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5 authors, including:

Marcelino Anguiano-Morales
Instituto Tecnológico de Chihuahua

Amalia Martínez
Centro de Investigaciones en Optica

Marcelo David Iturbe-Castillo
Instituto Nacional de Astrofísica, Óptica y Ele…

S. Chávez-Cerda
Instituto Nacional de Astrofísica, Óptica y Ele…

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Marcelino Anguiano-Morales,[1,*] Amalia Martínez,[1] M. David Iturbe-Castillo,[2] Sabino Chávez-Cerda,[2] and N. Alcalá-Ochoa[1]

[1]Centro de Investigaciones en Óptica, A. C. Lomas del Bosque No 115, Col Lomas del Campestre León, Guanajuato México 37150

[2]Instituto Nacional de Astrofisica, Óptica y Electrónica, Apartado Postal 51 y 216 Puebla, Puebla México 72000

*Corresponding author: anguiano@cio.mx

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It is well known that Bessel beams and the other families of propagation-invariant optical fields have the property of self-healing when obstructed by an opaque object. Here it is shown that there exists another kind of field distribution that can have an analog property. In particular, we demonstrate that a class of caustic wave fields, whose transverse intensity patterns change on propagation, when perturbed by an opaque object can reappear at a further plane as if they had not been obstructed. The physics of the phenomenon is fully explained and shown to be related to that of self-healing propagation invariant optical fields. © 2007 Optical Society of America

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1. Introduction

The concept of an axicon or a conical lens originally was introduced by MacLeod [1] in the mid 1950s. An important property of a conical lens is its ability to form a line image along the optical axis of the conical lens for a collimated incoming beam. The line image produced by an axicon has a transverse structure described by the J_0 Bessel function [2]. This image is very sensitive to aberrations, particularly if illuminated either with a cylindrical wave or an off axis source (e.g., a tilted plane wave); the outcome is an image defined by a caustic surface [3,4]. We will refer to this wave field as caustic beam.

Almost three decades after the introduction of the axicon, it was pointed out that from the scalar wave equation it was possible to obtain optical wave fields described by the J_0 Bessel function that could represent optical beams with an apparent lack of diffraction as they propagated [5,6]. For this reason they were called nondiffracting beams. Their propagation transverse invariance could be explained only if the whole Bessel beam propagated completely parallel to the propagation axis, similar to a plane wave. This apparent unphysical property yielded several controversial results until it was shown that Bessel beams are formed by the superposition of inward and outward conical Hankel waves [7]. A simple geometrical analysis shows that a positive axicon illuminated with a collimated beam produces uniform incoming conical wavefronts that, at the point where they reach the axis, become outgoing conical wavefronts. And, in the region of space where the outgoing waves superpose with those incoming ones, a transverse wave field with a J_0 Bessel profile will be created [7,8].

Within the strange properties of a nondiffracting Bessel beam is that of self-reconstruction or self-healing that occurs when part of the beam is blocked, and the beam reappears with practically the same transverse structure that it had before the obstruction. This phenomenon, which is not related to diffraction, as it can be first thought, is easily understood within the approach of conical waves, since their wave vectors are slanted with respect to the propagation axis [5–8].

In this work we demonstrate for the first time that caustic beams can present the self-healing property. By illuminating an axicon with a wavefront produced by a cylindrical lens, we produce a caustic beam that, when it is obstructed by an opaque object, may reappear as if there was no obstruction. The self-healing

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characteristics of the caustic beam will depend on the sign of the cylindrical wavefront, in other words, if it arrives at the axicon as a converging or diverging cylindrical wave.

2. Bessel Profile Line Image Produced by an Axicon

It has long been known that an axicon illuminated by a uniform plane wave produces conical waves. Conical waves are commonly described by a linear phase function of the form $\exp(ik_r r)$ where the constant k_r is related to the inclination angle of the normal to the cone surface with respect to the symmetry axis by $k_r = k \sin \theta$, where k is the wave number. The resulting transverse field intensity produced with such a conic phase function does not agree with the line image profile of the axicon that is described by the Bessel function of the first kind J_0 [2,9]. The required complex phase function is the Hankel function $H_0^{(2)}(k_r r)$ that describes incoming conical waves. By making a simple geometrical analysis we observe that, just after the tip of the axicon, the incoming conical wave transforms into an outgoing conical wave. The latter is described by the Hankel function $H_0^{(1)}(k_r r)$. In the region of space where both conical waves superpose, which includes the axis, the transverse intensity will have finite amplitude and a J_0 Bessel profile with radial spatial frequency given by k_r [7,8]. Neglecting edge diffraction effects, in the whole volume of superposition a Bessel beam is created that only changes its transverse extent maintaining its amplitude and frequency invariant [7,9]. Notice that the singularity on axis of the Hankel functions disappears when they superpose, and then there is no physical inconsistency. The phases of Hankel functions can be used to represent conical waves as shown in Fig. 1. The superposition of this very particular kind of conical wave gives rise to the Bessel beam. The conic wave nature of Bessel beams is the responsible for its self-reconstruction property.

It is easy to deduce that an axicon of refractive index n, with base angle γ illuminated with a uniform plane wave, will produce a conical wave whose cone of wave vectors makes an angle δ with respect to the propagation axis approximately given by $\delta = (n - 1)\gamma$. If the illuminating wave has wave number k, then the radial frequency of the resulting Bessel pattern will be

$$k_r = (n - 1)k\gamma. \tag{1}$$

If the radius of the axicon is a, the maximum distance where the conical waves will superpose is

$$z_{\max} = \frac{a}{\gamma(n - 1)}. \tag{2}$$

It can be obtained geometrically that the Bessel transverse intensity pattern will have its maximum transverse extent at half this distance. The numerical solution of the Helmholtz equation using an incoming conical Hankel wave $H_0^{(2)}(k_r r)$ as the initial condition is shown in Fig. 2. Since Gaussian beams are commonly used as a comparison reference, the Helmholtz equation has been normalized according to Gaussian beam units. This means that given a Gaussian beam of waist w_0 at $z = 0$, the transverse coordinate r is measured in units of w_0 and the longitudinal coordinate in units of the corresponding diffraction length $L_D = kw_0^2/2$. Translating to experimental data, we assume a laser source of 633 nm with Gaussian profile with a waist equal to 0.5 mm; then with these data in Fig. 2, a normalized transverse coordinate value of 15 is equal to 7.5 mm and a normalized propagation distance of 1 is equivalent to approximately 1.24 m. In the figure the incoming Hankel wave has a radius $a = 16$ with inclination such that $k_r = 16$ in normalized units. With these parameters the Bessel beam of maximum transverse extent occurs at the propagation distance of $z = 1$. We observe two main regions, the one where

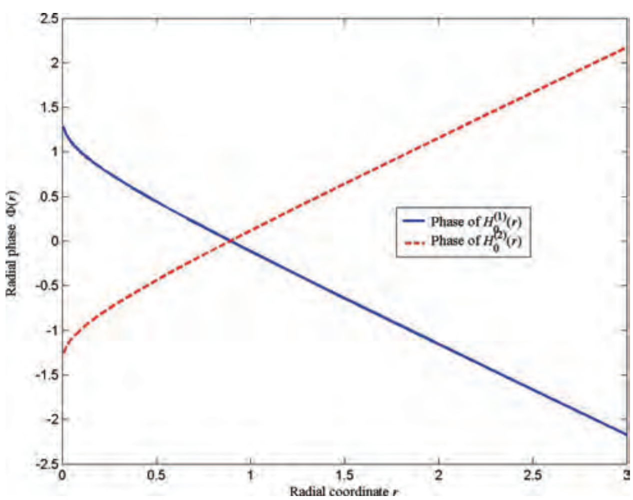

Fig. 1. (Color online) Phases of Hankel functions to represent conical waves.

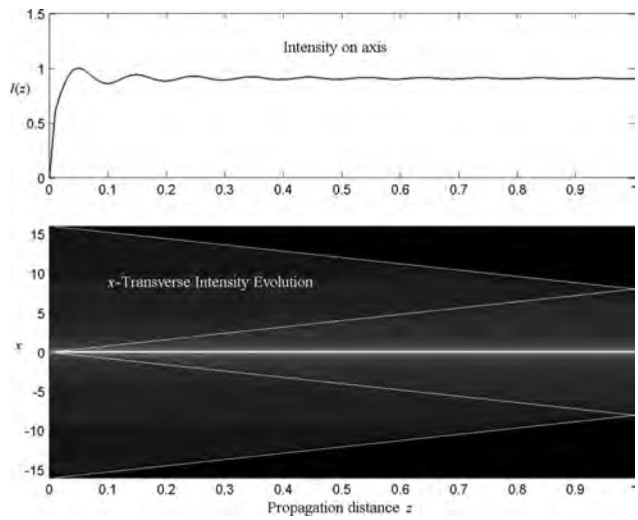

Fig. 2. Numerical solution of the Helmholtz equation using an incoming conical Hankel wave $H_0^{(2)}(k_r r)$ as the initial condition.

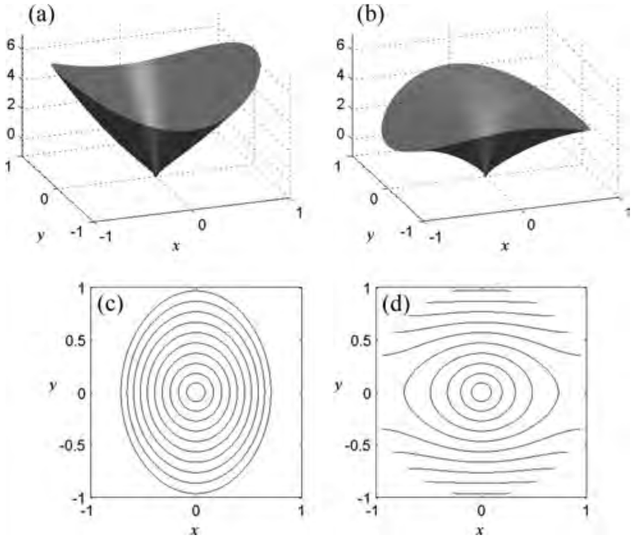

Fig. 3. Plots of the resulting wavefront for a cylindrical wave (a) convergent case and (b) divergent case.

only the propagating incoming conical wave exists (the two small triangles that in fact are one single volume in three dimensions) and the other where it transforms into the outgoing conical wave superposing with it and producing the Bessel beam (large centered triangle/cone). The lines obtained applying geometrical optics clearly agree with the limits of the conical wave propagation.

3. Formation and Propagation of Caustic Beams

As mentioned in the introduction, the line image produced by an axicon is very sensitive to tilt and astigmatism aberrations. Both produce similar intensity caustic patterns that expand (diffract) on propagation [3,4]. Astigmatism can be created by a cylindrical lens with transfer function

$$t_{\mathrm{CL}} = \exp\left(-i\,\frac{kx^2}{2f}\right), \tag{3}$$

where f is the focal distance of the lens. f is positive if the wavefront is converging and negative otherwise. Both situations can be achieved with a single positive lens along the optical axis at each side of the focal plane, respectively. When the axicon is illuminated with the cylindrical wave, the resulting phase front can be approximated by

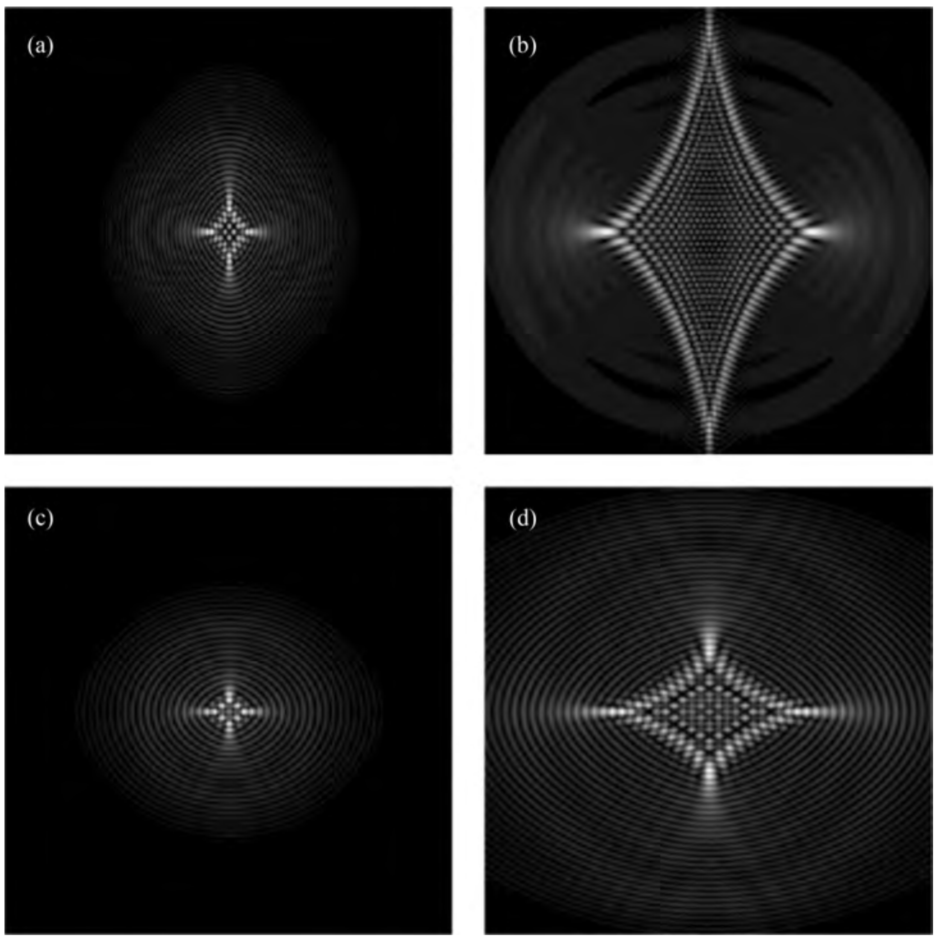

Fig. 4. Numerical simulation of caustic beams for positive cylindrical wave illumination: (a) $z = 0.5$ normalized units, (b) 1.5 normalized units; for negative cylindrical wave illumination, (c) $z = 0.5$ normalized units, (d) 1.5 normalized units.

$$\Phi(x,\ y) \cong \pm\frac{kx^2}{2f} + k_r\sqrt{x^2+y^2} + \frac{\pi}{4}, \qquad (4)$$

where the positive sign of the first term corresponds to a converging wave and the negative is for the opposite case. The plots of the resulting wavefront for both situations are shown in Figs. 3(a) and 3(b). We notice that for the positive case, Fig. 3(a), the wavefront is always converging, contrary to the negative case in which the wavefront may diverge due to the opposite signs of the quadratic and linear terms, Fig. 3(b). The closest points to the propagation axis at which the rays start to diverge are along the x axis and are at the maxima of the parabolas obtained when $y = 0$. In terms of the axicon and lens parameters, they are found to be at

$$x_c = \pm f\delta = \pm f\gamma(n-1). \qquad (5)$$

When a cylindrical wavefront impinges on the flat surface of the axicon, the intensity pattern is determined by the curvature of the corresponding wavefront. By looking at the contour plots of the wavefronts in Figs. 3(a) and 3(b), we can deduce that the intensity patterns will be caustics that will no longer be invariant on propagation [2]. For the positive case, Fig. 3(c), the contour plots are practically ellipses, which are known to produce astroid caustics [10]. For the negative case, the inner contours show that on propagation the caustic beam can also be approximated by an astroid. The intensity patterns of the caustic beams shown in Fig. 4 have been produced using a cylindrical lens of focal normalized distance $f = \pm 2$. Figures 4(a) and 4(b) show the caustic generated by the positive lens at the distances of $z = 0.5$ and $z = 1.0$, respectively. Figures 4(c) and 4(d) are the caustics formed at the same distance by the negative lens (in the units defined above the sides of the square images are equivalent to 7.5 mm). Astroid optical caustics are formed by the interference of two opposite cusp caustics that are aligned with respect to the major axis of the ellipse associated to the caustic; see Fig. 3.

The mathematical representation of the focal region of the astroid caustic beam is given by the parametric equations [10],

$$x = \frac{a^2 - b^2}{a}\cos^3\theta, \qquad y = \frac{b^2 - a^2}{b}\sin^3\theta, \qquad (8)$$

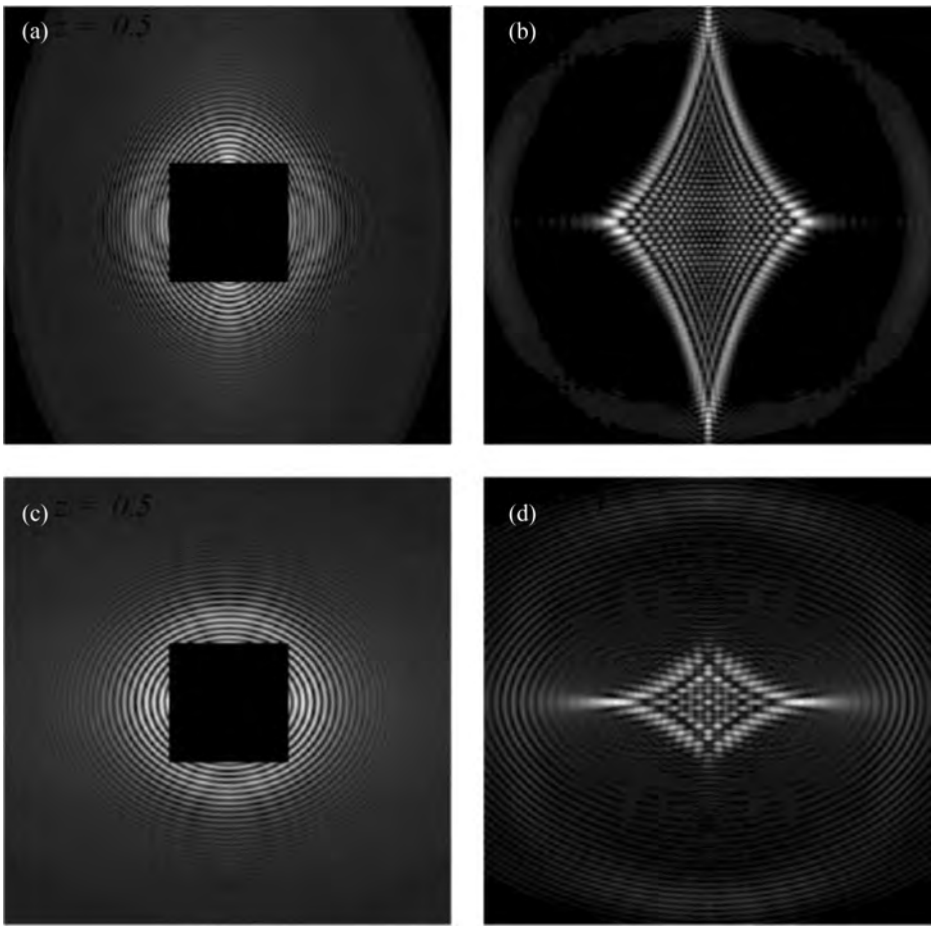

Fig. 5. Self-healing of a caustic beam for positive cylindrical wave illumination: (a) $z = 0.5$ normalized units, (b) 1.5 normalized units; for negative cylindrical wave illumination, (c) $z = 0.5$ normalized units, (d) 1.5 normalized units.

where a and b are the semiaxes of the ellipse, which can be combined and rewritten as

$$(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}. \qquad (9)$$

Similarly to the evolution of the conical Hankel wave above, it is expected that the focal volume be contained within an evolute; with further propagation on z the entire caustic pattern is increased as shown in Fig. 4, where the normal rays to the elliptic conical surface generated the evolute.

4. Self-Reconstruction of a Nondiffracting Beam

The property of self-reconstruction is usually associated with nondiffracting Bessel beams. However, it is not particular to them. There are other families of fundamental nondiffracting beams that are exact solutions of the Helmholtz equation, namely, Mathieu beams with elliptic and hyperbolic transverse intensity patterns, transverse parabolic beams, and also those created by the interference of a finite number of tilted plane waves. The spectrum of each fundamental family of nondiffracting wave fields has a respective function that is defined on a ring-shaped domain in frequency space. The wave vectors of all families of nondiffracting beams lie on a cone, i.e., all these families of beams are formed by the superposition of conical waves whose amplitude is modulated by their respective spectral function [11,12].

The physics of self-reconstruction of a nondiffracting beam is very clear under the light of the dynamics of conical wave fields [8]. Since the incident nondiffracting beam is formed by the superposition of conical waves, those parts traveling inward that were not obstructed will generate outgoing conical waves after passing through the axis reconstructing the original transverse intensity pattern. From this approach, it is noticed that diffraction does not play a role in the self-reconstruction of the beam, although it can be a perturbation in the resulting pattern at early stages of the reconstruction. By making a simple geometrical analysis, it is found that if a circular obstruction of radius R is placed on the path of a nondiffracting beam with a cone of wavevectors making an angle δ, it will start to regenerate at a distance given by

$$z_{\text{rec}} = \frac{R}{\gamma(n-1)}, \qquad (10)$$

where the parameters of the axicon have been used. Notice the analogy with Eq. (1); the reason is that geometrically their longitudinal sections form similar triangles. Equation (1) is for the cone of light illuminated by the conical wave, while Eq. (10) is for the conical shadow of the object. When the obstruction is off-axis, after this distance, the incoming conical wave that was not affected by the obstruction encounters again the outgoing conical wave reconstructing the nondiffracting beam.

5. Self-Healing of a Caustic Beam

Usually the terms self-healing and self-reconstructing are used indistinctly. Here we will refer to self-reconstruction as meaning the properly of a beam of fully regenerating a perturbed region when partially blocked by an opaque obstruction and then having its original transverse intensity pattern. Self-healing of a beam will be understood as the phenomenon of a beam being perturbed by an opaque object and reappearing at a further plane with practically the same transverse intensity distribution as if it had not been obstructed. In our definition, for self-healing, the transverse intensity before and after the obstruction might not be the same. In this sense self-reconstruction can be understood as self-healing but not the other way around.

We have deduced above from the contour plots [Figs. 3(c) and 3(d)] that the resulting wavefronts of illuminating an axicon with a positive or a negative cylindrical wave can produce caustic beams. Now,

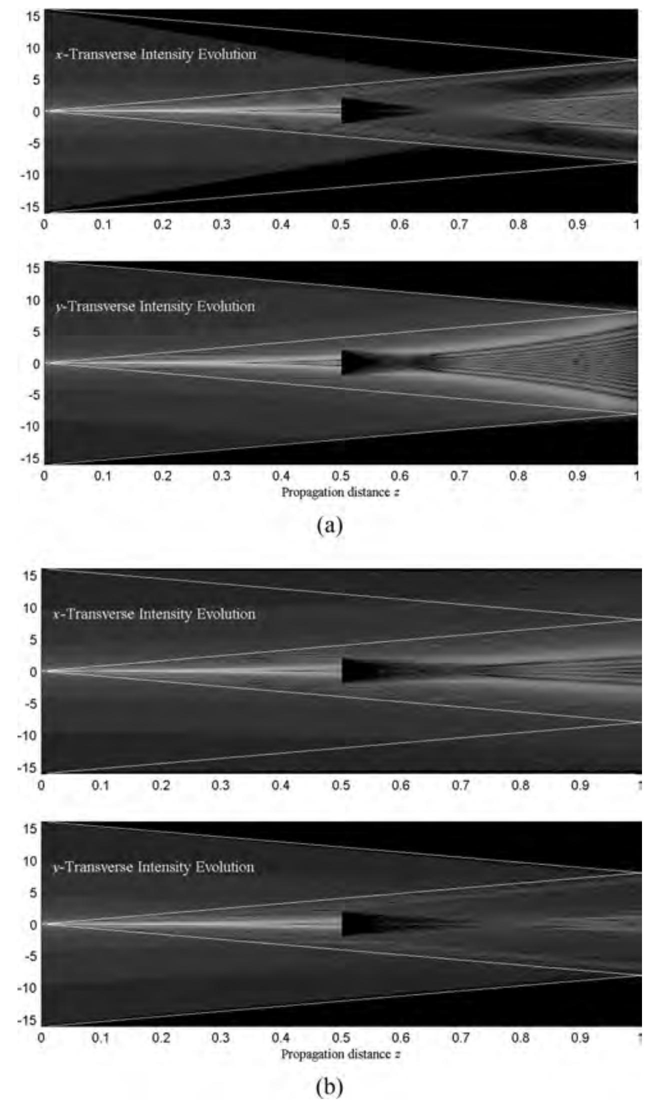

Fig. 6. Evolution of the caustic beam in the plane: (a) x–z (focusing) and (b) y–z (defocusing).

looking at the resulting wavefront obtained after illuminating the axicon with a positive cylindrical wave, Fig. 3(a), we see that it can still be considered conical. The same can be said for that obtained under negative cylindrical wave illumination if the base of the cone of the wavefront does not exceed a radius of $|x_c|$ given by Eq. (5). With this fact in mind and knowing that self-reconstruction is due to conical wavefronts, then we expect to have a similar effect. To investigate this, positive and negative caustic beams were obstructed at some point of their propagation, and then the transverse intensity patterns at a distance beyond the obstruction were studied. We notice that, in general, caustic beams produced by an axicon are not necessarily propagation invariant. A square obstruction of side $l = 4$ was placed at the distance $z = 0.5$, and the observation plane is at $z = 1.0$ in normalized units. The self-healing of caustic beams is clearly observed in Fig. 5 for the positive [Figs. 5(a) and 5(b)] and the negative [Figs. 5(c) and 5(d)] cylindrical wave illumination. In comparisons between Figs. 4(b) and 5(b) for the positive case, and Figs. 4(d) and 5(d) for the negative case, we see that the central parts of the transverse patterns are practically identical. After further propagation, the pattern is completely healed. The evolution of the caustics in the planes x–z and y–z is shown in Fig. 6. In each plot is seen the triangular (conic) shadow created by the obstruction illuminated by the conical waves. The vertex of that triangle (cone), where the process of self-healing of the beam begins, has a good agreement with the distance given by Eq. (10) of $z_{\mathrm{rec}} = 0.25$. For the positive case, by comparing the top and the bottom plots in Fig. 6(a), the focusing of the wave in the plane x–z is clearly seen in the external geometric triangles, and the light intensity is well within the geometrical region. The opposite occurs for the negative case in which the light intensity is defocused from the triangular regions; see top of Fig. 6(b).

For off-axis obstructions, caustic beams also present the self-healing property, but it might require further propagation distances (see Fig. 7). It can be easily deduced that the self-healing of the caustic beams depends on the size of the object and on the angle of the cone of wavevectors of the conical wave. In general, to avoid diffraction effects, the object has to be much smaller than the transverse extent of the base of the conical wave of the obstructed wave field. For the case studied here it was enough that the transverse extent of the obstruction was one fourth of the whole initial beam. By increasing the angle of wavevectors, the beam extent has to be increased. Notice that in propagation of optical beams, the angle of wavevectors is limited by the paraxial approximation.

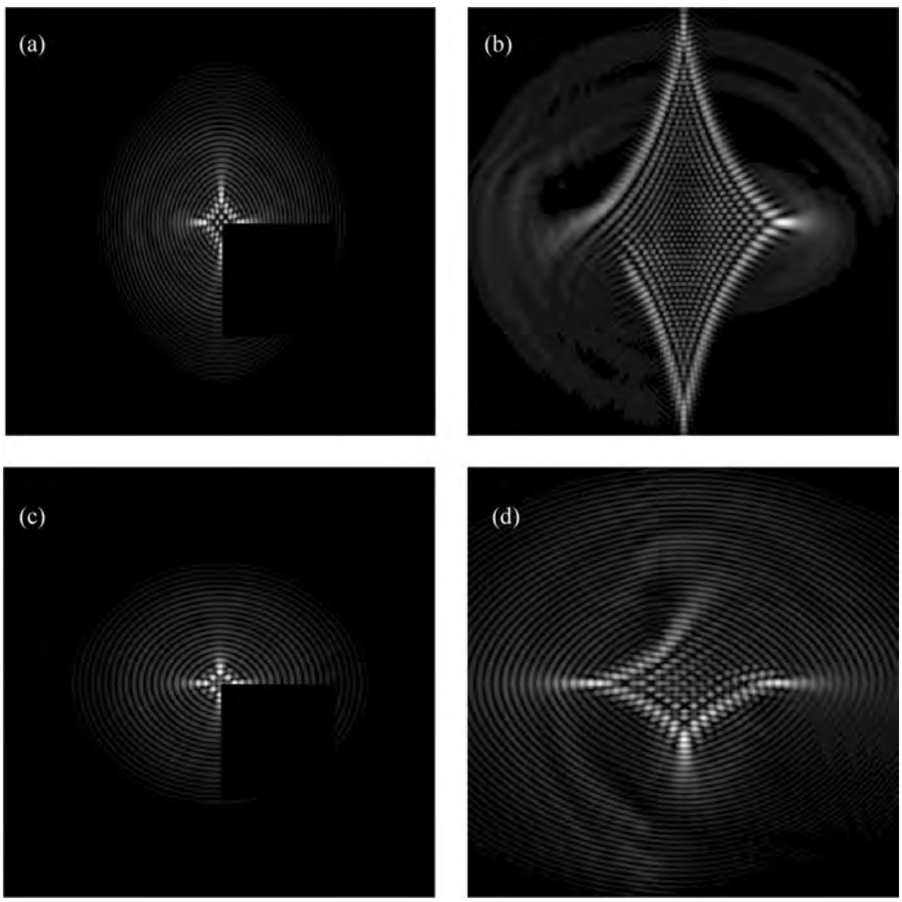

Fig. 7. Numerical simulation of a caustic beams off axis obstructed for positive cylindrical wave illumination: (a) $z = 0.5$ normalized units, (b) 1.5 normalized units; for negative cylindrical wave illumination, (c) $z = 0.5$ normalized units, (d) 1.5 normalized units.

6. Conclusions

We have reviewed the basis for the phenomenon of self-reconstruction that occurs with nondiffracting Bessel beams. Then, it was shown that caustic beams, produced by illuminating an axicon with a cylindrical wavefront, when obstructed by an opaque object, reappeared at a given plane with the same transverse intensity structure of that created by an unobstructed caustic beam. We referred to this phenomenon as self-healing. The physics of the phenomenon was fully explained, and we showed that it is related to the theory of self-reconstruction of general nondiffracting optical fields based on conical waves. In this work we demonstrated, for the first time to our knowledge, that caustic beams can present the self-healing property. Self-healing of the caustic beam occurs either when the cylindrical wavefront arriving at the axicon is a converging or a diverging cylindrical wave.

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