h=0.01e-6 it is necessary to tune the value of the linear resistor R, to generate a chaotic behavior similar to Fig. 3(a). Elsewhere, when Chua's diode is synthesized by CMOS compatible CFOAs, as shown in section 5, again it will be necessary to tune the value of R. Besides, the fourth-order RK method computes better chaotic behaviors than by applying the simple FE method [4].



(a)



(b)



(c)

Fig. 5 Numerical simulations of Chua's circuit by applying the third-order AM algorithm: Double-scroll and its error with (a) h=1e-6, (b) h=0.1e-6, and (c) h=0.01e-6 with 10,000 iterations.

4. Numerical Simulation by Multistep Algorithms

The Adams-Moulton (AM) algorithms are implicit multistep ones. For instance, the third-order algorithm is given by (12), where the term $f(x_{n+1}, t_{n+1})$ can be predicted using (9), i.e. FE. The evaluation of (6)-(8) using this implicit algorithm derives (13), where A and b are the state-matrix and Chua's diode characteristic, h, x_n and x_{n-1} are the step-size and state vectors evaluated at one and two past steps, and $\hat{x}_{n+1}$ is predicted using FE. The third order AM algorithm has the local error truncation given by (14) [5].

$$x_{n+1} = x_n + h\left\{ \tfrac{5}{12} f(x_{n+1}, t_{n+1}) + \tfrac{8}{12} f(x_n, t_n) - \tfrac{1}{12} f(x_{n-1}, t_{n-1}) \right\} \quad (12)$$

$$x_{n+1} = x_n + h\left\{ \tfrac{5}{12} (A\hat{x}_{n+1} + b) + \tfrac{8}{12} (Ax_n + b) - \tfrac{1}{12} (Ax_{n-1} + b) \right\} \quad (13)$$

$$\varepsilon_T = \left[C_k \hat{x}^{(k+1)}(\hat{\tau}) \right] h^{k+1} = \left[-\frac{1}{24} \hat{x}^{(4)}(\hat{\tau}) \right] h^4 \quad (14)$$

Again, by setting: C_1=450pF, C_2=1.5nF, L=1mH, g_1=1/1358, g_2=1/2464, g_3=1/1600, BP_1=0.114V, BP_2=0.4V, h=0.1e-6, $V_{C1}(0)$=0.01V, $V_{C2}(0)$=0V and $I_L(0)$=0A, Fig. 5 shows the double-scroll chaotic behavior at three different step sizes, and its associated error evaluating (14), but with R=1625.

As for the numerical simulation using RK, the error is larger when the step size is large using AM. Although the value of R in Fig. 3(a), Fig. 4, and Fig. 5 is different, both methods RK and AM are quite useful to estimate the values of the circuit elements in Chua's circuit at the ESL.

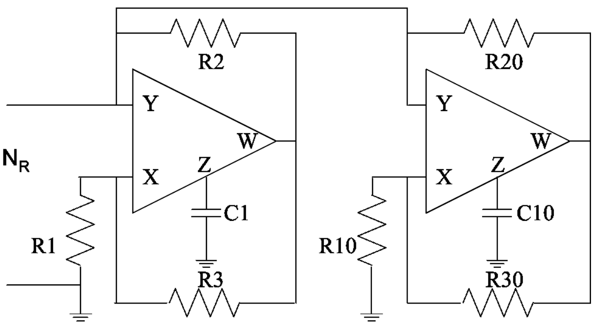


Fig. 6 Synthesis of Chua's diode by CMOS compatible CFOAs.

5. Synthesis of Chua's Diode

The simulation of Chua's circuit at the transistor level of abstraction is very time-consuming. In this manner, numerical simulation speed-up time simulation since it allows the use of ABMs for nonlinear systems instead of handling complex SPICE transistor models [1]. Furthermore, an electronic designer can apply numerical simulation to estimate the values of the circuit elements and ABMs, e.g. Chua's diode, at a level of abstraction higher than the transistor one, i.e. at the ESL. In [4] it has been proved the synthesis of Chua's circuit by adopting a top-down approach. On the other hand, this section describes the synthesis of Chua's diode by CMOS compatible CFOAs, as shown by Fig. 6.

In Fig. 7 are presented SPICE simulation results when varying both resistors R1 and R10, to appreciate the control on the negative slopes of Chua's diode. For instance, by using the CFOA designed in [3] with standard CMOS IC technology of 0.35µm, and by setting R1=2.2k, R2=R3=220, C1=C10=0.1p, R10=3.3k, and R20=R30=22k, then Fig. 6 allows the synthesis of Fig. 2 among the following values: g_1=-1/1358, g_2=-1/2464, g_3=1/1600, BP_1=0.114V, and BP_2=0.4V, which are the values used in sections 3 and 4. Further, by varying the value of R1 and R10, as shown in Fig. 7, g_1 can be synthesized with voltage values between -0.16V and 0.16V, which generates currents from ±50µA to ±120µA, while g_2 can be synthesized with voltage values from ±0.01V to ±0.61V, which generates currents from ±50µA to ±340µA. That way, an analog designer has a wide range of values to adjust the slopes and breaking points (BP_1, BP_2) of Chua's diode to make numerical simulations.

By using Fig. 6 to synthesize Chua's diode in Fig. 1, and by setting C_1=450pF, C_2=1.5nF, L=1mH, and $V_{C1}(0)$=0.01V, Fig. 8 shows the chaotic behavior with R=1625. As a result, the chaotic response is in good agreement with that shown in Fig. 4 and Fig. 5.

6. Conclusions

It has been presented the numerical simulation of Chua's circuit by applying polynomial approximation and multistep algorithms. It was shown that both methods, the fourth-order RK and the third-order AM presents similar behaviors when varying the step-size. For instance, computationally speaking, the fourth-order RK method requires more memory than the third-order AM algorithm, since RK needs the evaluation of four functions. Besides, both methods are quite useful and appropriate to make numerical simulation of chaotic systems, Chua's circuit in this case.

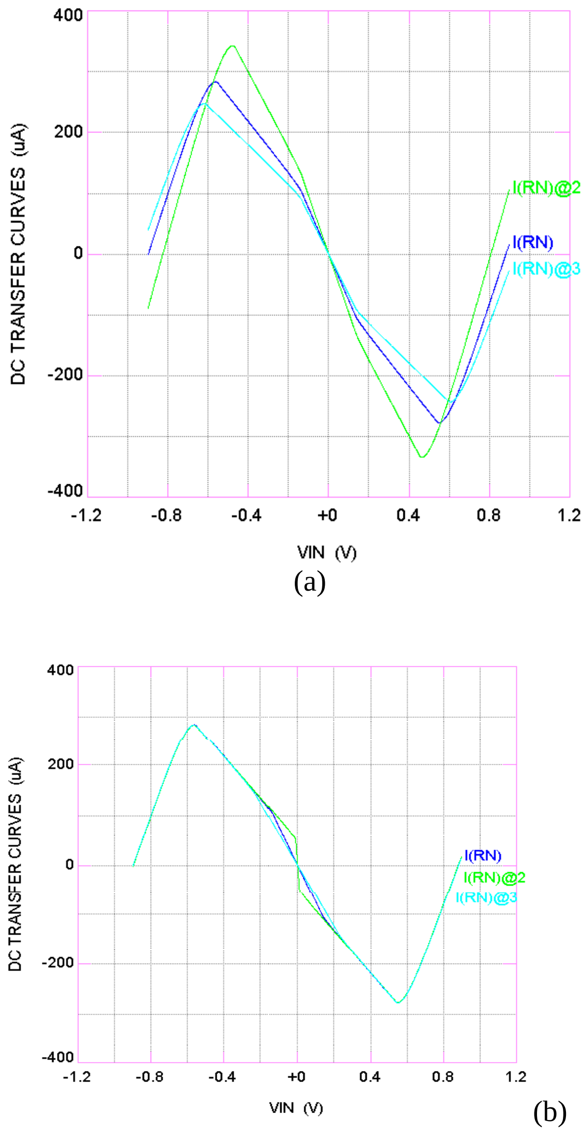


Fig. 7 SPICE simulations of Chua's diode by varying: (a) R1 and (b) R10 in Fig. 6.

Numerical simulation can be used to estimate the values of the elements of a nonlinear circuit, Chua's diode in this case, in order to perform analog circuit synthesis from the ESL down to the transistor level of abstraction. This process accomplishes the objectives of the EDA industry.

It has been shown that Chua's diode can be implemented with novel active devices, such as the CFOA, and that it can provide a wide range of values which fit much better to approximate the proposed I-V piecewise-linear characteristic used within the numerical simulations. Most important is that it has been shown that the results computed with SPICE are in good agreement with that computed from numerical simulation.

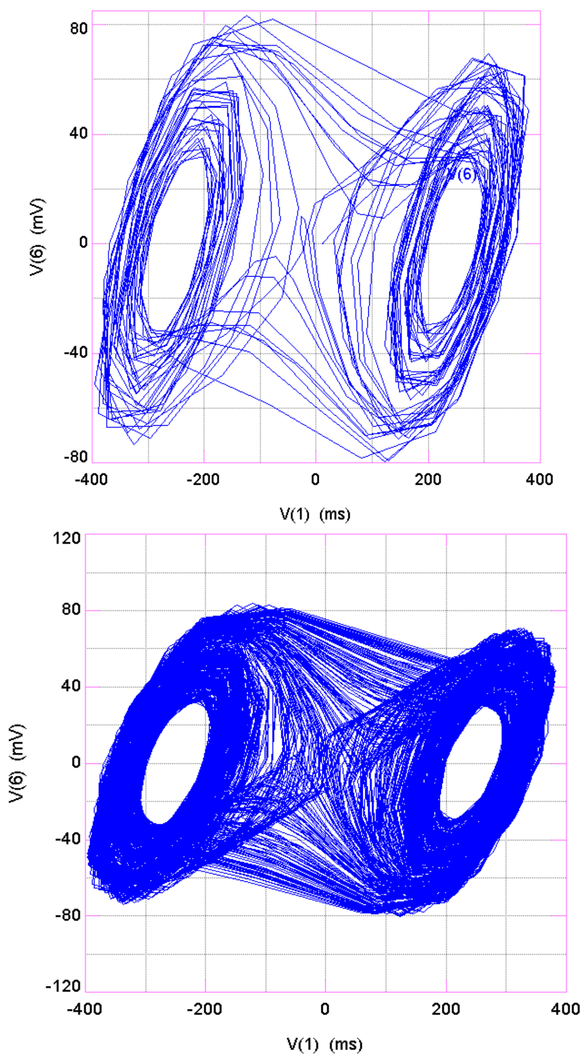


Fig. 8 SPICE simulations of Chua's circuit with R=1625: (a) 10,000 iterations and (b) 100,000 iterations.

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