

Designing Chua's circuit from the behavioral to the transistor level of abstraction

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Abstract

A MATLAB program is presented to design Chua's circuit by applying state-variables at the behavioral level of abstraction. The user can interact with the program to calculate state trajectories by modifying the values of five circuit-elements: a resistor, two capacitors, an inductor and a nonlinear resistor, i.e. Chua's diode (N_R). N_R is modeled by a piecewise-linear $i-v$ characteristic which is synthesized further by analog building blocks at the transistor level of abstraction. Finally, Chua's circuit is simulated by using SPICE and standard CMOS integrated-circuit technology, whose results are in good agreement with MATLAB.

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1. Introduction

The electronic design automation (EDA) industry still remains uncertain of its growth [1]. The EDA industry has always had a major shift when a new language is used for designing systems. Recently, much of the EDA community has hyped Electronic System Level (ESL) languages as the new frontier, e.g. the signal processing design community almost all use MATLAB, because it provides three major features to system designers: an intuitive interface that makes it easy for designers to quickly model their designs, a library of predeveloped mathematical functions, and powerful graphing and visualization capabilities [1,2]. However, there is currently no automated path to transition MATLAB's algorithms to an environment for physical implementation. In this manner, this work sets the guidelines to cover the gap between the ESL (behavioral level of abstraction), down to the transistor level by designing a chaotic oscillator through the combination of MATLAB and SPICE [2].

Chaos phenomenon has been studied extensively from three decades ago, in various areas of science such as biology, ecology, physics, and optics [3]. Chua's circuit is endowed with virtually every bifurcation phenomena because it is the only chaotic system which can be easily built, simulated, and tractable mathematically. Its

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supreme simplicity and robustness has made it the circuit of choice for practical applications including: secure communications [4], visual sensing, neural networks, dry turbulence, nonlinear waves and music.

In this manner, a design-approach is introduced in this paper for Chua’s circuit [3–6], which is modeled and simulated in MATLAB by applying state-variable approach [7]. Further, Chua’s circuit is implemented by using current feedback operational amplifiers (CFOAs) [6], and simulated by using SPICE with standard CMOS technology of 0.35 μm.

2. Modeling Chua’s circuit by MATLAB

In Fig. 1 is shown the simplest autonomous third-order chaotic circuit consisting of: a resistor, an inductor, two capacitors, and Chua’s diode (N_R) [3].

By applying circuit theory [7], Chua’s circuit can be described by (1)–(3). In (1), $f(v_{C_1})$ describes the current through Chua’s diode (i_{N_R}) [3], so that the voltage-controlled driving point characteristic of N_R can be modeled by (4), which is sketched in Fig. 2. For voltage signals less than BP₁ in absolute value the characteristic has a linear segment with negative slope g_1 . For absolute voltages larger than the breakpoint BP₁ the characteristic has two linear segments of negative slope g_2 . The negative piecewise-linear behavior is valid in the nominal range (–BP₂, BP₂), in which the diode is normally operated. For voltages outside this range the slope of the characteristic becomes positive g_3 .

$$C_1 \frac{dv_{C_1}}{dt} = \frac{v_{C_2} - v_{C_1}}{R} - f(v_{C_1}), \tag{1}$$

$$C_2 \frac{dv_{C_2}}{dt} = \frac{v_{C_1} - v_{C_2}}{R} + i_L, \tag{2}$$

$$L \frac{di_L}{dt} = -v_{C_2} \tag{3}$$

$$i_{N_R} = \begin{cases} g_2 v_{C_1} + (g_2 - g_1)BP_1 & v_{C_1} < -BP_1, \\ g_1 v_{C_1} & -BP_1 \leq v_{C_1} \leq BP_1, \\ g_2 v_{C_1} + (g_1 - g_2)BP_1 & v_{C_1} > BP_1. \end{cases} \tag{4}$$

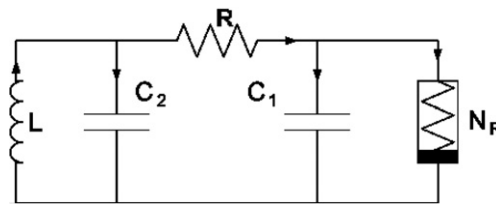


Fig. 1. Chua’s circuit.

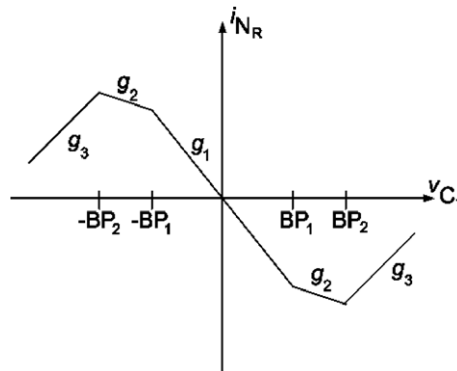


Fig. 2. Driving point characteristic of Chua’s diode.

To calculate state trajectories by MATLAB to show the generation of bifurcation and chaotic behaviors, (1)–(4) can be programmed into MATLAB, but they should be formulated as a system of first-order differential equations.

2.1. Calculation of state trajectories

By selecting v_{C_1}, v_{C_2}, i_L as state-variables, (1)–(3) can be rearranged as shown by (5)–(7), respectively. As one sees, since $f(v_{C_1})$ models the ideal piecewise-linear behavior of N_R , it can be replaced by a Norton equivalent [7], i.e. a conductance in parallel with a current source. The conductance is denoted by m , which should be updated to its corresponding negative-slope (g_1, g_2, g_3), according to the voltage ranges in Fig. 2. The current source (i_{N_R}) should be updated to its corresponding value according to (4).

$$\frac{dv_{C_1}}{dt} = -\frac{v_{C_1} - v_{C_2}}{RC_1} - \frac{mv_{C_1} - i_{N_R}}{C_1}, \tag{5}$$

$$\frac{dv_{C_2}}{dt} = \frac{v_{C_1} - v_{C_2}}{RC_2} + \frac{i_L}{C_2}, \tag{6}$$

$$\frac{di_L}{dt} = -\frac{v_{C_2}}{L}, \tag{7}$$

$$\begin{bmatrix} \dot{v}_{C_1} \\ \dot{v}_{C_2} \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_1} - \frac{m}{C_1} & \frac{1}{RC_1} & 0 \\ \frac{1}{RC_2} & -\frac{1}{RC_2} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_{C_1} \\ v_{C_2} \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{i_{N_R}}{C_1} \\ 0 \\ 0 \end{bmatrix}. \tag{8}$$

In this manner, (5)–(7) can be described by (8), i.e. as state-variables. This initial-value problem can be solved by applying the simple step-by-step method based on the iterative formula given by (9) [7, pp. 203], where A describes the state-matrix and h the integration step-size.

$$x_{j+1} = x_j + hAx_j. \tag{9}$$

In this manner, (8) was programmed in MATLAB and evaluated by (9) by setting the following values: $C_1 = 450$ pF, $C_2 = 1.5$ nF, $L = 3$ mH, $R = 1655 \Omega$, $g_1 = -1/1358$, $g_2 = -1/2464$, $g_3 = 1/1600$, $i_L(0) = 0$ A, $v_{C_1}(0) = 0.01$ V, $v_{C_2}(0) = 0$ V, $BP_1 = 0.114$ V, $BP_2 = 0.4$ V, and $h = 0.0000001$.

The resulting state trajectories can be plotted according to the user-requirements. For instance, in Fig. 3 is shown the chaotic behavior of each state-variable, in Fig. 4 the combinations among state-variables, and in Fig. 5 a 3D plot for the three state-variables. Most important is that the interactive program can help to

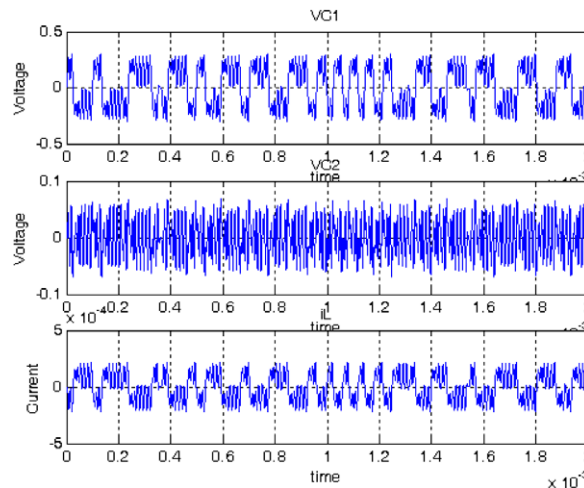


Fig. 3. Signals in each state-variable vs. time.

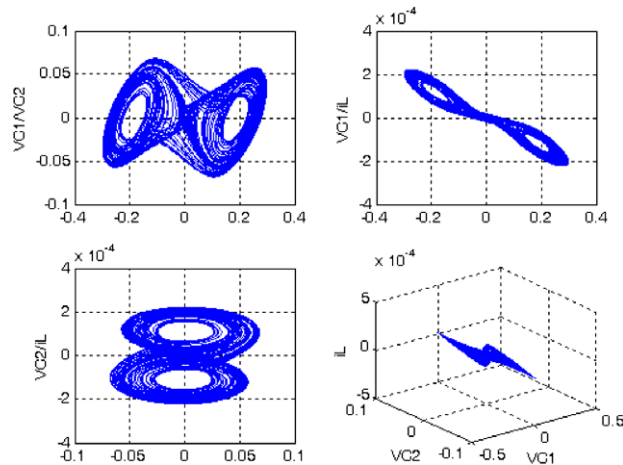


Fig. 4. Plotting two and three state-variables in a 2D and 3D graphs.

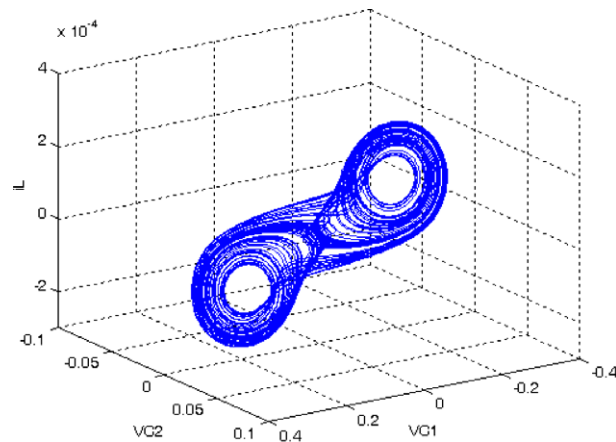


Fig. 5. Chaotic behavior by plotting the three state-variables in a 3D graph.

an analog integrated-circuit (IC) designer to explore on suitable values for the circuit elements in Fig. 1, to generate all chaotic behaviors [3–6], and to synthesize N_R at the transistor level of abstraction [6], as shown in the following section.

3. Design of Chua's circuit by SPICE

N_R has been realized by using operational transconductance amplifiers (OTAs) [2], and current-feedback operational amplifiers (CFOAs) [5]. However, according to the novel analog signal processing applications, it is quite recommended to use CFOAs since it has the advantage of wider bandwidth compared to OTAs or other more complex analog building blocks. An important thing is that a CFOA consist of voltage followers (VFs) [8,9] and current conveyors [10], which are quite useful to design oscillators.

N_R can be synthesized by using two CFOAs as shown by Fig. 6. The design of the CFOA at the transistor level is deeply described in [6], so that by using SPICE and standard CMOS technology of $0.35\ \mu\text{m}$, the circuit in Fig. 6 generates the piecewise-linear $i-v$ characteristic shown by Fig. 7. As one sees, the breaking points are located at $\pm 114\ \text{mV}$, and the negative behavior is performed between $\pm 400\ \text{mV}$. These values were used in MATLAB in Fig. 2.

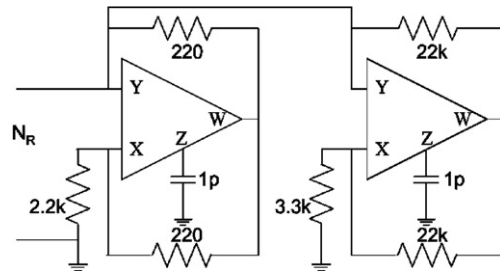


Fig. 6. Design of N_R by using CFOAs, taken from [6].

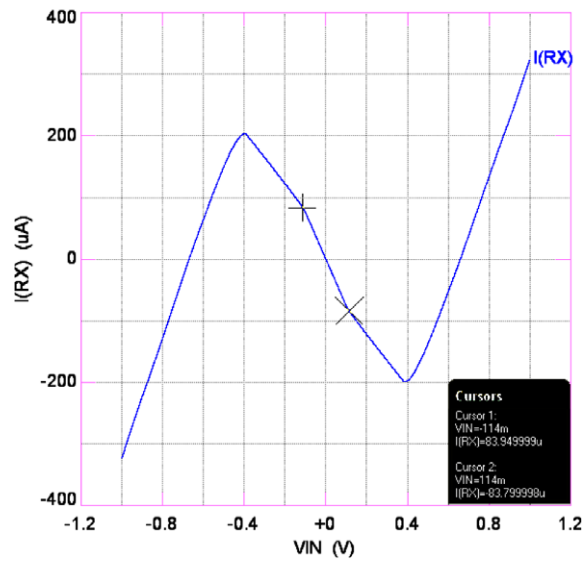


Fig. 7. Simulation of the $i-v$ characteristic of N_R by SPICE, taken from [6].

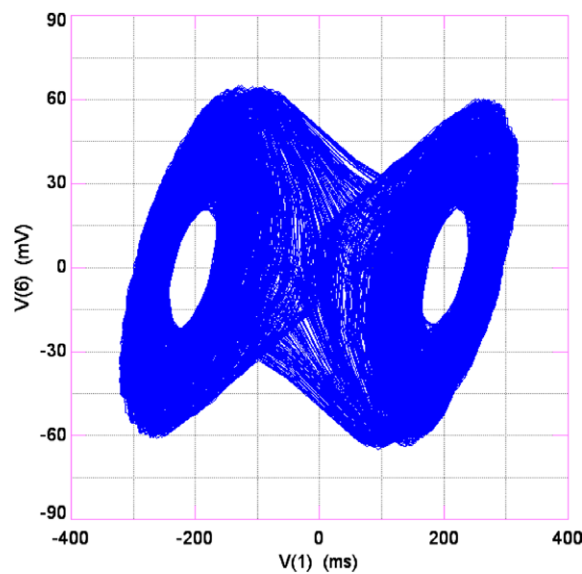


Fig. 8. Generation of the double-scroll attractor, taken from [6].

In this manner, by using Fig. 6 to synthesize N_R in Fig. 1, a bifurcation sequence can be obtained by varying the value of the linear resistor R , as already shown in [3]. For instance, the double-scroll behavior shown by Fig. 8 was generated by using the same values from Section 3, where $V(6)$ is associated to V_{C2} and $V(1)$ to V_{C1} . As a result, since Fig. 4 is in good agreement with Fig. 8, one can conclude on the suitability of the proposed design-approach to cover the gap between the behavioral level of abstraction by MATLAB (ESL), down to the transistor level one by SPICE.

4. Conclusion

It has been shown that the combination of MATLAB and SPICE is very useful to design analog ICs. In this manner, Chua's circuit has been modeled and simulated at the behavioral level of abstraction by applying state-variables. At this level of abstraction, the user can interact with the proposed MATLAB program to explore on the values of the elements in Chua's circuit, where N_R is modeled by a piecewise-linear $i-v$ characteristic. N_R can be synthesized further at the transistor level of abstraction by using CMOS CFOAs, so that by using SPICE one can observe the generation of bifurcation and chaotic behaviors by increasing the value of the linear resistance. The proposed design-approach has been demonstrated by designing Chua's circuit since the results from MATLAB (Fig. 4) are in good agreement with the SPICE simulations (Fig. 8).

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