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A SIMPLE PERIOD FINDING PROCEDURE FOR ASTRONOMICAL TIME SERIES WITH FEW OBSERVATIONAL DATA

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CONTENTS

Abstract

A procedure for finding periods in astronomical time series containing few observational data using simple mathematical operations is developed. Selecting data close to the maximum or minimum values of the time series differences among these values around the maximum or minimum are obtained to produce a set of intervals. Using a technique similar to the least common divisor and applying the maximum common denominator to the set of intervals approximate periods are found. The different ways to improve the periods found are presented. The procedure is applied to a simulated random sinusoidal data set and also to some data from binary and pulsating variable stars to show how the procedure works. The procedure is simple to use for any type of data spacing and with gaps and produces results in accordance with other methods.

1. Introduction

Time series data are ordered sequences of measurements and the analysis of time series is based on the assumption that successive values in the data represent consecutive measurements taken at equally or unequally spaced time intervals and with gaps. There are two main objectives of time series analysis: (a) identifying the nature of the phenomenon represented by the sequence of observations, and (b) predicting future values of the time series variables. Both of these objectives require that the pattern of observed time series data is identified and more or less formally described. Once the pattern is established, we can interpret and integrate it with other data (i.e., use it in our theory of the investigated phenomenon). Regardless of the depth of our understanding and the validity of our interpretation of the phenomenon, we can extrapolate the identified pattern to predict future events. Most time series patterns can be described in terms of two basic classes of components: trend and seasonality. The former represents a general systematic linear or nonlinear component that changes over time and does not repeat or at least does not repeat within the time range captured by our data. The latter may have a formally similar nature, however, it repeats itself in systematic intervals over time. Those two general classes of time series components may coexist in real-life data. The latter component can have periodic variations that is necessary to characterize to understand the phenomena at hand. The problem of finding periodicities in the time series of many types of observational and experimental data, and from a diversity of other phenomena have been studied in many papers in the past. There exist in the astronomical and time series analysis a great number of methods and procedures to solve the problem of periodicities in the observations of many types of applications. Petri (1962) wrote at his time that no method exists to determine the correct period of a spectroscopy binary form observations taken many periods apart. Aitken (1963) gives some reference and recipes to find periods using plots of parts of the data and reversing them with respect to a fixed points to find close coincidences and the interval between two point is equal to the period. The need for precisely determining periods of cyclic phenomena is well known and numerous methods have been produces for evenly spaced data. (Lafler and Kinman, 1965; Blackman and Tukey, 1959; Fahlman and Ulrych, 1982). Lately the attention is centered in phenomena observed at irregularly spaced intervals and with gaps (Gray and Desikachary, 1973; Deeming, 1975; Lomb, 1976; Scargle, 1982). To help in the acquisition of new data in changing time series in general it is necessary to have an approximate period to select judiciously the times of further acquisition of new data in order to determine a better period. A review of several techniques for uncovering periodicities in variable and binary stars can be found in an article by Fullerton (1986). A simple procedure using the correlation between the time series and the remainders of the series with respect to the tentative period for equally distributed intervals is described by (Whittaker and Robinson, 1944). There are several period search algorithms in the literature (Lafler and Kinman, 1965, Jurkevich, 1971; Marraco and Mussio, 1980), Least squares methods (Barning, 1963; Vaniĉek, 1971), String Length Statistics (Dworetsky, 1983), Fourier methods (Wehlau and Leung, 1964; Gray and Desikachary, 1973), Periodogram analysis (Shuster, 1898, Lomb 1976, Scargle, 1982, Press and Teukolsky, 1988, Press and Rybicki, 1989), Fast Fourier methods for data unevenly spaced and with gaps (Deeming, 1975), and Spline methods (Akerlof et al., 1994). In this article we present a simple procedure to find approximate periods in astronomical time series that complements the methods mentioned before when the number of observations is small.

2. Procedure

This procedure can be used when one has few point of the time series and it is necessary to have an idea of the period in order to obtain further data to be able to get a better value of the period as mentioned before. Taking values close to the maxima (minima) of a given observed or experimental series for the purposes, in the one hand to have few values for computational convenience and in the other hand to assure that the maxima are taken into account at least approximately, in order to define differences among these data. With these differences, it is possible to find with a variant of the least common denominator which will satisfy the time intervals between these observations. Any such interval between the approximate maxima is related to the period. Each pair of corresponding phases gives a relation t_l - t_m = nP, where $t_l – t_m$ is the interval, n is an integer, and P is the period. From those intervals one can find submultiples of them, and with the results one obtains a value approximately common to all of the intervals that will give the tentative period. This very approximate period is used in the next step of our procedure. The intervals found above are also used to find the common greatest divisor (CGD) between two such interval. With all the CGD's found before the average of them is calculated, because the points are close to but not necessarily in the maximum (minimum) of the series, and that result would represent the tentative period. Then from the given time series data one searches for the maximum (minimum) value of the amplitude and defining a small interval around the maximum (minimum) value that can include enough values to be able to find a good approximate period as was mentioned above. Then calculating the differences of the values found before starting with the first with respect to the rest of the values and then with the second one with respect to the remaining values, except the second, and so on. With these intervals it is possible to find a tentative value for the period by dividing the first set of intervals by two as many times as necessary to obtain a set of numbers and then by three, and so on. The result of those divisions show the numbers that are similar in size to each other giving the tentative period. This is the approximate period that will be used later as the stoping parameter in the quasi Euclidean procedure used to find the CGD. From the intervals obtained from the differences between all the values with respect to the first one are obtained. The first difference is used with all the other differences in the process of finding the CGD in order to obtain a series of number that are used to find the mean of those number that becomes the approximate period. In the process to find the CGD to stop the process, one uses the greatest number found above, in the function for that calculation. The pseudo code of the Euclidean algorithm for integer numbers is given by the function,

```
function gcd(a, b)
   while b ≠ 0
     t := b
     b := a mod b
     a := t
   return a
```

For real numbers the stoping factor is different from zero in the while statement and should be chosen judiciously to have the appropriate range, given by a similar technique to the least common divisor, values found before.

The best way to summarize the procedure is through a flow chart diagram that describes the different steps of the description given above.

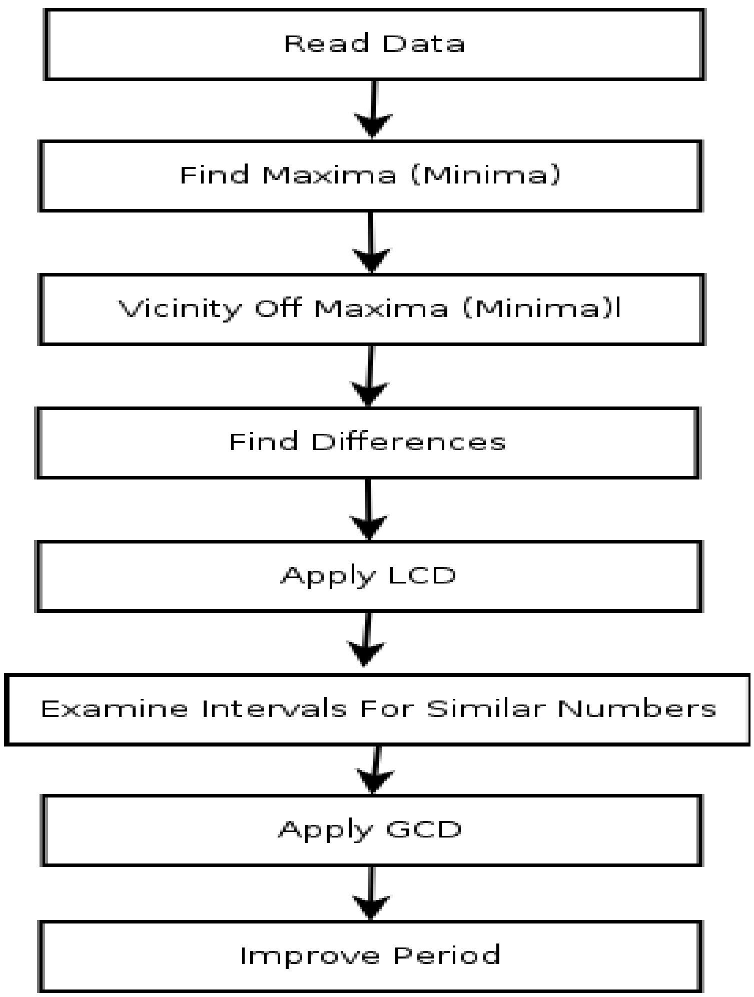

Diagram 1. Summary of the procedure.

3. Improvement of the Period

There are several way to improve the tentative period found above. With the value of the period found a phase diagram is plotted to see if it represents the observations correctly and changing the period slightly to one can appreciate the changes in the diagram until one is satisfied with the plot, for example when the minimum of the curve is close to half of the phase. We use an approximate least squares method (Bloomfield, 1976) to find a better period through a simple iteration procedure. Also, using more sophisticated method as the ones proposed by Lafler and Kinman (1965) (Jurkevich, 1971, Marraco and Mussio, 1980) one can find a better period, or using Spline (Akerlof, 1994), or Period04 (Lenz and Breger, 2005).

4. Examples of the Procedure

4.1 Numerical Simulation: Sinusoidal

In this section some analysis are made of some time series with the purpose of showing how the procedure is applied to some numerical simulations and several real observations of some known binary and variable stars reported in the literature. The numerical simulation is made for a sinusoidal variation with a given period with random data generated using a gaussian distribution.

$$y = R \sin(wt + \varphi) \quad , \tag{1}$$

with

$$w = \frac{2\pi}{P} \quad , \tag{2}$$

where P is the period, φ is the phase and R is the amplitude.

We have used a period of 2.5 days and an amplitude of 1.0 for the sinusoidal variation. Following our procedure we can recuperate the period with out any problem, as we will show. The data close to the maximum value are seven and are given in Table 1.

1	31.61425	0.9996352
2	16.63571	0.9996378
3	1.614091	0.9996241
4	19.13149	0.9998671
5	54.11985	0.9999163
6	59.13433	0.9997246
7	84.12874	0.9999558

Table 1. Values close to the maxima of the time series.

The differences between the first value and the other six, and of the second value with the other five and so on are given in Table 2.

1	14.97854
2	30.00016
3	12.48277
4	22.50560
5	27.52008
6	52.51449
7	15.02162
8	2.495779
9	37.48414
10	42.49863
11	67.49303
12	17.51740
13	52.50576
14	57.52024
15	82.51465
16	34.98837
17	40.00285
18	64.99725
19	5.014484
20	30.00889
21	24.99440

Table 2. Differences between the values forming groups of six, five, four, three, two, and one.

Divide the first six numbers of Table 2 by two and then by three, then four, five, and so on, the same could be done with the other numbers to produce a list of numbers where some of them are almost equal. One way to carry out this process is with the first and second numbers of Table 2 that can be divided by two and then by three and so on and then the other numbers are divided with those results to find the number of times they are divisible and then divide the number by those factors giving Table 3,

14.97854	30.00016	12.48277	22. 5056	27.520008	52.51449
6	12	5	9	11	21
2.49642	2.500013	2.496554	2.500622	2.501825	2.50069

Table 3. The first six values of Table 2. in the first row, in the second row the factor, and in the third row the results.

The greatest of these similar numbers in this case is around 2.5018. But at first sight one can see in Table 2 that the period could be around 2.495779. Applying the maximum common multiple with the pseudo Euclidean algorithm using the value of 2.5018 found in the last step multiplied by 0.8 as the stoping parameter to examine the results up to this quantity in order to have numbers of the order of 2.5018 in this procedure, Table 4 is generated.

1	2.523373	2.523373
2	2.539366	5.062738
3	2.562292	7.625031
4	2.598892	10.22392
5	2.542722	12.76664
6	2.495779	15.26242
7	2.543236	17.80566
8	2.566154	20.37181
9	2.602745	22.97456
10	2.542723	25.51728
11	2.590164	28.10745

12	2.613091	30.72054
13	2.649681	33.37022
14	2.543236	35.91345
15	2.566154	38.47961
16	2.602753	41.08236
17	2.518705	43.60107
18	2.555319	46.15639
19	2.532393	48.68878

Table 4. Results of the GCD in the second column and the sum in column three.

the average value using column three of Table 4, that is the sum of the number of the second column, divided by 19 is 2.562567. Plotting a phase diagram with this value for the period Figure 1 is obtained.

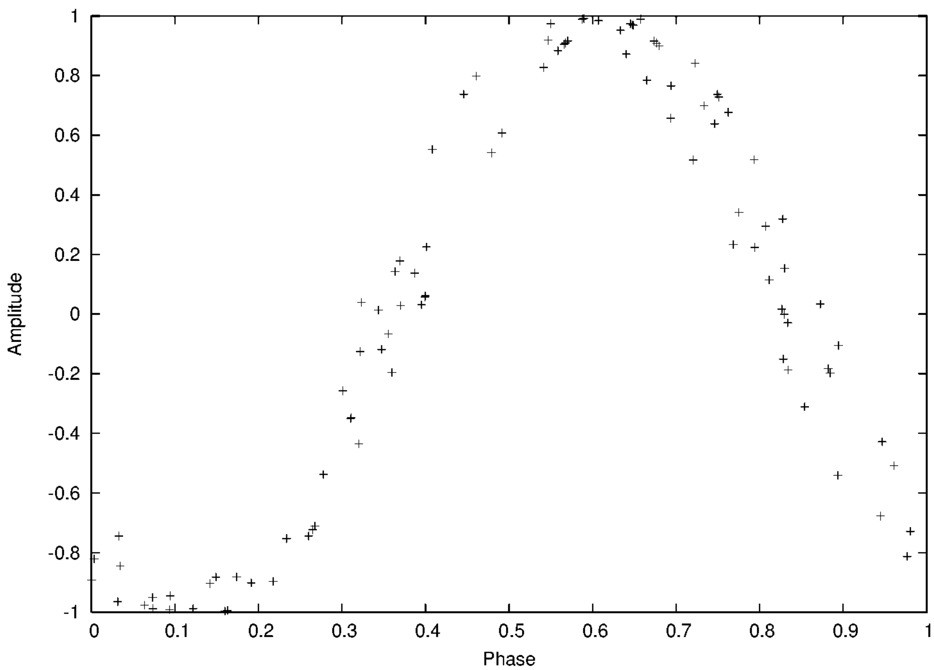

Figure 1. Phase diagram for the sinusoidal with the period 2.562567

This phase diagram shows some scatter of the points around the theoretical curve which means that the period is close to the theoretical one but must be corrected. The period can be improved in several ways as was mentioned before.

With the tentative period the method of minimum least squares in its simplest formulation can be applied to find a better period (Bloomfield, 1976). For a three-parameter model, following the notation of Bloomfield, given by

$$x_i = \mu + A \cos(wt_i) + B \sin(wt_i) + \epsilon_i \quad , \tag{3}$$

where x_i and t_i denote the *ith* values of the observations, w is the frequency and ϵ_i is the residual. The approximate solution of the equations of the estimates of least squares for the model are

$$\tilde{\mu} = \bar{x} = \frac{\sum x_i}{n} \quad , \tag{4}$$

$$\tilde{A} = 2\sum (x_i - \bar{x})\cos(wt_i) \quad , \tag{5}$$

$$\tilde{B} = \sum (x_i + \bar{x})\sin(wt_i) \quad . \tag{6}$$

To find R and φ, the amplitude and phase we solve the above equations with

$$A = -R\sin(\phi) \tag{7}$$

and

$$B = -R\cos(\phi) \quad , \tag{8}$$

therefore

$$R = \sqrt{A^2 + B^2} \tag{9}$$

and

$$\phi = \arctan\left(-\frac{B}{A}\right) \quad . \tag{10}$$

In this equations the frequency w is regarded as known. The method is extended to include the estimation of w following a simple iteration procedure starting with the approximate value found in the first part of the procedure presented in this article and defining the sum of squares of the residuals as

$$e = \frac{n}{2}R^2 \tag{11}$$

to carry out the iterations over frequency for all the equations given above by

$$w_{n+1} = w_n + e \times 10^{-(3)} \quad . \tag{12}$$

The criteria for stopping the iterations is the value found for the approximate period. Our procedures gives good results with respect to the theoretical period of 2.5.

4.2 Analysis of Some Observations

4.2.1 Binaries Stars

The analysis of three binary stars of different periods is presented to show the procedure for these type of time series.

4.2.1.1 26 Aquilae

This binary star has high orbital eccentricity where the primary component is of type G8 III-IV. There are fifty-one spectroscopic observations covering a 20 years interval. Figure 2 shows the plot of the 51 radial velocities listed by Franklin (1952)

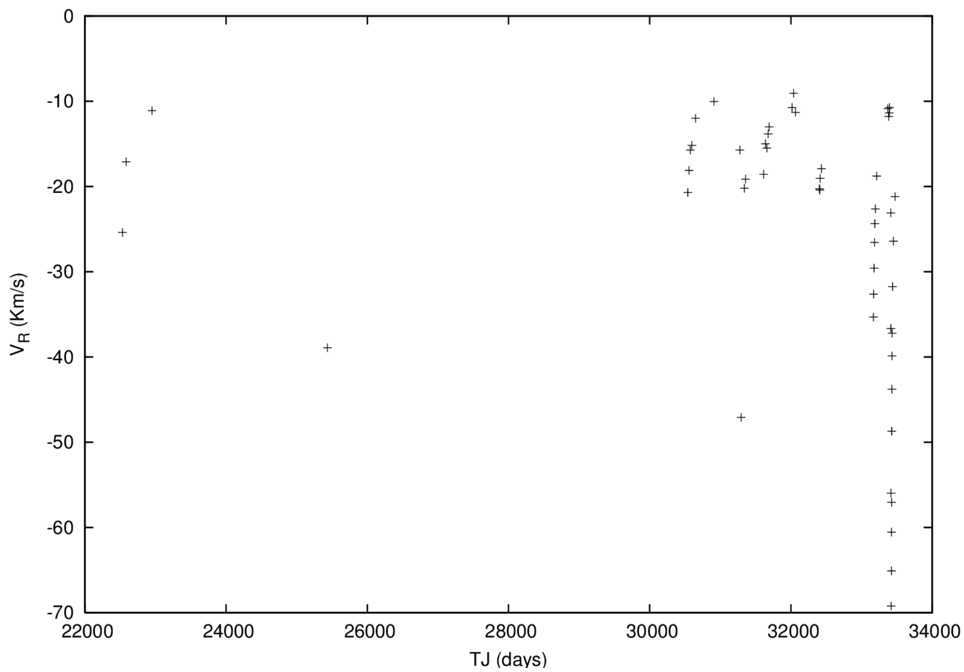

Figure 2. Observational Radial Velocity Curve for 26 Aquilae.

The data close to the maximum value are six and are given in table 5.

1	22951.695	-11.100
2	30908.668	-10.030
3	32015.820	-10.740
4	32036.832	-9.070
5	33372.023	-10.870
6	33397.984	-10.720

Table 5. Values close to the maxima of the time series.

The differences between the first value and the other five, and of the second value with the other four and so on are given in table 5.

1	1	7956.973
1	2	9064.125
1	3	9085.137
1	4	10420.33
1	5	10446.29
1	6	1107.152

1	7	1128.164
1	8	2463.355
1	9	2489.316
1	10	21.01172
1	11	1356.203
1	12	1382.164
1	13	1335.191
1	14	1361.152
1	15	25.96094

Table 6. Differences between the values forming groups of five, four, three, two, and one.

Divide the first five numbers of Table 6 using the same procedure as before, the same could be done with the other numbers to produce a list of numbers where some of them are almost equal.

7956.973	9064.125	9085.133	10420.33	10446.29
30	34	34	39	39
265.23	266.592	267.21	267.19	267.85

Table 7. The first five values of Table 6. in the first row, in the second row the factor, and in the third row the results.

The greatest of the similar numbers in this case is around 267.85. Applying the maximum common multiple with the pseudo Euclidean algorithm using the value of 267.85 found in the last step multiplied by 0.8 as the stoping parameter to have numbers of the order of 267.85 in this procedure, Table 8 is generated.

2	1	279.5273	279.5273
2	2	395.0742	674.6016
2	3	281.9219	956.5234
2	4	307.8828	1264.406
2	5	279.5273	1543.934

2	6	300.5391	1844.473
2	7	394.2930	2238.766
2	8	213.3477	2452.113
2	9	21.01172	2473.125
2	10	321.6719	2794.797
2	11	347.6328	3142.430
2	12	300.6602	3443.090
2	13	326.6211	3769.711
2	14	25.96094	3795.672

Table 8. Results of the GCD in the second column and the sum in column three.

the average value using column three of Table 8, that is the sum of the number of the second column, divided by 14 is 271.1194. Plotting a phase diagram with this value for the period Figure 3 is obtained.

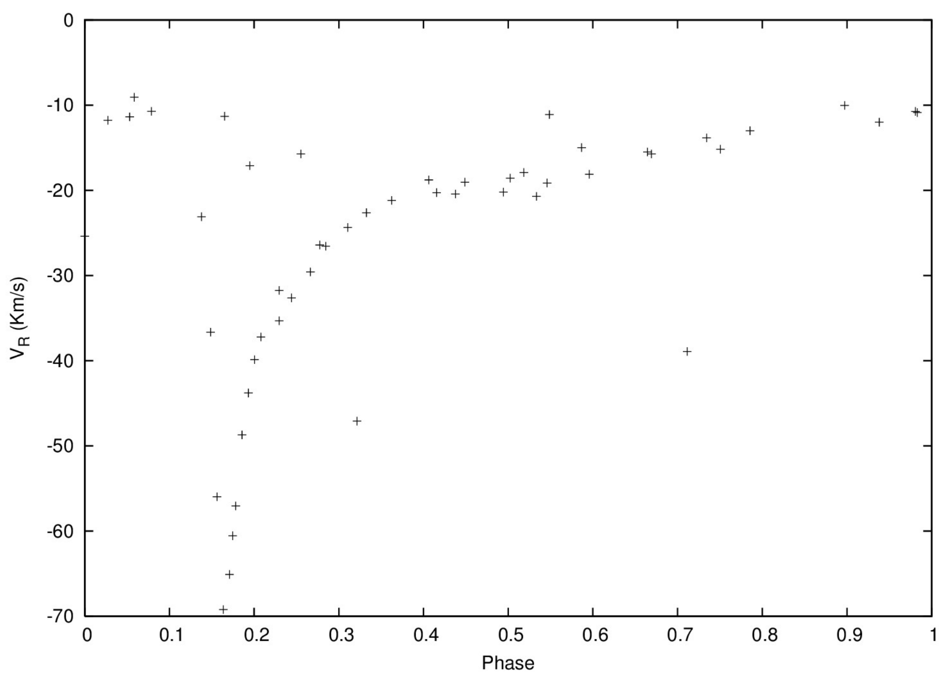

Figure 3. Phase Diagram for 26 Aquilae for the radial velocities.

This phase diagram shows some scatter of the points with a well defined curve which means that the period is close to the theoretical one and must be corrected. The period can be improved in several ways as was mentioned before. With the curve fitting method one finds a period of 266.995 days and the period given by Franklin and Wolfe et al. is 266.544 and 266.7 by Spline.

4.2.1.2 HD145425

This binary star located in Serpens Caput with magnitude 9.5 and spectral type K0 with forty-six radial velocities observed (Griffin, 1994). Table 4 show the plot of the forty-six radial velocities.

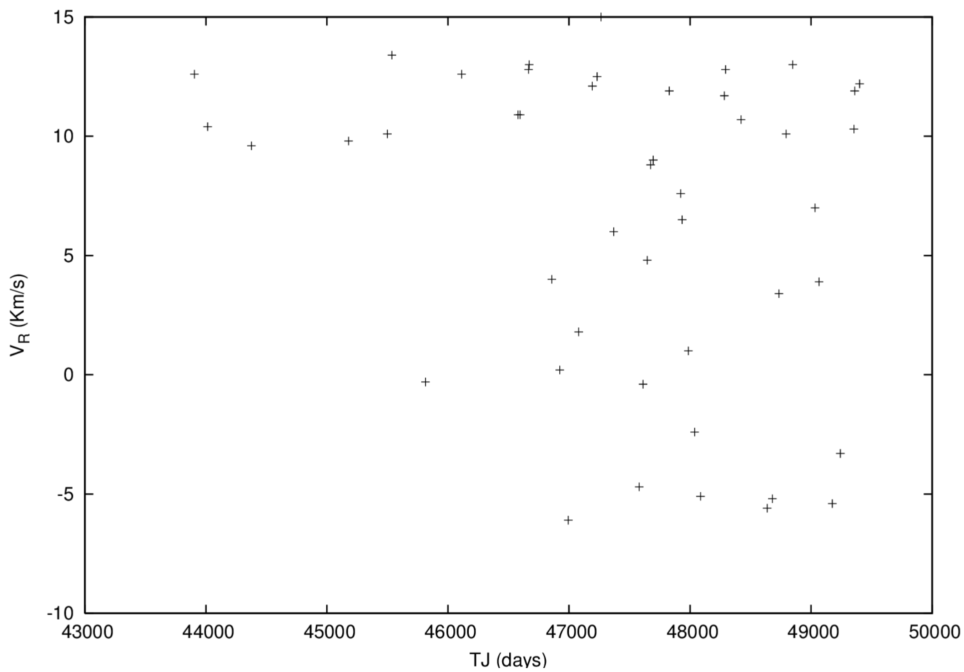

Figure 4. Observational Radial Velocities Data for HD145425.

The data close to the maximum value are eight and are given in Table 9.

1	43905.238	12.600
2	45536.922	13.400
3	46113.531	12.600
4	46665.859	12.800
5	46670.840	13.000
6	47264.121	15.000
7	48293.219	12.800
8	48847.879	13.000

Table 9. Values close to the maxima of the time series.

1	1	1631.684
1	2	2208.293
1	3	2760.621
1	4	2765.602
1	5	3358.883
1	6	4387.980
1	7	4942.641
1	8	576.6094
1	9	1128.938
1	10	1133.918
1	11	1727.199
1	12	2756.297
1	13	3310.957
1	14	552.3281
1	15	557.3086
1	16	1150.590
1	17	2179.688
1	18	2734.348
1	19	4.980469
1	20	598.2617
1	21	1627.359
1	22	2182.020
1	23	593.2812
1	24	1622.379
1	25	2177.039
1	26	1029.098
1	27	1583.758
1	28	554.6602

Table 10. Differences between the values forming groups of seven, six,five,....., and one.

Divide the first seven numbers of Table 10 using the same procedure as before, the same could be done with the other numbers to produce a list of numbers where some of them are almost equal.

1631.684	2208.293	2760.621	2765.602	3388.883	4387.98	4942.641
3	4	5	5	6	8	9
543.895	552.073	552.124	553.120	564.813	548.5	549.142

Table 11. The first nine values of Table 10. in the first row, in the second row the factor, and in the third row the results.

The greatest of the similar numbers in this case is around 564.813. But at first sight one can see in the Table 11 that the period could be around 552.3281. Applying the maximum common multiple with the pseudo Euclidean algorithm using the value of 564.813 found in the last step multiplied by 0.7 as the stoping parameter to have numbers of the order of 564.813 in this procedure, Table 11 is generated.

2	1	478.4648	478.4648
2	2	368.2969	846.7617
2	3	373.2773	1220.039
2	4	488.0938	1708.133
2	5	560.2617	2268.395
2	6	636.4570	2904.852
2	7	576.6094	3481.461
2	8	650.4727	4131.934
2	9	655.4531	4787.387
2	10	770.2695	5557.656
2	11	363.9727	5921.629
2	12	440.1680	6361.797
2	13	552.3281	6914.125
2	14	557.3086	7471.434
2	15	672.1250	8143.559
2	16	744.2930	8887.852
2	17	820.4883	9708.340
2	18	358.9336	10067.27
2	19	598.2617	10665.54
2	20	550.5586	11216.09
2	21	387.3516	11603.45
2	22	593.2812	12196.73
2	23	545.5781	12742.30
2	24	382.3711	13124.68
2	25	670.1641	13794.84
2	26	506.9570	14301.80
2	27	554.6602	14856.46

Table 12. Results of the GCD in the second column and the sum in column three.

the average value using column three of Table 12, that is the sum of the number of the second column, divided by 27 is 550.2391. Plotting a phase diagram with this value for the period Figure 5 is obtained.

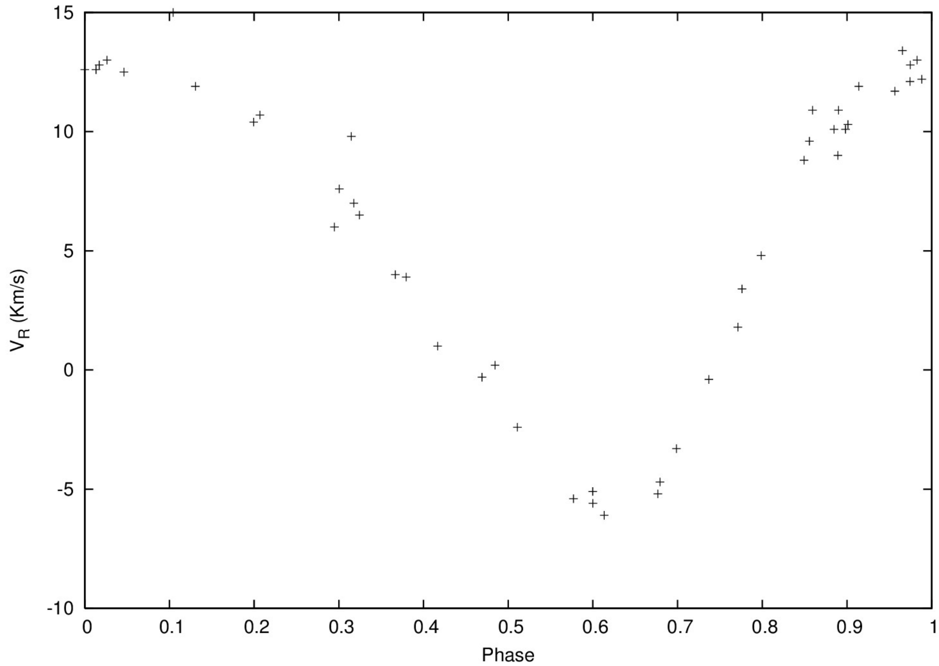

Figure 5. Phase Diagram for HD145425.

This phase diagram shows some scatter of the points and must be corrected. The period can be improved using the curve fitting method giving 550.963 and 550.134 with Spline. The value given by Griffin is 549.9.

4.2.1.3 HD217792

This spectroscopic binary star of magnitude V = 6.10 and spectral type F0V has fifty-two radial velocity observations (Bopp, Evans, and Laing, 1970) plotted in Figure 6.

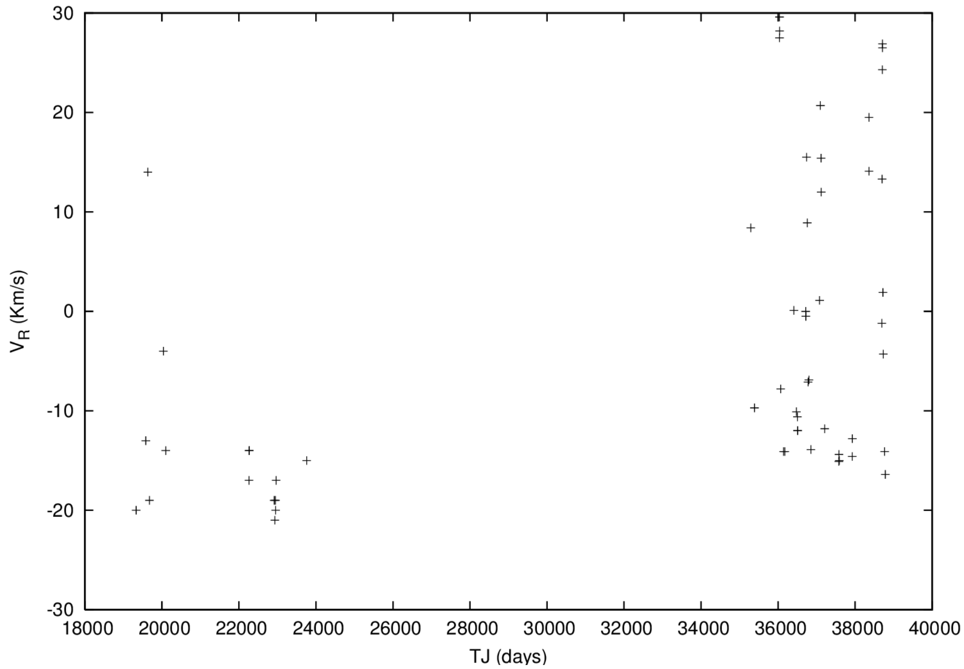

Figure 6. Observed Radial Velocities Data for HD217792.

The data close to the maximum value are 14 and are given in table 13.

1	19334.660	-20.000
2	19678.699	-19.000
3	22262.350	-17.000
4	22918.471	-19.000
5	22933.430	-21.000
6	22949.340	-19.000
7	22953.330	-20.000
8	22967.330	-17.000
9	23761.301	-15.000
10	37578.352	-15.100
11	37580.344	-14.400
12	37588.336	-15.000
13	37927.305	-14.600
14	38782.227	-16.400

Table 13. Values close to the maxima of the time series.

1	1	344.0391
1	2	2927.689
1	3	3583.811
1	4	3598.770
1	5	3614.680
1	6	3618.670

1	7	3632.670
1	8	4426.641
1	9	18243.69
1	10	18245.68
1	11	18253.68
1	12	18592.64
1	13	19447.57
1	14	2583.650
1	15	3239.771
1	16	3254.730
1	17	3270.641
1	18	3274.631
1	19	3288.631
1	20	4082.602
1	21	17899.65
1	22	17901.64
1	23	17909.64
1	24	18248.61
1	25	19103.53
1	26	656.1211
1	27	671.0801
1	28	686.9902
1	29	690.9805
1	30	704.9805
1	31	1498.951
1	32	15316.00
1	33	15317.99
1	34	15325.99
1	35	15664.96
1	36	16519.88
1	37	14.95898
1	38	30.86914
1	39	34.85938
1	40	48.85938
1	41	842.8301
1	42	14659.88
1	43	14661.87
1	44	14669.87
1	45	15008.83
1	46	15863.76
1	47	15.91016
1	48	19.90039
1	49	33.90039

1	50	827.8711
1	51	14644.92
1	52	14646.91
1	53	14654.91
1	54	14993.88
1	55	15848.80
1	56	3.990234
1	57	17.99023
1	58	811.9609
1	59	14629.01
1	60	14631.00
1	61	14639.00
1	62	14977.96
1	63	15832.89
1	64	14.00000
1	65	807.9707
1	66	14625.02
1	67	14627.01
1	68	14635.01
1	69	14973.97
1	70	15828.90
1	71	793.9707
1	72	14611.02
1	73	14613.01
1	74	14621.01
1	75	14959.97
1	76	15814.90
1	77	13817.05
1	78	13819.04
1	79	13827.04
1	80	14166.00
1	81	15020.93
1	82	1.992188
1	83	9.984375
1	84	348.9531
1	85	1203.875
1	86	7.992188
1	87	346.9609
1	88	1201.883
1	89	338.9688

1	90	1193.891
1	91	854.9219

Table 14. Differences between the values forming groups of thirteen,....., and one.

Divide the first 13 numbers of Table 14 using the same procedure as before, the same could be done with the other numbers to produce a list of numbers where some of them are almost equal.

344.0391	2927.689	3583.811	3398.77	3614.68	3618.67	3632.67	4726.641	18243.69	18245.68
2	16	20	19	20	20	20	26	102	102
172.02	182.98	179.191	178.883	180.734	189.934	181.634	181.794	178.86	178.88

18253.68	18592.64	19447.57
102	104	109
178.958	178.754	178.418

Table 15. The first thirteen values of Table 14. in the first row, in the second row the factor, and in the third row the results.

The greatest of the similar numbers in this case is around 181.794. Applying the maximum common multiple with the pseudo Euclidean algorithm using the value of 181.794 found in the last step multiplied by 0.8 as the stoping parameter to have numbers of the order of 181.794 in this procedure, Table 16 is generated.

2	1	168.6621	168.6621
2	2	210.5684	379.2305
2	3	225.5273	604.7578
2	4	241.4375	846.1953
2	5	245.4277	1091.623
2	6	259.4277	1351.051
2	7	210.0879	1561.139
2	8	196.8457	1757.984
2	9	198.8379	1956.822
2	10	206.8301	2163.652
2	11	208.4746	2372.127

2	12	220.0859	2592.213
2	13	222.3809	2814.594
2	14	203.8535	3018.447
2	15	218.8125	3237.260
2	16	234.7227	3471.982
2	17	238.7129	3710.695
2	18	252.7129	3963.408
2	19	203.3730	4166.781
2	20	190.1309	4356.912
2	21	192.1230	4549.035
2	22	200.1152	4749.150
2	23	201.7598	4950.910
2	24	213.3711	5164.281
2	25	150.1348	5314.416
2	26	165.0938	5479.510
2	27	181.0039	5660.514
2	28	184.9941	5845.508
2	29	198.9941	6044.502
2	30	149.6543	6194.156
2	31	136.4121	6330.568
2	32	138.4043	6468.973
2	33	146.3965	6615.369
2	34	148.0410	6763.410
2	35	159.6523	6923.062
2	36	138.7441	7061.807
2	37	30.86914	7092.676
2	38	34.85938	7127.535
2	39	48.85938	7176.395
2	40	149.1094	7325.504
2	41	230.4902	7555.994
2	42	232.4824	7788.477
2	43	240.4746	8028.951
2	44	163.2109	8192.162
2	45	185.6680	8377.830
2	46	15.91016	8393.740
2	47	19.90039	8413.641
2	48	33.90039	8447.541
2	49	272.8945	8720.436
2	50	215.5312	8935.967
2	51	217.5234	9153.490
2	52	225.5156	9379.006
2	53	148.2520	9527.258
2	54	170.7090	9697.967
2	55	134.7539	9832.721
2	56	17.99023	9850.711
2	57	138.1914	9988.902
2	58	210.3438	10199.25

2	59	212.3359	10411.58
2	60	220.3281	10631.91
2	61	155.0352	10786.95
2	62	201.4336	10988.38
2	63	14.00000	11002.38
2	64	268.9551	11271.33
2	65	206.3535	11477.69
2	66	208.3457	11686.03
2	67	216.3379	11902.37
2	68	151.0449	12053.42
2	69	197.4434	12250.86
2	70	254.9551	12505.81
2	71	192.3535	12698.17
2	72	194.3457	12892.51
2	73	202.3379	13094.85
2	74	137.0449	13231.90
2	75	183.4434	13415.34
2	76	206.9062	13622.25
2	77	208.8984	13831.14
2	78	216.8906	14048.04
2	79	151.5977	14199.63
2	80	197.9961	14397.63
2	81	1.992188	14399.62
2	82	9.984375	14409.61
2	83	214.1992	14623.80
2	84	260.5977	14884.40
2	85	7.992188	14892.39
2	86	212.2070	15104.60
2	87	258.6055	15363.21
2	88	204.2148	15567.42
2	89	250.6133	15818.04
2	90	181.1523	15999.19

Table 16. Results of the GCD in the second column and the sum in column three.

the average value using column three of Table 16, that is the sum of the number of the second column, divided by 90 is 177.7688. Plotting a phase diagram with this value for the period Figure 7 is obtained.

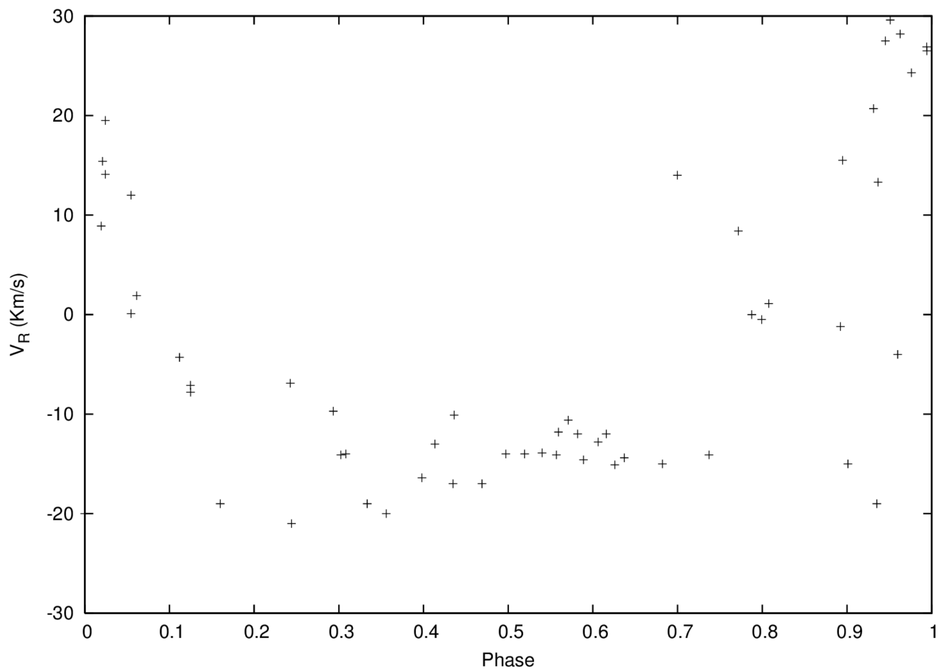

Figure 7. Phase Diagram for HD217792.

This phase diagram shows some scatter of the points which means that the period is not that far from the best period and must be corrected. The period can be improved giving 178.053 with the curve fitting method and 178.316 with Spline. The value give Bopp et al. is 178.3177.

4.3 Variables Stars

4.3.1 BK Centaurus

This classical Cepheid has 49 radial velocity observations that show a beat period.

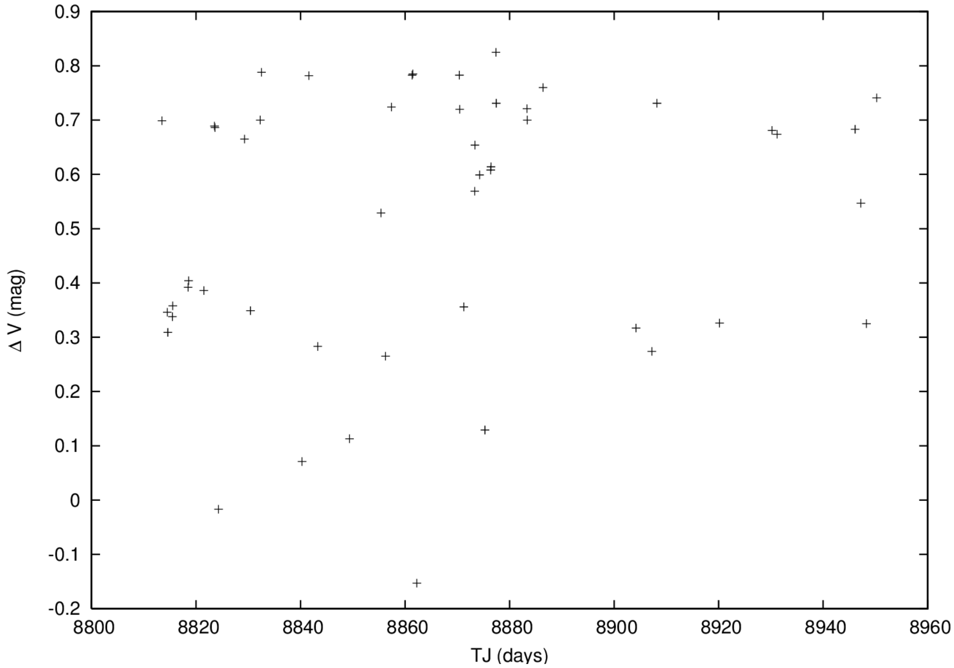

Figure 8. Visual Light Curve for BK Centaurus.

The data close to the maximum value are 12 and are given in table 17.

1	8832.520	0.788
2	8841.580	0.782
3	8857.380	0.724
4	8861.360	0.783
5	8861.480	0.785
6	8870.390	0.783
7	8877.380	0.825
8	8877.430	0.731
9	8883.320	0.721
10	8886.400	0.760
11	8908.200	0.731
12	8950.240	0.741

Table 17. Values close to the maxima of the time series.

1	1	9.060547
1	2	24.86035
1	3	28.84082
1	4	28.96094
1	5	37.87012
1	6	44.86035
1	7	44.91016

1	8	50.80078
1	9	53.88086
1	10	75.68066
1	11	117.7207
1	12	15.79980
1	13	19.78027
1	14	19.90039
1	15	28.80957
1	16	35.79980
1	17	35.84961
1	18	41.74023
1	19	44.82031
1	20	66.62012
1	21	108.6602
1	22	3.980469
1	23	4.100586
1	24	13.00977
1	25	20.00000
1	26	20.04980
1	27	25.94043
1	28	29.02051
1	29	50.82031
1	30	92.86035
1	31	0.1201172
1	32	9.029297
1	33	16.01953
1	34	16.06934
1	35	21.95996
1	36	25.04004
1	37	46.83984
1	38	88.87988
1	39	8.909180
1	40	15.89941
1	41	15.94922
1	42	21.83984
1	43	24.91992
1	44	46.71973
1	45	88.75977
1	46	6.990234
1	47	7.040039
1	48	12.93066
1	49	16.01074

1	50	37.81055
1	51	79.85059
1	52	4.9804688E-02
1	53	5.940430
1	54	9.020508
1	55	30.82031
1	56	72.86035
1	57	5.890625
1	58	8.970703
1	59	30.77051
1	60	72.81055
1	61	3.080078
1	62	24.87988
1	63	66.91992
1	64	21.79980
1	65	63.83984
1	66	42.04004

Table 18. Differences between the values forming groups of eleven,....., and one.

Divide the first 11 numbers of Table 18 using the same procedure as before, the same could be done with the other numbers to produce a list of numbers where some of them are almost equal.

9.060547	24.86035	28.84082	28.96094	37.87012	44.86035	44.91016	50.80078	53.88086
3	8	9	9	12	15	15	17	18
3.02018	3.10754	3.20454	3.21788	3.14584	2.99069	2.99401	2.98828	2.99338
75.68066	117.7207							
25	39							
3.02723	3.01845							

Table 19. The first thirteen values of Table 14. in the first row, in the second row the factor, and in the third row the results.

The greatest of the similar numbers in this case is around 3.218. Applying the maximum common multiple with the pseudo Euclidean algorithm using the value of 3.218 found in the last step multiplied by 0.8 as the stoping parameter to have numbers of the order of 3.0 in this procedure, Table 20 is generated.

2	1	2.096680	2.096680
2	2	3.306641	5.403320
2	3	3.426758	8.830078
2	4	3.050781	11.88086
2	5	3.077148	14.95801
2	6	3.126953	18.08496
2	7	2.053711	20.13867
2	8	2.812500	22.95117
2	9	3.720703	26.67188
2	10	3.977539	30.64941
2	11	4.193359	34.84277
2	12	3.531250	38.37402
2	13	3.651367	42.02539
2	14	3.275391	45.30078
2	15	3.301758	48.60254
2	16	3.351562	51.95410
2	17	2.278320	54.23242
2	18	3.037109	57.26953
2	19	3.945312	61.21484
2	20	4.202148	65.41699
2	21	3.980469	69.39746
2	22	4.100586	73.49805
2	23	3.724609	77.22266
2	24	3.750977	80.97363
2	25	3.800781	84.77441
2	26	2.727539	87.50195
2	27	3.486328	90.98828
2	28	2.073242	93.06152
2	29	2.330078	95.39160
2	30	2.081055	97.47266
2	31	2.786133	100.2588
2	32	3.533203	103.7920
2	33	3.583008	107.3750
2	34	3.230469	110.6055
2	35	2.148438	112.7539
2	36	3.137695	115.8916
2	37	3.556641	119.4482
2	38	2.666016	122.1143
2	39	3.413086	125.5273

2	40	3.462891	128.9902
2	41	3.110352	132.1006
2	42	2.028320	134.1289
2	43	3.017578	137.1465
2	44	3.436523	140.5830
2	45	2.828125	143.4111
2	46	2.877930	146.2891
2	47	2.525391	148.8145
2	48	3.524414	152.3389
2	49	2.432617	154.7715
2	50	2.851562	157.6230
2	51	2.031250	159.6543
2	52	3.909180	163.5635
2	53	2.926758	166.4902
2	54	2.382812	168.8730
2	55	3.797852	172.6709
2	56	3.859375	176.5303
2	57	2.876953	179.4072
2	58	2.333008	181.7402
2	59	3.748047	185.4883
2	60	3.080078	188.5684
2	61	2.536133	191.1045
2	62	3.951172	195.0557
2	63	3.518555	198.5742
2	64	2.902344	201.4766
2	65	3.446289	204.9229

Table 20. Results of the GCD in the second column and the sum in column three.

the average value using column three of Table 20, that is the sum of the number of the second column, divided by 65 is 3.152659. Plotting a phase diagram with this value for the period Figure 9 is obtained.

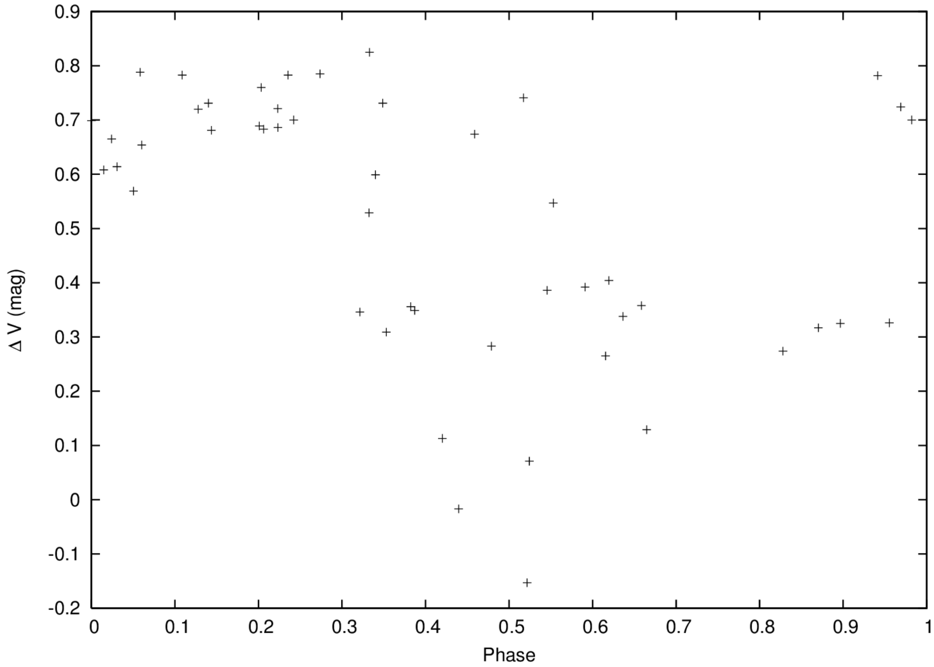

Figure 9. Phase Diagram Curve for BK Centaurus.

This phase diagram shows some scatter of the points and must be corrected. The period can be improved in several ways giving 3.17389 by Leotta-Janin, 3.218 with the curve fitting method and 3.166 with Spline.

5. Comparisons with Other Methods

This simple procedure produces approximate periods using elementary mathematical operations as are the analogues of the least common divisor and the greatest common divisor, hence can not be compared with more elaborated methods, but even so one can find approximate periodicities in unevenly spaced data containing gaps for few data points of the observational series. With the approximate periods found one can use any of the other methods to improve these tentative periods as we have done with the curve fitting by the approximate least squares method. The results for the cases considered in this article compare well with the results obtained with other methods. An also one can consider it is a complement to some of the other methods because the period found could be used as the starting search for periodicities for those methods.

6. Conclusion and Commentaries

The procedure for finding period for few observational data uses the values close to the maximum of the observational time series to apply something like the least common divisor to find an approximation of the period that can be used in a procedure similar to de Euclid's procedure to find the greatest common divisor but with a stoping parameter different from zero that can be obtained form the approximate period found before multiplied by a small fraction to produce values close to the approximate period. As the points close to the maximum are approximations to the maxima (minima) of the series and can fall in either side of the maximum, therefore it is necessary to take an average of the values found with the GCD to obtain a value close to the true period. This value can be improved in different ways as mentioned previously. We use the curve fitting method by least squares with a three-parameter model iteratively. The procedure produces results good enough for predicting in an approximate way the periods necessary for forecasting the evolution of a observed time series with the purpose of aiding in choosing the future observational times of the phenomena under study. The procedure produces results in agreement with the results produced by other more sophisticated methods.

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